This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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001 10.0 points

Evaluate the integral

$$I = \int_0^2 te^{-t} dt .$$

1. $I = 1 + \frac{3}{e^3}$

2. $I = 1 + \frac{2}{e^3}$

3. $I = 1 - \frac{2}{e^2}$

4. $I = 1 - \frac{2}{e^3}$

5. $I = 1 + \frac{3}{e^2}$

6. $I = 1 - \frac{3}{e^2}$ **correct**

Explanation:

After Integration by Parts,

$$I = \Big[-te^{-t}\Big]_0^2 + \int_0^2 e^{-t} dt$$

$$= \Big[-te^{-t} - e^{-t}\Big]_0^2 .$$

Consequently,

$$\boxed{I = -2e^{-2} - e^{-2} + 1 = 1 - \frac{3}{e^2}} .$$

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002 10.0 points

Evaluate the integral

$$I = \int_0^1 6xe^{2x}\, dx.$$

1. $I = 3\Big(2e^2 + 1\Big)$

2. $I = \frac{3}{2}e^2$

3. $I = 3\Big(e^2 + 1\Big)$

4. $I = 3e^2$

5. $I = \frac{3}{2}\Big(2e^2 + 1\Big)$

6. $I = \frac{3}{2}\Big(e^2 + 1\Big)$ **correct**

Explanation:

After integration by parts,

$$I = \Big[3xe^{2x}\Big]_0^1 - 3\int_0^1 e^{2x}\, dx$$

$$= \Big[3xe^{2x} - \frac{3}{2}e^{2x}\Big]_0^1 .$$

Consequently,

$$\boxed{I = \frac{3}{2}\Big(e^2 + 1\Big)} .$$

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003 10.0 points

Evaluate the definite integral

$$I = \int_0^{\ln(3)} 5(3 - xe^x)\, dx .$$

1. $I = 4$

2. $I = 2$

3. $I = 8$

4. $I = 6$

5. $I = 10$ **correct**

Explanation:

After integration by parts,

$$\int_0^{\ln(3)} xe^x\, dx = \Big[xe^x\Big]_0^{\ln(3)} - \int_0^{\ln(3)} e^x\, dx$$

$$= \Big[xe^x - e^x\Big]_0^{\ln(3)} = (3\ln(3) - 3) + 1$$

since $e^{\ln(3)} = 3$. On the other hand,

$$\int_0^{\ln(3)} 3\,dx = 3\ln(3).$$

Thus

$$I = 5\Big(3\ln(3) - (3\ln(3) - 3) - 1\Big).$$

Consequently,

$$\boxed{I = 10}.$$

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004 10.0 points

Determine the integral

$$I = \int (6x+7)e^{2x}\,dx.$$

1. $I = (3x-5)\,e^{2x} + C$

2. $I = 2\,(3x+5)\,e^{2x} + C$

3. $I = (3x+2)\,e^{2x} + C$ **correct**

4. $I = 2\,(3x+2)\,e^{2x} + C$

5. $I = (3x+5)\,e^{2x} + C$

6. $I = (3x-2)\,e^{2x} + C$

Explanation:

After integration by parts,

$$\begin{aligned} I &= \frac{1}{2}(6x+7)e^{2x} \\ &\quad - \frac{1}{2}\int e^{2x}\frac{d}{dx}(6x+7)\,dx \\ &= \frac{1}{2}(6x+7)e^{2x} - 3\int e^{2x}\,dx \\ &= \frac{1}{2}(6x+7)e^{2x} - \frac{3}{2}e^{2x} + C \end{aligned}$$

Consequently,

$$\boxed{I = (3x+2)\,e^{2x} + C}.$$

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005 10.0 points

Evaluate the integral

$$I = \int_0^1 (7x^2-5)\,e^x\,dx.$$

1. $I = 2e + 9$

2. $I = 2(e-1)$

3. $I = 9e - 2$

4. $I = 2e - 9$ **correct**

5. $I = 9e + 2$

Explanation:

After Integration by Parts once,

$$\int_0^1 (7x^2-5)e^x\,dx$$

$$= \Big[(7x^2-5)e^x\Big]_0^1 - 14\int_0^1 xe^x\,dx.$$

To evaluate this last integral we Integrate by Parts once again. For then

$$14\int_0^1 xe^x\,dx = \Big[14xe^x\Big]_0^1 - 14\int_0^1 e^x.$$

Consequently,

$$I = \Big[(7x^2 - 5 - 14x + 14)e^x\Big]_0^1,$$

and so

$$\boxed{I = 2e - 9}.$$

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006 10.0 points

Evaluate the definite integral

$$I = \int_1^9 e^{\sqrt{t}}\,dt.$$

1. $I = 4e^3 - 2e$

2. $I = 4e^3 + 2e$

3. $I = 6e^9$

4. $I = 6e^3$

5. $I = 6e^9 + 2e$

6. $I = 4e^3$ **correct**

Explanation:

Let $w = \sqrt{t}$, so that

$$t = w^2, \quad dt = 2w\,dw\,.$$

Then

$$I = \int_1^3 2w\,e^w\,dw\,.$$

To evaluate this last integral we use now use integration by parts:

$$I = \Big[2w\,e^w\Big]_1^3 - 2\int_1^3 e^w\,dw$$
$$= 6e^3 - 2\,e - 2(e^3 - e)\,.$$

Consequently,

$$\boxed{I = 4e^3}.$$

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007 10.0 points

Determine the integral

$$I = \int e^{-4x}\cos x dx\,.$$

1. $I = \dfrac{1}{17}e^{-4x}\Big(\cos x - 4\sin x\Big) + C$

2. $I = \dfrac{1}{5}e^{-4x}\Big(\sin x - 4\cos x\Big) + C$

3. $I = -\dfrac{1}{5}e^{-4x}\Big(\cos x + 4\sin x\Big) + C$

4. $I = \dfrac{1}{17}e^{-4x}\Big(\sin x + 4\cos x\Big) + C$

5. $I = \dfrac{1}{5}e^{-4x}\Big(\cos x + 4\sin x\Big) + C$

6. $I = \dfrac{1}{17}e^{-4x}\Big(\sin x - 4\cos x\Big) + C$ **correct**

Explanation:

After integration by parts,

$$I = -\frac{1}{4}e^{-4x}\cos x + \frac{1}{4}\int e^{-4x}\frac{d}{dx}\cos x\,dx$$
$$= -\frac{1}{4}e^{-4x}\cos x - \frac{1}{4}\int e^{-4x}\sin x\,dx\,.$$

To reduce this last integral to one having the same form as I, we have to integrate by parts again since then

$$\int e^{-4x}\sin x\,dx$$
$$= -\frac{1}{4}e^{-4x}\sin x + \frac{1}{4}\int e^{-4x}\frac{d}{dx}\sin x\,dx$$
$$= -\frac{1}{4}e^{-4x}\sin x + \frac{1}{4}\int e^{-4x}\cos x\,dx$$
$$= -\frac{1}{4}\Big\{e^{-4x}\sin x - I\Big\}\,.$$

Thus

$$I = -\frac{1}{4}e^{-4x}\cos x + \frac{1}{16}\Big\{e^{-4x}\sin x - I\Big\}\,.$$

Solving for I we see that

$$\Big(1 + \frac{1}{16}\Big)I = -\frac{1}{4}e^{-4x}\cos x + \frac{1}{16}e^{-4x}\sin x\,.$$

Consequently

$$\boxed{I = \frac{1}{17}e^{-4x}\Big(\sin x - 4\cos x\Big) + C}$$

with C an arbitrary constant.

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008 10.0 points

Evaluate the integral

$$I = \int_0^{\pi} 2x\cos x\,dx\,.$$

1. $I = \pi - 4$

2. $I = 2\pi$

3. $I = -2$

4. $I = -4$ **correct**

5. $I = \pi - 2$

6. $I = 2$

Explanation:

After integration by parts we see that

$$I = \Big[2x\sin x\Big]_0^{\pi} - \int_0^{\pi} 2\sin x \frac{d}{dx}(x)\,dx$$

$$= \Big[2x\sin x\Big]_0^{\pi} - \int_0^{\pi} 2\sin x\,dx$$

$$= 2\Big[x\sin x + \cos x\Big]_0^{\pi}.$$

Consequently,

$$\boxed{I = -4}.$$

009 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} (x^2+4)\sin x\,dx.$$

1. $I = \frac{\pi}{2} - 4$

2. $I = \pi - 2$

3. $I = \pi + 4$

4. $I = \frac{\pi}{2} + 4$

5. $I = \pi + 2$ **correct**

6. $I = \frac{\pi}{2} + 2$

Explanation:

After integration by parts,

$$I = -\Big[(x^2+4)\cos x\Big]_0^{\pi/2} + \int_0^{\pi/2} \cos x\Big\{\frac{d}{dx}(x^2+4)\Big\}\,dx$$

$$= 4 + 2\int_0^{\pi/2} x\cos x\,dx.$$

To evaluate this last integral we need to integrate by parts once again. For then

$$\int_0^{\pi/2} x\cos x\,dx = \Big[x\sin x\Big]_0^{\pi/2} - \int_0^{\pi/2}\sin x\,dx = \frac{\pi}{2} + \Big[\cos x\Big]_0^{\pi/2}.$$

Consequently,

$$\boxed{I = \pi + 2}.$$

010 10.0 points

Determine the indefinite integral

$$I = \int e^{-x}\sin 3x\,dx.$$

1. $I = \frac{1}{10}e^{-x}\Big(\sin 3x + 3\cos 3x\Big) + C$

2. $I = -\frac{1}{10}e^{-x}\Big(\sin 3x + 3\cos 3x\Big) + C$ **correct**

3. $I = -\frac{1}{9}e^{x}\Big(\sin 3x - 3\cos 3x\Big) + C$

4. $I = \frac{1}{9}e^{x}\Big(\sin 3x + 3\cos 3x\Big) + C$

5. $I = -\frac{1}{9}e^{-x}\Big(\sin 3x - 3\cos 3x\Big) + C$

6. $I = \frac{1}{10}e^{x}\Big(\sin 3x - 3\cos 3x\Big) + C$

Explanation:

After integration by parts,

$$I = -e^{-x}\sin 3x + \int e^{-x}\frac{d}{dx}\sin 3x\,dx$$
$$= -e^{-x}\sin 3x + 3\int e^{-x}\cos 3x\,dx\,.$$

To reduce this last integral to one having the same form as I, we integrate by parts again for then

$$\int e^{-x}\cos 3x\,dx$$
$$= -e^{x}\cos 3x + \int e^{-x}\frac{d}{dx}\cos 3x\,dx$$
$$= -e^{-x}\cos 3x - 3\int e^{-x}\sin 3x\,dx$$
$$= -(e^{-x}\cos 3x + 3I)\,.$$

Thus

$$I = -e^{-x}\sin 3x + 3\Big\{e^{-x}\cos 3x - 3I\Big\}\,.$$

Solving for I we see that

$$\Big(1+9\Big)I = -e^{-x}\sin 3x - 3e^{-x}\cos 3x\,.$$

Consequently

$$\boxed{I = -\frac{1}{10}e^{-x}\Big(\sin 3x + 3\cos 3x\Big) + C}$$

with C an arbitrary constant.

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011 10.0 points

Evaluate the definite integral

$$I = \int_1^e 4x^3\ln(x)\,dx.$$

1. $I = \frac{1}{4}(3e^4+1)$ **correct**

2. $I = (3e^4-1)$

3. $I = (3e^4+1)$

4. $I = \frac{1}{4}(3e^4-1)$

5. $I = \frac{3}{4}e^4$

Explanation:

After integration by parts,

$$I = \Big[x^4\ln(x)\Big]_1^e - \int_1^e x^3\,dx$$
$$= e^4 - \int_1^e x^3\,dx\,,$$

since $\ln(e) = 1$ and $\ln(1) = 0$. But

$$\int_1^e x^3\,dx = \frac{1}{4}(e^4-1).$$

Consequently,

$$\boxed{I = e^4 - \frac{1}{4}(e^4-1) = \frac{1}{4}(3e^4+1)}\,.$$

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012 10.0 points

Evaluate the definite integral

$$I = \int_0^2 \sin^{-1}\Big(\frac{x}{2}\Big)\,dx\,.$$

1. $I = -1$

2. $I = \pi - 1$

3. $I = \frac{1}{2}(\pi - 2\ln(2))$

4. $I = \frac{1}{2}(\pi + 2\ln(2))$

5. $I = 2$

6. $I = \pi - 2$ **correct**

Explanation:

Let $x = 2u$; then $dx = 2\,du$ while

$$x = 0 \implies u = 0,$$
$$x = 2 \implies u = 1\,.$$

In this case,

$$I = 2\int_0^1 \sin^{-1}(u)\,du\,,$$

so after integration by parts,

$$I = 2\Big[u\,\sin^{-1}(u)\Big]_0^1 - 2\int_0^1 \frac{u}{\sqrt{1-u^2}}\,du$$

$$= 2\Big[u\,\sin^{-1}(u) + \big(1-u^2\big)^{1/2}\Big]_0^1.$$

Consequently,

$$\boxed{I = 2\left(\frac{\pi}{2}-1\right) = \pi - 2}\,.$$

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013 10.0 points

Evaluate the integral

$$I = \int_1^e 2x\,\ln(x)\,dx\,.$$

1. $I = e+1$

2. $I = e-1$

3. $I = e^2+1$

4. $I = \frac{1}{2}(e^2+1)$ **correct**

5. $I = \frac{1}{2}(e^2-1)$

6. $I = \frac{1}{2}(e-1)$

Explanation:

After integration by parts,

$$I = \Big[x^2\ln(x)\Big]_1^e - \int_1^e x^2\left(\frac{1}{x}\right)dx$$

$$= e^2\ln(x) - \int_1^e x\,dx\,.$$

Consequently,

$$\boxed{I = e^2 - \left[\frac{1}{2}x^2\right]_1^e = \frac{1}{2}(e^2+1)}\,.$$

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014 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/4} x\sec^2 x\,dx\,.$$

1. $I = \frac{1}{4}\pi - \frac{1}{2}\ln 2$ **correct**

2. $I = \frac{1}{4}\pi + \frac{1}{2}\ln 2$

3. $I = \frac{1}{2}\pi + \frac{1}{4}\ln 2$

4. $I = \frac{1}{4}\pi - \ln 2$

5. $I = \frac{1}{2}\pi - \frac{1}{4}\ln 2$

6. $I = \frac{1}{2}\pi + \ln 2$

Explanation:

Since

$$\frac{d}{dx}\tan x = \sec^2 x\,,$$

integration by parts is suggested. For then,

$$I = \Big[x\tan x\Big]_0^{\pi/4} - \int_0^{\pi/4}\tan x\,dx$$

$$= \frac{1}{4}\pi - \int_0^{\pi/4}\tan x\,dx\,.$$

On the other hand,

$$\int_0^{\pi/4}\tan x\,dx = \Big[\ln|\sec x|\Big]_0^{\pi/4} = \ln\sqrt{2}\,.$$

Consequently,

$$\boxed{I = \frac{1}{4}\pi - \ln\sqrt{2} = \frac{1}{4}\pi - \frac{1}{2}\ln 2}\,.$$

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015 10.0 points

Evaluate the integral

$$I = \int_0^1 x\,f(x)\,dx$$

when

$$f(1) = 7, \quad f'(1) = 6\,.$$

1. $I = \dfrac{5}{4} - \dfrac{1}{6}\displaystyle\int_0^1 x^3 f''(x)\,dx$

2. $I = 5 - \dfrac{1}{2}\displaystyle\int_0^1 x^2 f'(x)\,dx$

3. $I = \dfrac{5}{2} + \dfrac{1}{6}\displaystyle\int_0^1 x^3 f''(x)\,dx$ **correct**

4. $I = \dfrac{15}{4} - \dfrac{1}{2}\displaystyle\int_0^1 x^2 f''(x)\,dx$

5. $I = 5 + \dfrac{1}{2}\displaystyle\int_0^1 x^2 f'(x)\,dx$

Explanation:

After integration by parts,

$$\int_0^1 x f(x)\,dx$$

$$= \Big[\frac{1}{2}x^2 f(x)\Big]_0^1 - \frac{1}{2}\int_0^1 x^2 f'(x)\,dx$$

$$= \frac{1}{2}f(1) - \frac{1}{2}\int_0^1 x^2 f'(x)\,dx\,.$$

To evaluate this last integral we have to integrate by parts once again. But then

$$\frac{1}{2}\int_0^1 x^2 f'(x)\,dx$$

$$= \Big[\frac{1}{6}x^3 f'(x)\Big]_0^1 - \frac{1}{6}\int_0^1 x^3 f''(x)\,dx$$

$$= \frac{1}{6}f'(1) - \frac{1}{6}\int_0^1 x^3 f''(x)\,dx\,.$$

When

$$f(1) = 7, \quad f'(1) = 6,$$

therefore,

$$\boxed{I = \frac{5}{2} + \frac{1}{6}\int_0^1 x^3\, f''(x)\,dx}\,.$$