This print-out should have 13 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Evaluate the integral

$$I = \int_{\pi/6}^{\pi/3} (6\sin 2x + 2\cos 2x)\, dx\,.$$

1. $I = \frac{7}{2}\sqrt{3}$

2. $I = 3\sqrt{3}$

3. $I = 6$

4. $I = 5$

5. $I = 3$ **correct**

6. $I = \sqrt{3}$

Explanation:

To reduce the integral to one involving just $\sin u$ and $\cos u$, set $u = 2x$.

Then $du = 2\,dx$, so

$$I = \frac{1}{2}\int_{\pi/3}^{2\pi/3} (6\sin u + 2\cos u)\, du$$

$$= \frac{1}{2}\Big[-6\cos u + 2\sin u\Big]_{\pi/3}^{2\pi/3}.$$

But

$$\cos\frac{2\pi}{3} = -\frac{1}{2}, \quad \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2},$$

while

$$\cos\frac{\pi}{3} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Consequently,

$$\boxed{I = 3}\,.$$

002 10.0 points

Evaluate the integral

$$I = \int_0^1 5x\,(1-x^2)^4\, dx\,.$$

1. $I = \frac{5}{4}$

2. $I = -1$

3. $I = \frac{1}{2}$ **correct**

4. $I = -\frac{5}{4}$

5. $I = \frac{5}{8}$

6. $I = -\frac{5}{8}$

Explanation:

Set $u = 1-x^2$. Then $du = -2x\,dx$ while

$$x = 0 \implies u = 1\,,$$

$$x = 1 \implies u = 0\,.$$

In this case,

$$I = -\frac{5}{2}\int_1^0 u^4\, du = \frac{5}{2}\int_0^1 u^4\, du\,.$$

Consequently,

$$\boxed{I = \frac{5}{2}\Big[\frac{1}{5}u^5\Big]_0^1 = \frac{1}{2}}\,.$$

003 10.0 points

Determine the integral

$$I = \int 4\,x\,(3+2x^2)^4\, dx.$$

1. $I = (3+2x^2)^5 + C$

2. $I = \frac{1}{5}(3+2x^2)^5 + C$ **correct**

3. $I = -\frac{1}{5}(3+2x^2)^5 + C$

4. $I = -(3+2x^2)^5 + C$

5. $I = \frac{1}{4}(3+2x^2)^4 + C$

Explanation:
Set $u = 3 + 2\,x^2$. Then

$$du = 4x\,dx,$$

in which case

$$I = \int u^4\,du = \frac{1}{5}u^5 + C\,.$$

Thus

$$\boxed{I = \frac{1}{5}(3+2x^2)^5 + C}$$

with C an arbitrary constant.

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004 10.0 points

The graph of f has slope

$$\frac{df}{dx} = x\sqrt{2x^2+1}$$

and passes through the point (2, 2). Find the y-intercept of this graph.

1. y-intercept $= -\frac{5}{3}$

2. y-intercept $= -2$

3. y-intercept $= -\frac{8}{3}$

4. y-intercept $= -\frac{4}{3}$

5. y-intercept $= -\frac{7}{3}$ **correct**

Explanation:
The function f satisfies the equations

$$f(x) = \int x\sqrt{2x^2+1}\,dx, \quad f(2) = 2.$$

To evaluate the integral set $u = 2x^2 + 1$. For then $du = 4x\,dx$, in which case

$$f(x) = \frac{1}{4}\int u^{1/2}\,du = \frac{1}{6}u^{3/2} + C$$
$$= \frac{1}{6}(2x^2+1)^{3/2} + C\,,$$

