This print-out should have 9 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} 3\,\sin^3(x)\cos^2(x)\,dx\,.$$

1. $I = \frac{1}{5}$

2. $I = \frac{2}{5}$ **correct**

3. $I = \frac{8}{5}$

4. $I = \frac{6}{5}$

5. $I = \frac{4}{5}$

Explanation:

Since

$$\sin^3(x)\cos^2(x) = \sin(x)(\sin^2(x)\cos^2(x))$$

$$= \sin(x)(1-\cos^2(x))\cos^2(x)$$

$$= \sin(x)(\cos^2(x) - \cos^4(x))\,,$$

the integrand is of the form $\sin(x)f(\cos(x))$, suggesting use of the substitution $u = \cos(x)$. For then

$$du = -\sin(x)\,dx\,,$$

while

$$x = 0 \implies u = 1$$

$$x = \frac{\pi}{2} \implies u = 0\,.$$

In this case

$$I = -\int_1^0 3(u^2 - u^4)\,du\,.$$

Consequently,

$$\boxed{I = \left[-u^3 + \frac{3}{5}u^5\right]_1^0 = \frac{2}{5}}\,.$$

keywords: Stewart5e, indefinite integral, powers of sin, powers of cos, trig substitution,

002 10.0 points

Determine the indefinite integral

$$I = \int \sin^2 x \cos^3 x\,dx\,.$$

1. $I = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$ **correct**

2. $I = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

3. $I = \frac{1}{5}\cos^3 x + \frac{1}{3}\sin^5 x + C$

4. $I = -\frac{1}{5}\sin^3 x - \frac{1}{3}\cos^5 x + C$

5. $I = \frac{1}{5}\cos^3 x - \frac{1}{3}\sin^5 x + C$

6. $I = \frac{1}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

Explanation:

Since

$$\sin^2 x \cos^3 x = \sin^2 x \cos^2 x \,\cos x$$

$$= \sin^2 x\,(1-\sin^2 x)\cos x\,,$$

we see that I can be written as the sum

$$I = \int \sin^2 x\,(1-\sin^2 x)\cos x\,dx$$

$$= \int \sin^2 x \cos x\,dx$$

$$- \int \sin^4 x \cos x\,dx\,,$$

of two integrals, both of which can be evaluated using the substitution $u = \sin x$. For then

$$du = \cos x\,dx\,,$$

in which case

$$I = \int u^2\,du - \int u^4\,du$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C\,.$$

Consequently,

$$\boxed{I = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}\,.$$

003 10.0 points

Evaluate the indefinite integral

$$I = \int 8\cos^4 2t\,dt\,.$$

1. $I = 3t - \cos 4t + \frac{1}{8}\cos 8t + C$

2. $I = 3t + \cos 4t + \frac{1}{8}\cos 8t + C$

3. $I = 3t + \sin 4t - \frac{1}{8}\sin 8t + C$

4. $I = 3t + \sin 4t + \frac{1}{8}\sin 8t + C$ **correct**

5. $I = 3t - \sin 4t + \frac{1}{8}\sin 8t + C$

6. $I = 3t + \cos 4t - \frac{1}{8}\cos 8t + C$

Explanation:

Since

$$\cos^2\theta = \frac{1}{2}\Big(1 + \cos 2\theta\Big)\,,$$

the integrand can be rewritten as

$$8\cos^4 2t = 2\Big(1 + \cos 4t\Big)^2$$

$$= 2\Big(1 + 2\cos 4t + \cos^2 4t\Big)\,.$$

But in turn, this last expression can be rewritten as

$$2\Big(1 + 2\cos 4t + \frac{1}{2}\Big\{1 + \cos 8t\Big\}\Big)\,.$$

Thus

$$8\cos^4 2t = 2\Big(\frac{3}{2} + 2\cos 4t + \frac{1}{2}\cos 8t\Big)\,,$$

and so

$$I = 2\int\Big(\frac{3}{2} + 2\cos 4t + \frac{1}{2}\cos 8t\Big)\,dt\,.$$

Consequently,

$$\boxed{I = 3t + \sin 4t + \frac{1}{8}\sin 8t + C}$$

with C an arbitrary constant.

