

$$\lim_{b \rightarrow \infty} \frac{1}{b} \ln|b+4| - (\ln|-2+4|)$$

$\frac{\infty}{\infty} \rightarrow \frac{\infty}{\infty}$
 $\lim_{b \rightarrow \infty} \ln|2b| \rightarrow \infty$
 Divergent!

$$du = dx$$

$$\int u^{-3/2}$$

$$\int \frac{1}{\frac{3}{2}-2} u^{-\frac{3}{2}+2}$$

$$-2 u^{-1/2}$$

$$\lim_{b \rightarrow \infty} \left[-2(x-2)^{-1/2} \right]_3^b$$

$$\lim_{b \rightarrow \infty} (-2(b-2)^{1/2}) - (-2(3-2)^{1/2})$$

$$0 + 2$$

2 Convergent

Question #13: $\int_{-\infty}^0 \frac{1}{(x^2+1)^2} dx$

$$\lim_{a \rightarrow -\infty} \int_a^0 \frac{x}{(x^2+1)^2} dx$$

Let $u = x^2 + 1$
 $\frac{1}{2} du = 2x dx$
 $\frac{1}{2} du = x dx$

$$\lim_{a \rightarrow -\infty} \int_a^0 \frac{1}{2} (u^{-2}) du$$

$$\frac{1}{2} \left(\frac{1}{-1} u^{-1} \right)$$

$$-\frac{1}{4} u^{-2} \rightarrow \lim_{a \rightarrow -\infty} -\frac{1}{4} (a^2+1)^{-2}$$

$$\left(-\frac{1}{4} \frac{(a^2+1)^{-2}}{1} \right) - \lim_{a \rightarrow -\infty} -\frac{1}{4} (a^2+1)^{-2}$$

$$-\frac{1}{4} - 0 = -\frac{1}{4}$$

-1/4 convergent!

Question #17: $\int_0^{\infty} \frac{e^x}{(1+e^x)^2} dx$

$$\lim_{b \rightarrow \infty} \int_0^b \frac{e^x}{(1+e^x)^2} dx$$

Let $u = 1+e^x$
 $du = e^x dx$

$$\lim_{b \rightarrow \infty} \int_0^b \frac{1}{u^2} du$$

$$\frac{1}{-2} u^{-2+1}$$

$$-\frac{1}{2} u^{-1}$$

$$\lim_{b \rightarrow \infty} \left[-\frac{1}{2(1+e^x)} \right]_0^b$$

$$\left(-\frac{1}{2} \frac{1}{1+e^b} \right) - \left(-\frac{1}{2} \frac{1}{1+e^0} \right)$$

$$-\frac{1}{2} \cdot 0 - \left(-\frac{1}{2} \right)$$

Convergent 1/2

Question #19: $\int_{-\infty}^{\infty} x e^{-x^2} dx$

$$\int_{-\infty}^0 x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx$$

Let $u = -x^2$
 $du = -2x dx$
 $\text{Let } \frac{1}{2} du = -x dx$

$$-\frac{1}{2} \int e^u du$$

$$\lim_{a \rightarrow -\infty} -\frac{1}{2} e^{-x^2} \Big|_a^0 = \left(\lim_{a \rightarrow -\infty} \left(\frac{1}{2} e^{-a^2} \right) \right) - \left(\frac{1}{2} e^0 \right)$$

$$-\frac{1}{2} + \frac{1}{2}$$

$$\lim_{b \rightarrow \infty} -\frac{1}{2} e^{-x^2} \Big|_0^b = \left(\lim_{b \rightarrow \infty} -\frac{1}{2} e^{-b^2} \right) - \left(-\frac{1}{2} e^{-0} \right)$$

$$0 + \frac{1}{2} = \frac{1}{2}$$

$$-\frac{1}{2} + \frac{1}{2} = 0 = I$$

Question #27: $\int_{-\infty}^0 z e^{2z} dz$

Let $u = z$ $v = \frac{1}{2} e^{2z}$
 $du = dz$ $dv = e^{2z} dz$

$$\frac{1}{2} z e^{2z} - \int \frac{1}{2} e^{2z} dz$$

via u v formula: Let $w = 2z$
 $\frac{1}{2} dw = dz$

$$\frac{1}{4} \int w e^w dw$$

$$\lim_{a \rightarrow -\infty} \left[\frac{1}{4} w e^w - \frac{1}{4} e^w \right]_a^0$$

$$-\frac{1}{4} - 0 = -\frac{1}{4}$$

Convergent!

$$\frac{1}{2} z e^{2z} - \frac{1}{4} e^{2z} = 0$$

Question #4: $\int_0^9 \frac{1}{(x-1)^{3/2}} dx$

$$\lim \int_0^1 \frac{1}{(x-1)^{3/2}} + \int_1^9 \frac{1}{(x-1)^{3/2}}$$

Let $u = x-1$
 $du = dx$

$$\frac{1}{(u)^{3/2}} \rightarrow \frac{1}{2} u^{-3/2+1}$$

$$\frac{2}{3} (x-1)^{3/2} \Big|_0^1 - \frac{2}{3} (x-1)^{3/2} \Big|_1^9$$

$$0 - \left(\frac{2}{3} \right) \rightarrow -\frac{2}{3}$$

$$\frac{2}{3} (9-1)^{3/2} - \frac{2}{3} (1-1)^{3/2}$$

$$0 - 0 = 0$$

Convergent! $-\frac{2}{3} + 0 = -\frac{2}{3} + \frac{18}{3} = \frac{16}{3}$

Question #92: $\int_0^5 \frac{w}{w-2} dw$

$$\lim_{c \rightarrow 2} \left(\int_0^c \frac{w}{w-2} dw + \int_c^5 \frac{w}{w-2} dw \right)$$

From 2 to 2+
 $\text{Let } u = w-2$
 $w = u+2$
 $dw = du$

$$\int \frac{u+2}{u} du$$

$$\int 1 + \frac{2}{u} du$$

$$\int 1 + \frac{2}{u} du$$

$$u + 2 \ln|u|$$

$$\lim_{c \rightarrow 2} \left[w-2 + 2 \ln|w-2| \right]_0^5$$

-∞? Divergent? ✓

Ask about the properties of infinity!