An improvement on EWD912

In my final comments on EWD912 I suggested that Theorem 5.0 would admit a shorter proof. Here is a suggestion.

Proof Th. 5.0 With f enjoying some type of conjunctivity, f is monotonic; hence – Lemma 5.8 – h is monotonic.

In order to show that h is conjunctive over some V, i.e.

$$[h.(\underline{A}X: X\in V: X) \equiv (\underline{A}X: X\in V: h.X)]$$

we show that either side implies the other.

(i) Because h is monotonic, we have

$$[h.(\underline{A}X: X\in V: X) \Rightarrow (\underline{A}X: X\in V: h.X)] \quad .$$

(ii) To show the implication in the other direction, it suffices to show in view of (11) that

$$[(\underline{A}X: X\in V: h.X) \Rightarrow f.((\underline{A}X: X\in V: X), (\underline{A}X: X\in V: h.X))] .$$

or, since quantification distributes over pair forming

(14') $$[(\underline{A}X: X\in V: h.X) \Rightarrow f.(\underline{A}X: X\in V: (X, h.X))] \quad .$$

To this end we construct a bag W of predicate pairs by

(15') $\quad (X,Y) \in W \;\equiv\; X \in V \;\wedge\; [Y \equiv h.X]$

and observe that — see Lemma to be inserted in Ch.3 — V and W are of the same junctivity type since h is monotonic. Hence, suffices to show (14') under the assumption that f is conjunctive over W .

$$
\begin{array}{ll}
 & (\underline{A}X{:}\ X \in V{:}\ h.X) \\
= & \{(10)\} \\
 & (\underline{A}X{:}\ X \in V{:}\ f.(X, h.X)) \\
= & \{\text{one-point rule}\} \\
 & (\underline{A}X{:}\ X \in V{:}\ (\underline{A}Y{:}\ [Y \equiv h.X]{:}\ f.(X,Y))) \\
= & \{(15')\} \\
 & (\underline{A}X,Y{:}\ (X,Y) \in W{:}\ f.(X,Y)) \\
= & \{f \text{ conjunctive over } W\} \\
 & f.(\underline{A}X,Y{:}\ (X,Y) \in W{:}\ (X,Y)) \\
= & \{(15')\} \\
 & f.(\underline{A}X{:}\ X \in V{:}\ (\underline{A}Y{:}\ [Y \equiv h.X]{:}\ (X,Y))) \\
= & \{\text{one-point rule}\} \\
 & f.(\underline{A}X{:}\ X \in V{:}\ (X, h.X))
\end{array}
$$

(End of Proof Th. 5.0)

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