004 10.0 points

Determine the integral

$$I = \int (3\sin(\theta) - 2\sin^3(\theta))\,d\theta\,.$$

1. $I = \cos(\theta) - \frac{2}{3}\cos^3(\theta) + C$

2. $I = \cos(\theta) + \frac{2}{3}\sin^3(\theta) + C$

3. $I = -\cos(\theta) - \frac{2}{3}\cos^3(\theta) + C$ **correct**

4. $I = -\cos(\theta) + \frac{2}{3}\cos^3(\theta) + C$

5. $I = \cos(\theta) - \frac{2}{3}\sin^3(\theta) + C$

6. $I = -\cos(\theta) + \frac{2}{3}\sin^3(\theta) + C$

Explanation:

After simplification, we see that

$$3\sin(\theta) - 2\sin^3(\theta) = \sin(\theta)(3 - 2\sin^2(\theta))\,.$$

On the other hand,

$$\sin^2(\theta) = 1 - \cos^2(\theta)\,.$$

Thus the integrand can be rewritten as

$$\sin(\theta)(3 - 2(1 - \cos^2(\theta)))$$

$$= \sin(\theta)(1 + 2\cos^2(\theta))\,.$$

As this is now of the form $\sin(\theta)\, f(\cos(\theta))$, the subsitution $x = \cos(\theta)$ is suggested. For then

$$dx = -\sin(\theta)\, d\theta\,,$$

in which case

$$I = -\int (1 + 2x^2)\, dx = -\Big(x + \frac{2}{3}x^3\Big) + C\,.$$

Consequently,

$$\boxed{I = -\cos(\theta) - \frac{2}{3}\cos^3(\theta) + C}$$

with C an arbitrary constant.

keywords: trig identity, trig function, integral

005 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/2} (4\cos^2(x) + \sin^2(x))\, dx$$

1. $I = \frac{5}{4}\pi$ **correct**

2. $I = 5$

3. $I = 5\pi$

4. $I = \frac{5}{2}$

5. $I = \frac{5}{4}$

6. $I = \frac{5}{2}\pi$

Explanation:

Since

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x)),\ \ \sin^2(x) = \frac{1}{2}(1-\cos(2x))\,,$$

we see that

$$4\cos^2(x) + \sin^2(x) = \frac{1}{2}\Big(5 + 3\cos(2x)\Big)\,.$$

Thus

$$I = \frac{1}{2}\int_0^{\pi} \Big(5 + 3\cos(2x)\Big)\, dx$$

$$= \frac{1}{2}\Big[\, 5x + \frac{3}{2}\sin(2x)\,\Big]_0^{\pi/2}\,.$$

Consequently,

$$\boxed{I = \frac{5}{4}\pi}\,.$$

006 10.0 points

Find the value of the integral

$$I = \int_0^{\frac{\pi}{4}} \sec^2 x(3 - 2\tan x)\, dx\,.$$

Enter your answer as a decimal with four significant digits.

Correct answer: 2.

Explanation:

Set $u = \tan x$. Then

$$\frac{du}{dx} = \sec^2 x\,,$$

while

$$x = 0 \implies u = 0\,,$$

$$x = \frac{\pi}{4} \implies u = 1\,.$$

In this case

$$I = \int_0^1 (3 - 2u)\, du = \Big[\, 3u - u^2 \Big]_0^1\,.$$

Consequently,

$$\boxed{I = 2}\,.$$

keywords: substitution, trig substitution

007 10.0 points

Find the value of the definite integral

$$I = \int_0^{\pi/4} \Big(8\sec^4(x) - 5\sec^2(x)\Big)\tan(x)\, dx\,.$$

1. $I = \dfrac{7}{2}$ **correct**

2. $I = \dfrac{11}{2}$

3. $I = 5$

4. $I = 4$

5. $I = \dfrac{9}{2}$

Explanation:
Since

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x),$$

the substitution $u = \sec(x)$ is suggested. For then

$$du = \sec(x)\tan(x)\,dx,$$

while

$$x = 0 \implies u = 1,$$

$$x = \frac{\pi}{4} \implies u = \sqrt{2}\,.$$

In this case

$$I = \int_0^{\pi/4}\Big(8\sec^3(x) - 5\sec(x)\Big)\sec(x)\tan(x)\,dx$$

$$= \int_1^{\sqrt{2}}(8u^3 - 5u)\,du$$

$$= \Big[2u^4 - \frac{5}{2}u^2\Big]_1^{\sqrt{2}}.$$

Consequently,

$$\boxed{I = \frac{7}{2}}.$$

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008 10.0 points

Evaluate the integral

$$I = \int_0^{\pi/3}\frac{\sec(x)\tan(x)}{5 + 2\sec(x)}\,dx\,.$$

1. $I = -2\ln\left(\dfrac{7}{5}\right)$

2. $I = \dfrac{1}{2}\ln\left(\dfrac{7}{5}\right)$

3. $I = -2\ln\left(\dfrac{9}{10}\right)$

4. $I = \dfrac{1}{2}\ln\left(\dfrac{9}{7}\right)$ **correct**

5. $I = -\dfrac{1}{2}\ln\left(\dfrac{9}{7}\right)$

6. $I = 2\ln\left(\dfrac{9}{10}\right)$

Explanation:
Since

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)\,,$$

use of the substitution

$$u = 5 + 2\sec(x)$$

is suggested. For then

$$du = 2\,\sec(x)\tan(x)dx\,,$$

while

$$x = 0 \implies u = 7\,,$$

$$x = \frac{\pi}{3} \implies u = 9\,.$$

Thus

$$I = \frac{1}{2}\int_7^9 \frac{1}{u}\,du = \frac{1}{2}\Big[\ln(u)\Big]_7^9.$$

Consequently,

$$\boxed{I = \frac{1}{2}\ln\left(\frac{9}{7}\right)}.$$

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009 10.0 points

Find the value of

$$I = \int_0^{\pi/4} 4\tan^4 x\,dx\,.$$

1. $I = \frac{1}{3}(3\pi - 8)$ **correct**

2. $I = \frac{1}{3}(3\pi - 4)$

3. $I = \frac{1}{2}(3\pi - 2)$

4. $I = \frac{1}{2}(3\pi - 8)$

5. $I = \frac{1}{2}(3\pi - 4)$

6. $I = \frac{1}{3}(3\pi - 2)$

Explanation:

Since

$$\tan^2 x = \sec^2 x - 1,$$

we see that

$$\begin{aligned}\tan^4 x &= \tan^2 x(\sec^2 x - 1)\\ &= \tan^2 x \sec^2 x - \tan^2 x\\ &= \tan^2 x \sec^2 x - (\sec^2 x - 1).\end{aligned}$$

Thus

$$\tan^4 x = (\tan^2 x - 1)\sec^2 x + 1.$$

In this case,

$$I = 4\int_0^{\pi/4} (\tan^2 x - 1)\sec^2 x\, dx + \int_0^{\pi/4} 4\, dx.$$

To evaluate the first of these integrals, set $u = \tan x$. Then

$$4\int_0^1 (u^2 - 1)\, du = 4\Big[\frac{1}{3}u^3 - u\Big]_0^1 = -\frac{8}{3}.$$

On the other hand

$$\int_0^{\pi/4} 4\, dx = \Big[4x\Big]_0^{\pi/4} = \pi.$$

Consequently,

$$\boxed{I = \frac{1}{3}(3\pi - 8)}.$$