where C has to be chosen so that

$$f(2) = 2\,, \quad i.e., \quad C + \frac{9}{2} = 2\,.$$

Thus

$$f(x) = \frac{1}{6}\Big((2x^2+1)^{3/2} - 27\Big) + 2.$$

Consequently, the graph has

$$\boxed{y\text{-intercept} = -\frac{7}{3}}.$$

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005 10.0 points

Evaluate the integral

$$I = \int x^2\sqrt{x^3+7}\,dx\,.$$

1. $I = \frac{1}{9}\left(x^3+7\right)^{3/2} + C$

2. $I = 3\left(x^3+7\right)^{1/2} + C$

3. $I = 3\left(x^3+7\right)^{3/2} + C$

4. $I = \frac{1}{9}\left(x^3+7\right)^{1/2} + C$

5. $I = \frac{2}{9}\left(x^3+7\right)^{1/2} + C$

6. $I = \frac{2}{9}\left(x^3+7\right)^{3/2} + C$ **correct**

Explanation:
Set $u = x^3 + 7$. Then

$$du = 3x^2\,dx\,,$$

in which case

$$I = \frac{1}{3}\int \sqrt{u}\,du = \frac{2}{9}u^{3/2} + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = \frac{2}{9}\left(x^3+7\right)^{3/2} + C}.$$

006 10.0 points

Determine the integral

$$I = \int \frac{2}{(1+4x)^3}\,dx\,.$$

1. $I = \dfrac{1}{8(1+4x)^4} + C$
2. $I = -\dfrac{1}{8(1+4x)^4} + C$
3. $I = \dfrac{1}{4(1+4x)^2} + C$
4. $I = -\dfrac{1}{4(1+4x)^2} + C$ **correct**
5. $I = -\dfrac{1}{8(1+4x)^2} + C$
6. $I = \dfrac{1}{4(1+4x)^4} + C$

Explanation:

Set $u = 1+4x$. Then

$$du = 4\,dx\,,$$

in which case

$$I = \frac{1}{2}\int u^{-3}\,du = -\frac{1}{4}u^{-2} + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = -\frac{1}{4(1+4x)^2} + C}.$$

007 10.0 points

Evaluate the definite integral

$$I = \int_1^5 \frac{2x-7}{\sqrt{7x-x^2}}\,dx\,.$$

Correct answer: -1.42558.

Explanation:

Set $u = 7x - x^2$. Then

$$du = (7-2x)\,dx\,,$$

while

$$x = 1 \implies u = 6\,,$$

$$x = 5 \implies u = 10\,.$$

In this case,

$$I = -\int_6^{10} \frac{1}{\sqrt{u}}\,du = -\Big[2\sqrt{u}\Big]_6^{10}\,.$$

Consequently,

$$\boxed{I = -2(\sqrt{10}-\sqrt{6}) = -1.42558}.$$

008 10.0 points

Determine the integral

$$I = \int t^2\cos\left(3-t^3\right)\,dt\,.$$

1. $I = 3\cos(3-t^3) + C$
2. $I = -\dfrac{1}{3}\sin(3-t^3) + C$ **correct**
3. $I = \cos(3-t^3) + C$
4. $I = -\sin(3-t^3) + C$
5. $I = -3\cos(3-t^3) + C$
6. $I = \dfrac{1}{3}\sin(3-t^3) + C$

Explanation:

Set $u = 3 - t^3$, Then

$$du = -3t^2\,dt\,,$$

in which case

$$I = -\frac{1}{3}\int \cos u\,du = -\frac{1}{3}\sin u + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = -\frac{1}{3}\sin(3-t^3)+C}.$$

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009 10.0 points

Determine the integral

$$I = \int \cos^5 x\, \sin x\, dx\,.$$

1. $I = \frac{1}{4}\sin^4 x + C$

2. $I = -\frac{1}{5}\cos^5 x + C$

3. $I = -\frac{1}{6}\cos^6 x + C$ **correct**

4. $I = \frac{1}{5}\sin^5 x + C$

5. $I = -\frac{1}{4}\cos^4 x + C$

6. $I = \frac{1}{6}\sin^6 x + C$

Explanation:

Set $u = \cos x$. Then

$$du = -\sin\, dx\,,$$

in which case

$$I = -\int u^5\, du = -\frac{1}{6}u^6 + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = -\frac{1}{6}\cos^6 x + C}.$$

