$$\int \cos^6 y \sin^3 y \, dy$$

$u = \cos y \, dy$

$du = \underline{-\sin y \, dy}$

$$= \int \cos^6 y \, \underline{\sin^2 y} \, \sin y \, dy$$

$$= \int \cos^6 y \, (1-\cos^2 y) \, \sin y \, dy.$$

$$= \int u^6 (1-u^2)(-du)$$

$\boxed{\star \ \sin^2 y + \cos^2 y = 1 \ \star}$

↳ divide by $\sin^2 y$

$$1 + \frac{\cos^2 y}{\sin^2 y} = \frac{1}{\sin^2 y}$$

$$\rightarrow 1 + \cot^2 y = \csc^2 y$$

~~u~~-sub trig

$$= -\int u^6 - u^8 \, du$$

$$= -\left[\frac{1}{7}u^7 - \frac{1}{9}u^9\right]$$

$$= -\frac{1}{7}\cos^7 y + \frac{1}{9}\cos^9 y + C.$$

(7) $\displaystyle\int_0^{\pi/2} \cos^2\theta \, d\theta$

$$\frac{1 + \cancel{\sin}\, 2x}{2} \quad \cos$$

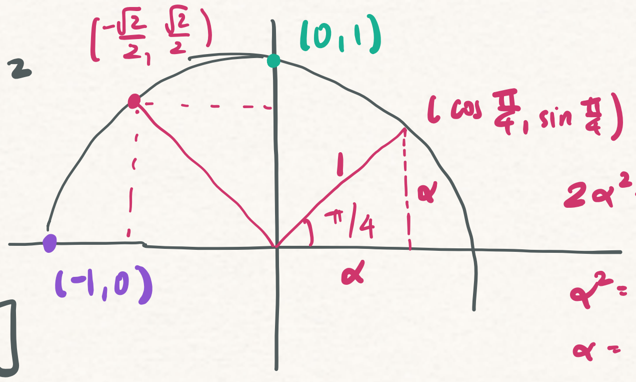
$$\star = \int_0^{\pi/2} \frac{1+\cos 2\theta}{2} \, d\theta$$

$\star \ \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2} \ \star$

$$= \frac{1}{2}\int_0^{\pi/2} 1 + \cos 2\theta \, d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} \checkmark$$




$(\cos\frac{\pi}{4}, \sin\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$2\alpha^2 = 1$

$\alpha^2 = \frac{1}{2}$

$\alpha = \sqrt{\frac{1}{2}}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

$$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - \frac{1}{2}\sin 0\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi}{4}$$

(17) $\int \cot x \cos^2 x \, dx = \int \frac{\cos x}{\sin x} \cos^2 x \, dx$ $\quad \tan x = \frac{\sin x}{\cos x}$

$$= \int \frac{\cos x}{\sin x}(1-\sin^2 x)\,dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{1-u^2}{u}\,du.$$

21. $\int \tan x \sec^3 x \, dx$ $= \int u^2 du$

$$\boxed{\begin{array}{l} u = \sec x \\ du = \sec x \tan x \, dx \end{array}}$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^3 x}\,dx$$

$u \neq \sin x$ b/c $du = \cos x \, dx$ in denom.

$$= \int \frac{-du}{u^4}$$

* $u = \cos x$ : $du = -\sin x \, dx$

(36) $\int \frac{\sin\theta + \tan\theta}{\cos^3\theta}\,d\theta = \int \frac{\sin\theta}{\cos^3\theta}\,d\theta + \int \frac{\tan\theta}{\cos^3\theta}\,d\theta$

*

$$\int \frac{-du}{u^3} + (21)$$

(35) $\int_0^{\pi/4} \frac{\sin^3 x}{\cos x}\,dx$ $\quad u = \cos x \quad du = \underbrace{-\sin x \, dx}$

$$= \int_0^{\pi/4} \frac{\sin^2 x}{\cos x} \cdot \sin x \, dx$$

$$= \int_0^{\pi/4} \frac{1-\cos^2 x}{\cos x} \sin x \, dx = \int_0^{\pi/4} \frac{1-u^2}{u} \cdot -du$$

