APPENDIX

A. FUNCTION CM VALIDITY

The following is a more in depth analysis of the *CM validity* of the role assignment functions MMDR and MMD+MSD^2 described in Section 3.

A.1 Minimizing Longest Distance

It is trivial to determine that both MMDR and MMD+MSD^2 select a mapping of agents to role positions that minimizes the time for all agents to have reached their target destinations. The total time it takes for all agents to move to their desired positions is determined by the time it takes for the last agent to reach its target position. As the first comparison between mapping costs for both role assignment functions is the maximum distance that any single agent in a mapping must travel, and it is assumed that all agents move toward their targets at the same constant rate, the property of minimizing the longest distance holds for both MMDR and MMD+MSD^2.

A.2 Avoiding Collisions

Given the assumptions that no two agents and no two role positions occupy the same position on the field, and that all agents move toward role positions along a straight line at the same constant speed, if two agents collide it means that they both started moving from positions that are the same distance away from the collision point. Furthermore if either agent were to move to the collision point, and then move to the target of the other agent, its total path distance to reach that target would be the same as the path distance of the other agent to that same target. Considering that we are working in a Euclidean space, by the triangle inequality we know that the straight path from the first agent to the second agent's target will be less than the path distance of the first agent moving to the collision point and then moving on to the second agent's target (which is equal to the distance of the second agent moving on a straight line to its target). Thus if the two colliding agents were to switch targets the maximum distance either is traveling will be reduced (along with the sum of the squared distances traveled), thereby reducing the cost of the mapping for both MMDR and MMD+MSD^2, and the collision will be avoided. Figure 1 illustrates an example of this scenario.

The following is a proof sketch related to Figure 1 that no collisions will occur.

Assumption. *Agents A1 and A2 move at constant velocity v on straight line paths to static positions P2 and P1 respectively. A1 ≠ A2 and P1 ≠ P2. Agents collide at point C at time t.*

Claim. *A1→P2 and A2→P1 is an optimal mapping returned by MMDR.*

Case 1. *P1 and P2 ≠ C.*
By assumption:
$\overline{A_1C} = \overline{A_2C} = vt$
$\overline{A_1P_2} = \overline{A_1C} + \overline{CP_2} = \overline{A_2C} + \overline{CP_2}$
$\overline{A_2P_1} = \overline{A_2C} + \overline{CP_1} = \overline{A_1C} + \overline{CP_1}$

By triangle inequality:
$\overline{A_1P_1} < \overline{A_1C} + \overline{CP_1} = \overline{A_2P_1}$
$\overline{A_2P_2} < \overline{A_2C} + \overline{CP_2} = \overline{A_1P_2}$

$\max(\overline{A_1P_1}, \overline{A_2P_2}) < \max(\overline{A_1P_2}, \overline{A_2P_1})$

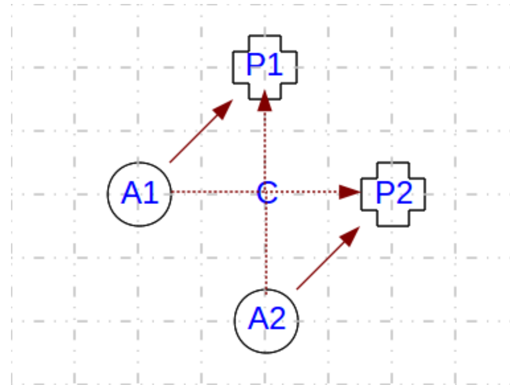

Figure 1: Example collision scenario. If the mapping (A1→P2,A2→P1) is chosen the agents will follow the dotted paths and collide at the point marked with a C. Instead both MMDR and MMD+MSD^2 will choose the mapping (A1→P1,A2→P2), as this minimizes both maximum path distance and sum of distances squared, and the agents will follow the paths denoted by the solid arrows thereby avoiding the collision.

$\overline{A_1P_1}^2 + \overline{A_2P_2}^2 < \overline{A_1P_2}^2 + \overline{A_2P_1}^2$
$\therefore \mathtt{cost}(A1 \rightarrow P1, A2 \rightarrow P2) < \mathtt{cost}(A1 \rightarrow P2, A2 \rightarrow P1)$
and claim is False.

