$Q_1: f(x,y) = ye^{xy}$

$\frac{\partial f}{\partial x} = e^{xy} \cdot \frac{\partial f}{\partial x}[xy] \cdot y = y^2 e^{xy}$

$\frac{\partial f}{\partial y} = e^{xy} + xye^{xy}$

$f'g + g'f = \frac{\partial f}{\partial y}[ye^{xy}]$

$e^{xy} + xye^{xy} = \frac{\partial f}{\partial y}$

$e^{xy} \cdot [x] \cdot y$

$Q_2: g(x,y) = \overset{f}{y}\overbrace{(x+x^2y)^5}^{g}$

$\frac{\partial g}{\partial x} = 5y(x+x^2y)^4(1+2xy)$

$\frac{\partial g}{\partial y} = (x+x^2y)^5 + (5y(x+x^2y)^4(x^2))$

$f'g + g'f$

$(x+x^2y)^5 + (5y(x+x^2y)^4(x^2)) = \frac{\partial g}{\partial y}$

$Q_3: f(x,y) = \frac{ax+by}{cx+dy}$

$\frac{\partial f}{\partial x} = \frac{ady - cby}{(cx+dy)^2}$

$\frac{\partial f}{\partial y} = \frac{bcx - adx}{(cx+dy)^2}$

$\frac{f'g - g'f}{g^2} = \frac{(a)(cx+dy) - ((c)(ax+by))}{(cx+dy)^2}$

$= \frac{\cancel{acx} + ady - \cancel{acx} + cby}{(cx+dy)^2}$

$= \frac{ady + cby}{(cx+dy)^2}$

$\frac{f'g - g'f}{g^2} = \frac{(b)(cx+dy) - ((d)(ax+by))}{(cx+dy)^2}$

$= \frac{bcx + \cancel{bdy} - dax - \cancel{dby}}{(cx+dy)^2}$

$= \frac{bcx - dax}{(cx+dy)^2}$

$Q_4: g(u,v) = (u^2v - v^3)^5$

$\frac{\partial g}{\partial u} = 5(u^2v - v^3)^4(2uv)$

$\frac{\partial g}{\partial v} = 5(u^2v - v^3)^4(u^2 - 3v^2)$

$Q_5: R(p,q) = [\tan^{-1}(pq^2)]' = \frac{1}{1+(pq^2)^2}$

$\frac{\partial R}{\partial p} = \frac{q^2}{1+(pq^2)^2}$

$\frac{\partial R}{\partial q} = \frac{2pq}{1+(pq^2)^2}$

$Q_6: F(x,y) = \int_y^x \cos(e^t)\,dt$

$\frac{\partial F}{\partial x} = \cos(e^x)$

$\frac{\partial F}{\partial y} = -\cos(e^y)$

FTC?

Ask Martines on Monday

Q_{11}: Find $f_x(x,y)$ and $f_y(x,y)$ when

$f(x,y) = xy^2 - x^3y$

$f_x(x,y) = [f(x,y)]\frac{\partial f}{\partial x} = \boxed{y^2 - 3x^2y}$

$f_y(x,y) = [f(x,y)]\frac{\partial f}{\partial y} = \boxed{2xy - x^3}$

Verifying the conclusion of Clairaut's Theorem holds

$Q_7: u = x^4y^3 - y^4$

$[u_{yx}]\ \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}(u) = \frac{\partial u}{\partial y}\frac{\partial u}{\partial x}(u)\ [u_{xy}]$

$\frac{\partial u}{\partial x}[3x^4y^2 - 4y^3] = \frac{\partial u}{\partial y}[4x^3y^3]$

✓ $12x^3y^2 = 12x^3y^2$ ✓

$\therefore$ Clairaut's Theorem holds $\leftrightarrow u_{xy} = u_{yx}$

$Q_8: u = \cos(x^2y)$

$f'g + g'f$

$-2x\sin(x^2y) + (-x^2\cos(x^2y)(-2xy))$

$-2x\sin(x^2y) + (-2xy\cos(x^2y)\cdot x^2)$

$\frac{\partial u}{\partial y}\frac{\partial u}{\partial x}[u] = \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}[u]$

$\frac{\partial u}{\partial y}[\overset{f}{-2xy}\ \overset{g}{\sin(x^2y)}] = \frac{\partial u}{\partial x}[-x^2\sin(x^2y)]$

✓ $-2x\sin(x^2y) - 2x^3y\cos(x^2y) = -2x\sin(x^2y) - 2x^3y\cos(x^2y)$ ✓

$\therefore$ Clairaut's Theorem holds for $u_{xy} = u_{yx}$

$Q_9: w = \frac{x}{y+2z}$, $\left[\underset{a}{\frac{\partial^3 w}{\partial z\,\partial y\,\partial x}}, \underset{b}{\frac{\partial^3 w}{\partial x^2\,\partial y}}\right]$

$\frac{f'g - g'f}{(g^2)}$

(a) $\left[\frac{-((2)(x))^f}{(y+2z)^2_g}\right]\frac{\partial w}{\partial y}\frac{\partial w}{\partial x} \rightarrow \frac{-(2\cancel{(y+2z)}(1)(-2x))}{(y+2z)^{\cancel{4}\,3}}$

$\left[\frac{4x^f}{(y+2z)^3_g}\right]\frac{\partial w}{\partial x} \rightarrow \frac{(4)\cancel{(y+2z)^3}}{(y+2z)^{\cancel{6}\,3}}$

$= \boxed{\frac{4}{(y+2z)^3}}$

(b) $\left[\cancel{\frac{(y+2z)}{(y+2z)^2}} \rightarrow \frac{1}{y+2z}\right]\frac{\partial w}{\partial x}\frac{\partial w}{y} \rightarrow \boxed{\emptyset}$

$Q_{10}: u = x^a y^b z^c\ \left[\frac{\partial^6 u}{\partial x\,\partial y\,\partial z^3}\right]$

$\frac{\partial u}{\partial y}\frac{\partial u}{\partial z^3}\left[a(x^{a-1}y^b z^c)\right]$

$\downarrow$

$\frac{\partial u}{\partial z^3}\left[ab(x^{a-1}y^{b-1}z^c)\right] \rightarrow \boxed{abc(c-1)(c-2)(c-3)(x^{a-1}y^{b-1}z^{c-4})}$