(41) $\int \csc x \, dx = \int \frac{1}{\sin x} dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int \frac{1}{\sin x} \cdot \frac{\cos x}{\cos x} dx$

$\sin 2x = 2 \sin x \cos x$

see bottom of note. ✱

$= \int \frac{\cos x \, dx}{\frac{1}{2} \sin 2x}$

$u = \sin 2x$

$du = \frac{1}{2} \cos 2x \, dx$

---

7.3

(10) $\int \frac{x^2}{\sqrt{9-x^2}} dx$

$x = a \sin\theta$

$x = 3 \sin\theta$

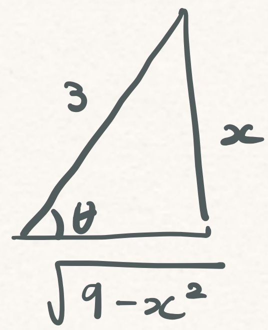
$dx = 3 \cos\theta \, d\theta$

$9 = a^2$

$3 = a$

$\frac{x}{3} = \sin\theta$

$\arcsin \frac{x}{3} = \theta$.




$= \int \frac{9 \sin^2\theta}{\sqrt{9 - 9\sin^2\theta}} \cdot 3 \cos\theta \, d\theta$

$= \int \frac{9 \quad \sin^2\theta}{3 \cdot \sqrt{1-\sin^2\theta}} \cdot 3 \cos\theta \, d\theta$

$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$= \int \frac{9 \quad \sin^2\theta}{\sqrt{\cos^2\theta}} \cos\theta \, d\theta$

$= 9 \int \sin^2\theta \, d\theta$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$= \frac{9}{2} \int 1 - \cos 2\theta \, d\theta$

$= \frac{9}{2} \left[ \theta - \frac{1}{2} \sin 2\theta \right]$ ✱ $+ C$

$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$= \frac{9}{2} \left( \arcsin\left(\frac{x}{3}\right) - \frac{1}{2} \left( 2 \sin\theta \cos\theta \right) \right)$

$$= \frac{9}{2}\left[\arcsin\left(\frac{x}{3}\right) - \left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)\right]$$

$$= \frac{9}{2}\arcsin\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} \quad + C$$