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010 10.0 points

Determine the integral

$$I = \int \frac{x-4}{(x^2-8x-6)^4}\, dx\,.$$

1. $I = -\frac{1}{3}\Big(\frac{1}{x^2-8x-6}\Big)^3 + C$

2. $I = \frac{1}{3}\Big(\frac{1}{x^2-8x-6}\Big)^3 + C$

3. $I = -\frac{1}{8}\Big(\frac{1}{x^2-8x-6}\Big)^3 + C$

4. $I = -\frac{1}{6}\Big(\frac{1}{x^2-8x-6}\Big)^3 + C$ **correct**

5. $I = \frac{1}{6}\Big(\frac{1}{x^2-8x-6}\Big)^3 + C$

Explanation:

Set $u = x^2 - 8x - 6$. Then

$$du = 2(x-4)\, dx,$$

so

$$\int \frac{x-4}{(x^2-8x-6)^4}\, dx = \int \frac{1}{2u^4}\, du = -\frac{1}{6u^3} + C.$$

Thus

$$\int \frac{x-4}{(x^2-8x-6)^4}\, dx = \boxed{-\frac{1}{6(x^2-8x-6)^3} + C}$$

with C an arbitrary constant.

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011 10.0 points

Determine the integral

$$I = \int \frac{1}{\theta^2}\left(5\cos\Big(\frac{1}{\theta}\Big) - \frac{2}{\theta}\right) d\theta$$

1. $I = -\frac{1}{\theta^2} - 5\cos\Big(\frac{1}{\theta}\Big) + C$

2. $I = \frac{1}{\theta^2} + 5\sin\Big(\frac{1}{\theta}\Big) + C$

3. $I = \frac{1}{\theta^2} - 5\cos\Big(\frac{1}{\theta}\Big) + C$

4. $I = -\frac{1}{\theta^2} - 5\sin\Big(\frac{1}{\theta}\Big) + C$

5. $I = \dfrac{1}{\theta^2} - 5\sin\Big(\dfrac{1}{\theta}\Big) + C$ **correct**

6. $I = \dfrac{1}{\theta^2} + 5\cos\Big(\dfrac{1}{\theta}\Big) + C$

Explanation:
Set $u = 1/\theta$. Then

$$du = -\frac{1}{\theta^2}\,d\theta\,,$$

so

$$I = -\int (5\cos(u) - 2u)\,du$$
$$= -5\sin(u) + u^2 + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = \frac{1}{\theta^2} - 5\sin\Big(\frac{1}{\theta}\Big) + C}\,.$$

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012 10.0 points

Evaluate the integral

$$I = \int 3\sec^6 x\,\tan x\,dx\,.$$

1. $I = \dfrac{3}{7}\sec^7 x + C$

2. $I = \dfrac{1}{2}\csc^6 x + C$

3. $I = \dfrac{1}{2}\sec^6 x + C$ **correct**

4. $I = \dfrac{3}{5}\sec^5 x + C$

5. $I = \dfrac{3}{5}\csc^5 x + C$

6. $I = \dfrac{3}{7}\csc^7 x + C$

Explanation:
Set $u = \sec x$. Then

$$du = \sec x\tan x\,dx\,,$$

in which case

$$I = 3\int u^5\,du = \frac{1}{2}u^6 + C$$

with C an arbitrary constant. Consequently,

$$\boxed{I = \frac{1}{2}\sec^6 x + C}\,.$$

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013 10.0 points

Find the value of the integral

$$I = \int_0^{\pi/4} \frac{\tan x - 3}{\cos^2 x}\,dx\,.$$

1. $I = -3$

2. $I = -4$

3. $I = -\dfrac{9}{2}$

4. $I = -\dfrac{7}{2}$

5. $I = -\dfrac{5}{2}$ **correct**

Explanation:
Since

$$\frac{1}{\cos^2 x} = \sec^2 x, \quad \frac{d}{dx}\tan x = \sec^2 x\,,$$

set $u = \tan x$. Then

$$du = \sec^2 x\,dx\,,$$

while

$$x = 0 \implies u = 0\,,$$
$$x = \frac{\pi}{4} \implies u = 1\,.$$

In this case

$$I = \int_0^1 (u-3)\,dx = \Big[\frac{1}{2}u^2 - 3u\Big]_0^1.$$

Consequently,

$$\boxed{I = -\frac{5}{2}}\,.$$

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keywords: IntSubst,