Case 2. *P2 = C, P2 ≠ C.*
By assumption:
$\overline{CP_2} > \overline{CP_1} = 0$
$\overline{A_2C} \leq \overline{A_1C} = vt$
$\overline{A_1P_1} = \overline{A_1C} < \overline{A_1C} + \overline{CP_2} = \overline{A_1P_2}$

By triangle inequality:
if $\overline{A_1C} = \overline{A_2C}$
$\overline{A_2P_2} < \overline{A_2C} + \overline{CP_2} = \overline{A_1C} + \overline{CP_2} = \overline{A_1P_2}$
otherwise $\overline{A_2C} < \overline{A_1C}$
$\overline{A_2P_2} \leq \overline{A_2C} + \overline{CP_2} < \overline{A_1C} + \overline{CP_2} = \overline{A_1P_2}$

$\max(\overline{A_1P_1}, \overline{A_2P_2}) < \max(\overline{A_1P_2}, \overline{A_2P_1})$
$\overline{A_1P_1}^2 + \overline{A_2P_2}^2 < \overline{A_1P_2}^2 + \overline{A_2P_1}^2$
$\therefore \mathtt{cost}(A1 \rightarrow P1, A2 \rightarrow P2) < \mathtt{cost}(A1 \rightarrow P2, A2 \rightarrow P1)$
and claim is False

Case 3. *P2 = C, P1 ≠ C.*
Claim False by corollary to Case 2.

Case 4. *P1, P2 = C.*
Claim False by assumption.

As claim is False for all cases MMDR does not return mappings with collisions. □

B. DYNAMIC CONSISTENCY

Dynamic consistency is important such that as agents move toward fixed target role positions they do not continually switch or thrash between roles thus impeding their progress in reaching target positions. Given the assumption that all agents move toward target positions at the same constant rate, all distances to targets in a MMDR mapping of agents to role positions will decrease at the same constant rate as the agents move until becoming 0 when an agent reaches its destination. Considering that agents move toward their target positions on straight line paths, it is not possible for the distance between any agent and any role position to decrease faster than the distance between an agent and the role position it is assigned to move toward. This means that the cost of any MMDR mapping can not improve

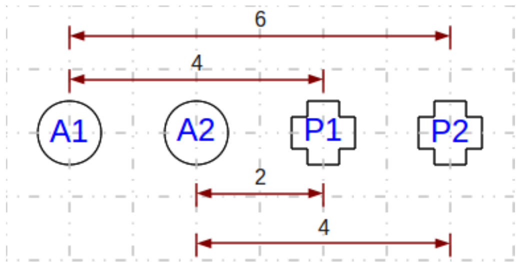

Figure 2: Example where minimizing the sum of path distances fails to hold desired properties. Both mappings of (A1→P1,A2→P2) and (A1→P2,A2→P1) have a sum of distances value of 8. The mapping (A1→P2,A2→P1) will result in a collision and has a longer maximum distance of 6 than the mapping (A1→P1,A2→P2) whose maximum distance is 4. Once a mapping is chosen and the agents start moving the sum of distances of the two mappings will remain equal which could result in thrashing between the two.

over time any faster than the lowest cost MMDR mapping being followed, and thus dynamic consistency is preserved. Note that it is possible for two mappings of agents to role positions to have the same MMDR cost as the case of two agents being equidistant to two role positions. In this case one of the mappings may be arbitrarily selected and followed by the agents. As soon as the agents start moving the selected mapping will acquire and maintain a lower cost than the unselected mapping. The only way that the mappings could continue to have the same MMDR cost would be if the two role positions occupy the same place on the field, however, as stated in the given assumptions, this is not allowed.

MMD+MSD^2 is not dynamically consistent as minimizing the sum of distances squared (MSD^2) is not dynamically consistent. (MSD^2) is shown to be not dynamically consistent in Appendix C.

C. OTHER ASSIGNMENT FUNCTIONS

Other potential ordering heuristics for mappings of agents to target positions include minimizing the sum of all distances traveled (MSD), minimizing the sum of all path distances squared (MSD^2), and assigning agents to targets in order of shortest distances (Greedy). None of these heuristics preserve both required properties listed in Section 2 for *CM validity* which are true for both MMDR and MMD+MSD^2. Also none of them are dynamically consistent.