(13) $$\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$$

$x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$x = \tan\theta$

$= \frac{\sin\theta}{\cos\theta}$

$dx = \frac{+\cos^2\theta + \sin^2\theta}{\cos^2\theta}\, d\theta$

$= \frac{1}{\cos^2\theta}\, d\theta$

$= \sec^2\theta\, d\theta$

$$= \int_0^a \frac{a\sec^2\theta\, d\theta}{(a^2 + a^2\tan^2\theta)^{3/2}}$$

$$= \int \frac{a\sec^2\theta\, d\theta}{(a^2)^{3/2}(1+\tan^2\theta)^{3/2}}$$

$\sin^2 x + \cos^2 x = 1$

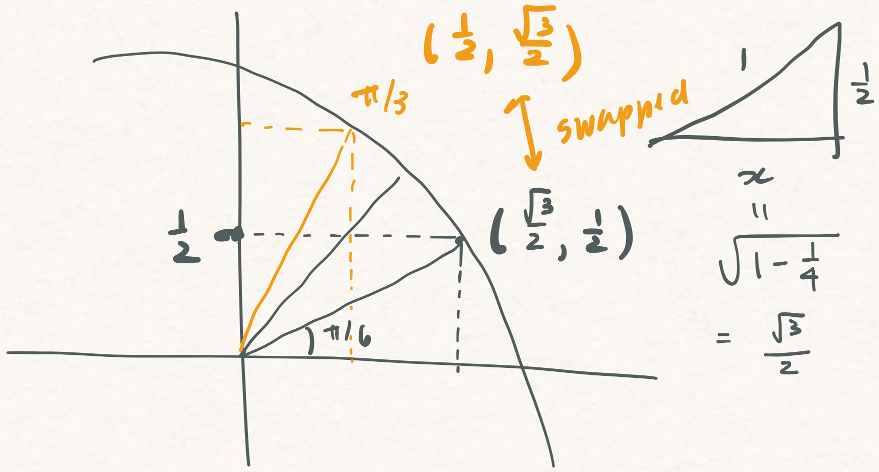
divide by $\cos^2 x$

$\tan^2 x + 1 = \sec^2 x$

$$= \int \frac{a\sec^2\theta\, d\theta}{a^3(\sec^2\theta)^{3/2}}$$

$$= \int \frac{\sec^2\theta\, d\theta}{a^2\sec^3\theta}$$

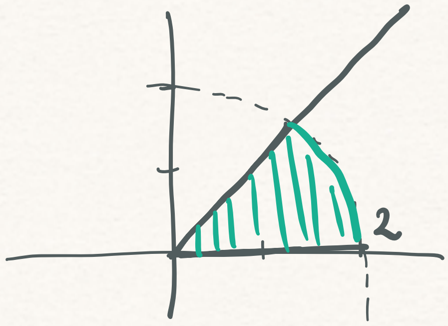
$$= \int_0^a \frac{1}{a^2}\cos\theta\, d\theta$$




$x = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

15.3

(8) $\iint_R (2x-y)\, dA$    $R$ = region in $1^{st}$ quad enclosed by



$x^2+y^2=4$

$x=0$

$y=x.$

$0 \le \theta \le \frac{\pi}{4}$

$$\int_0^{\pi/4}\int_0^{2} (2r\cos\theta - r\sin\theta)\, r\,dr\,d\theta$$

$0 \le r \le 2$

(29)

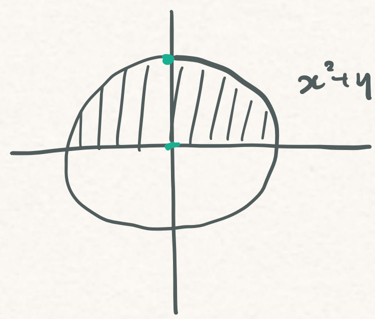
$\boxed{(30)}$

$$\int_0^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} (2x+y)\,dx\,dy.$$

$x = \sqrt{a^2-y^2}$

$x^2+y^2=a^2$ top bd

$-x = \sqrt{a^2-y^2}$





$0 \le \theta \le \pi$

$0 \le r \le a$

#

#41 revisited.

My original idea was right, but I should have multiplied by $\frac{\sin x}{\sin x}$ rather than $\frac{\cos x}{\cos x}$.

$$\int \csc x \, dx$$

$$= \int \frac{1}{\sin x} \, dx$$

$$= \int \frac{1}{\sin x} \cdot \frac{\sin x}{\sin x} \, dx$$

$$= \int \frac{\sin x \, dx}{\sin^2 x}$$

$$= \int \frac{\sin x \, dx}{1 - \cos^2 x}.$$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int \frac{-du}{1-u^2}$$

$$= \int \frac{du}{u^2-1}$$

PFD: $\frac{1}{u^2-1} = \frac{A}{u-1} + \frac{B}{u+1}$

~~for~~ $1 = A(u+1) + B(u-1)$

if $u = -1$ then $1 = -2B \Rightarrow B = -\frac{1}{2}$

$u = 1$ then $1 = 2A \Rightarrow \frac{1}{2} = A$

$$= \frac{1}{2} \int \frac{1}{u-1} - \frac{1}{u+1} \, du$$

$$= \frac{1}{2} \left[ \ln|u-1| - \ln|u+1| \right] + C$$

$$= \frac{1}{2} \left[ \ln|\cos x - 1| - \ln|\cos x + 1| \right] + c$$

$$= \frac{1}{2} \left[ \ln \left| \boxed{\frac{\cos x - 1}{\cos x + 1}} \right| \right] + c$$

$$-\int \frac{du}{1-u^2}$$

simplify this : $\dfrac{\cos x - 1}{\cos x + 1} \cdot \dfrac{\cos x - 1}{\cos x - 1}$

$$= \frac{(\cos x - 1)^2}{\cos^2 x - 1}$$

$$= \frac{(\cos x - 1)^2}{\sin^2 x}$$

$$= \frac{1}{2}\left[\ln\left|\frac{(\cos x - 1)^2}{\sin^2 x}\right|\right] + C$$

$$= \ln\left|\frac{\cos x - 1}{\sin x}\right| + C$$

$$= \ln|\cot x - \csc x| + C.$$