A supplement to EWD1002

For an f with a natural argument, "the $f.i$ form a weakening sequence" means

(84) $\quad (\underline{A}\, i,j: 0 \leq i < j: [f.i \Rightarrow f.j])$.

For a weakening sequence that is non-empty -i.e. such that $i=0$ is included in the range- we have (not surprisingly)

$$[(\underline{A}\, i :: f.i) \equiv f.0] \quad .$$

Proof We observe for any non-empty weakening sequence $f.i$

$$\begin{array}{ll} & (\underline{A}\, i :: f.i) \\ = & \quad \{\text{instantiation } i:=0 \text{ ; } i=0 \text{ in range}\} \\ & f.0 \wedge (\underline{A}\, i :: f.i) \\ = & \quad \{\text{range non-empty}\} \\ & (\underline{A}\, i :: f.0 \wedge f.i) \\ = & \quad \{(84) \text{ with } i:=0, \text{ which is in range; pred. calc.: } (\underline{A}\, j :: [f.0 \wedge f.j \equiv f.0])\} \\ & (\underline{A}\, i :: f.0) \\ = & \quad \{\text{range non-empty}\} \\ & f.0 \quad . \end{array}$$

(End of Proof.)

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