As can be seen in the example given in Figure 2, none of the properties necessarily hold for MSD.

The first property of all agents having reached their target destinations in as little time as possible is not always true for MSD^2 as shown in the example in Figure 3. MSD^2 does avoid collisions as explained in Appendix A.2. The following is an example in which MSD^2 is not dynamically consistent:

At time $t = 0$:
$A_1 = (3, 0)$
$A_2 = (2, 999)$
$P_1 = (0, 0)$
$P_2 = (1, 0)$

$A1 \to P1, A2 \to P2$
$\overline{A_1P_1} = 3,\ \overline{A_2P_2} = \sqrt{998002};\ \overline{A_1P_1}^2 + \overline{A_2P_2}^2 = 998011$

$A1 \to P2, A2 \to P1$

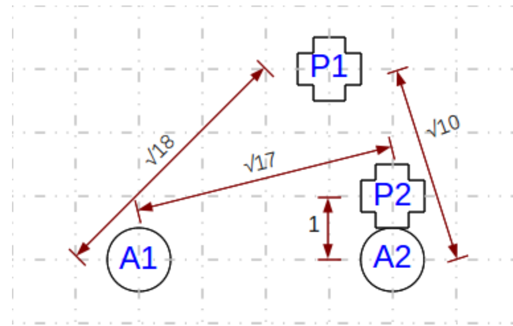

Figure 3: Example where minimizing the sum of path distances squared fails to hold desired property of minimizing the time for all agents to have reached their target destinations. The mapping (A1→P1,A2→P2) has a path distance squared sum of 19 which is less than the mapping (A1→P2,A2→P1) for which this sum is 27. Both MMDR and MMD+MSD^2 will choose the mapping with the greater sum as its maximum path distance (proportional to the time for all agents to have reached their targets) is $\sqrt{17}$ which is less than the other mapping's maximum path distance of $\sqrt{18}$.

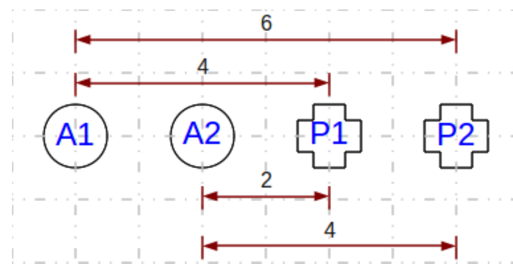

Figure 4: Example where greedily choosing shortest paths fails to hold desired properties. The shortest distance is from A2→P1 resulting in a mapping of (A2→P1,A1→P2) to be chosen. The mapping (A2→P1,A1→P2) will result in a collision and has a longer maximum distance of 6 than the mapping (A1→P1,A2→P2) whose maximum distance is 4. Once the agents collide it is possible that A1 will move on top of P1 thus pushing A2 off of P1 and towards P2. This displacement of A2 may result in a switch between mappings and potential thrashing.

$\overline{A_1P_2} = 2,\ \overline{A_2P_1} = \sqrt{998005};\ \overline{A_1P_2}^2 + \overline{A_2P_1}^2 = 998009$

MSD^2 mapping $(A1 \to P2, A2 \to P1) \because 998009 < 998011$

At time $t = 2$:
$A_1 = (1, 0)$
$A_2 = (\tilde{}2, \tilde{}997)$
$P_1 = (0, 0)$
$P_2 = (1, 0)$

$A1 \to P1, A2 \to P2$
$\overline{A_1P_1} = 1,\ \overline{A_2P_2} = \sqrt{994010};\ \overline{A_1P_1}^2 + \overline{A_2P_2}^2 = 994011$

$A1 \to P2, A2 \to P1$
$\overline{A_1P_2} = 0,\ \overline{A_2P_1} = \sqrt{994013};\ \overline{A_1P_2}^2 + \overline{A_2P_1}^2 = 994013$

MSD^2 mapping $(A1 \to P1, A2 \to P2) \because 994011 < 994013$

As the mapping switched MSD^2 is not dynamically consistent.

As can be seen in the example given in Figure 4, none of the properties necessarily hold for Greedy.