

Q1: Find a power series representation for the function

$$f(z) = \frac{1}{z-4}$$

$$P_S = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\frac{1}{-4+2}$$

$$Z_s = \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z}{4}\right)^n$$

$$\boxed{-\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{n+1} z^n} \quad \checkmark \quad \frac{1}{-4\left(1-\frac{z}{4}\right)}$$

Q2: Find a power series representation for the function

$$f(x) = \frac{1}{6+x}$$

$$P_S = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\frac{1}{6(1-\left(\frac{-x}{6}\right))}$$

$$X_S = \frac{1}{6} \sum_{n=0}^{\infty} \left(-\frac{x}{6}\right)^n$$

$$= \frac{1}{6} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{6}\right)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{6}\right)^{n+1} x^n \xrightarrow{\text{Simp.}} \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{6^{n+1}} x^n}$$

Q3: Find a power series representation for the function

$$f(x) = \frac{1}{6-x^3}$$

$$\frac{1}{6(1-\frac{x^3}{6})}$$

$$X_S = \frac{1}{6} \sum_{n=0}^{\infty} \left(\frac{x^3}{6}\right)^n$$

$$= \frac{1}{6} \sum_{n=0}^{\infty} x^{3n} \left(\frac{1}{6}\right)^n$$

$$\boxed{\sum_{n=0}^{\infty} \frac{x^{3n}}{6^{n+1}}}$$

Q4: Find a power series representation for

$$\frac{4+3x}{1+x}$$

$$\frac{4}{1+x} + \frac{3x}{1+x}$$

$$4 \cdot \frac{1}{1-(-x)} + 3x \frac{1}{1-(-x)}$$

$$4 \sum_{n=0}^{\infty} (-x)^n + 3x \sum_{n=0}^{\infty} (-x)^n$$

$$4 \sum_{n=0}^{\infty} (-1)^n x^n + 3x \sum_{n=0}^{\infty} (-1)^n x^n$$

$$+ 3 \sum_{n=0}^{\infty} (-1)^n x^{n+1}$$

Referenced ChatGPT for this question

Q5: Evaluate the integral

$$f(t) = \int_0^+ \frac{s}{1-s^4} \rightarrow \int_0^+ s \frac{1}{1-s^4}$$

$$\left[ \sum_{n=0}^{\infty} \frac{1}{4n+2} s^{4n+2} \right]_0^+$$

$$\boxed{\sum_{n=0}^{\infty} \frac{4n+2}{4n+2}}$$

$$\int_0^+ s \sum_{n=0}^{\infty} (s^4)^n$$

$$\int_0^+ \sum_{n=0}^{\infty} s^{4n+1}$$