This print-out should have 15 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+8}}{\sqrt[4]{n}}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent **correct**

2. divergent

3. absolutely convergent

Explanation:

By the Alternating Series test, the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+8}}{\sqrt[4]{n}}$$

converges. On the other hand, by the p-series test with $p = \frac{1}{4} \le 1$, the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$$

is divergent. Consequently, the series is

$$\boxed{\text{conditionally convergent}}\,.$$

002 10.0 points

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{2n^2+5}$$

have?

1. conditionally convergent **correct**

2. divergent

3. absolutely convergent

Explanation:

The given series is an alternating series

$$\sum_{n=1}^{\infty} (-1)^n f(n)$$

where f is the function defined on $(0, \infty)$ by

$$f(x) = \frac{x}{2x^2+5} > 0\,.$$

Now by the Quotient Rule,

$$f'(x) = \frac{2x^2+5-4x^2}{(2x^2+5)^2}$$

$$= \frac{5-2x^2}{(2x^2+5)^2} < 0$$

for all large x $\left(\text{for } x^2 > \frac{5}{2}, \text{ in fact}\right)$; in particular, f is decreasing for all large x, so

$$f(n) > f(n+1)$$

for all large n. In addition, $f(x) \to 0$ as $n \to \infty$. The Alternating series test thus applies and says that the series $\sum_{n=1}^{\infty} (-1)^n f(n)$ converges, so the given series converges.

On the other hand,

$$f(x) \ge \frac{1}{2x}, \quad x \ge 1\,.$$

But then by the Comparison test and the p-series test, the series $\sum_{n=1}^{\infty} f(n)$ diverges, so the given series is not absolutely convergent.

Consequently, the given series is

$$\boxed{\text{conditionally convergent}}\,.$$

003 10.0 points

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$$

have?

1. conditionally convergent **correct**

2. absolutely convergent

3. divergent

Explanation:

This series is not absolutely convergent as shown:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$$

=

$$|\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}|$$

=

$$\sum_{n=1}^{\infty} \frac{(1)^n}{3n+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{3n+1}$$

where p ≤ 1, this series is divergent.

However,

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$$

is convergent, thus the series as a whole is conditionally convergent.

---

004 10.0 points

Determine whether the series

$$\sum_{m=1}^{\infty} (-1)^{m-1} \frac{3}{\sqrt{1+m^2}}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent **correct**

2. absolutely convergent

3. divergent

Explanation:

The given series has the form of an alternating series

$$\sum_{m=1}^{\infty} (-1)^{m-1} a_m\,, \quad a_m = \frac{3}{\sqrt{1+m^2}}\,.$$

To check for absolute convergence we apply the Limit Comparison Test with

$$a_m = \frac{3}{\sqrt{1+m^2}}, \qquad b_m = \frac{1}{m}\,.$$

For then

$$\lim_{m\to\infty} \frac{a_m}{b_m} = \lim_{m\to\infty} \frac{3m}{\sqrt{1+m^2}} = 3 > 0.$$

Thus the series $\sum a_m$ converges if and only if the series

$$\sum_{m=1}^{\infty} \frac{1}{m}$$

converges. But, by the p-series test with $p = 1$, this last series diverges; in particular, the given series does not converge absolutely.

To check if the given series converges conditionally, consider first the function

$$f(x) = \frac{3}{\sqrt{1+x^2}}\,.$$

Then, by the Chain Rule,

$$f'(x) = -\frac{3x}{(1+x^2)^{3/2}} < 0$$

for all $x > 0$, while

$$\lim_{x\to\infty} f(x) = 0\,.$$

But $a_m = f(m)$. Consequently,

$$a_{m+1} < a_m\,, \quad \lim_{m\to\infty} a_m = 0\,.$$

By the Alternating series test, therefore, the series

$$\sum_{m=1}^{\infty} (-1)^{m-1} a_m$$

converges, and so the given series is

$$\boxed{\text{conditionally convergent}}\,.$$

---

005 10.0 points

Determine whether the series

$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln(n)}$$

is conditionally convergent, absolutely convergent, or divergent.

1. series is absolutely convergent

2. series is conditionally convergent

3. series is divergent **correct**

Explanation:
By the Divergence Test, a series

$$\sum_{n=N}^{\infty} (-1)^n a_n$$

will be divergent for each fixed choice of N if

$$\lim_{n\to\infty} a_n \neq 0$$

since it is only the behaviour of a_n as $n \to \infty$ that's important. Now, for the given series, $N = 2$ and

$$a_n = \frac{n}{\ln(n)}\,.$$

But by L'Hospital's Rule,

$$\lim_{x\to\infty} \frac{x}{\ln(x)} = \lim_{x\to\infty} \frac{1}{1/x} = \infty\,.$$

Consequently, by the Divergence Test, the given series is

$$\boxed{\text{divergent}}\,.$$

---

006 10.0 points

Which one of the following properties does the series

$$\sum_{n=1}^{\infty} \frac{n\,(-8)^n}{5^{n-1}}$$

have?

1. divergent **correct**

2. conditionally convergent

3. absolutely convergent

Explanation:
With

$$a_n = \frac{n\,(-8)^n}{5^{n-1}}$$

we see that

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)8^{n+1}}{5^n}\left(\frac{5^{n-1}}{n8^n}\right) = \frac{n+1}{n}\left(\frac{8}{5}\right).$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{8}{5}\,,$$

so by the Ratio Test the given series is

$$\boxed{\text{divergent}}\,.$$

---

007 10.0 points

Determine whether the series

$$\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent

2. absolutely convergent **correct**

3. divergent

Explanation:

We use the Ratio Test with

$$a_n = \frac{(-4)^n}{(2n)!}.$$

For then

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-4)^{n+1}}{(2n+2)!}\frac{(2n)!}{(-4)^n}\right|$$

$$= \left|\frac{(-4)^{n+1}}{(2n+2)!}\frac{(2n)!}{(-4)^n}\right| = \left|\frac{-4}{(2n+1)(2n+2)}\right|.$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty}\frac{4}{(2n+1)(2n+2)} = 0 < 1.$$

Consequently, the

$$\boxed{\text{series is absolutely convergent}}.$$

---

008 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty}\frac{2^n}{(3n+1)\,2^{2n+1}}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent
2. absolutely convergent **correct**
3. divergent

Explanation:

With

$$a_n = \frac{2^n}{(3n+1)\,2^{2n+1}}$$

we see that

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{n+1}(3n+1)\,2^{2n+1}}{(3(n+1)+1)\,2^{2(n+1)+1}2^n}$$

$$= \frac{3n+1}{3n+4}\left(\frac{2}{4}\right)$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{2}{4} < 1,$$

so by the Ratio Test the given series is

$$\boxed{\text{absolutely convergent}}.$$

---

009 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n!}{8^n}$$

is absolutely convergent, conditionally convergent, or divergent.

1. absolutely convergent
2. conditionally convergent
3. divergent **correct**

Explanation:

With

$$a_n = (-1)^n\frac{n!}{8^n}$$

we see that

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!8^n}{8^{n+1}n!} = \frac{n+1}{8}.$$

Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = +\infty,$$

and so by the Ratio Test the original series is

$$\boxed{\text{divergent}}.$$

---

010 10.0 points

Decide whether the series

$$\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}5^n$$

converges or diverges.

1. diverges **correct**

2. converges

Explanation:

The given series has the form

$$\sum_{n=1}^{\infty} a_n\,, \qquad a_n = \frac{(n!)^2}{(2n)!}\,5^n\,.$$

But then

$$\frac{a_{n+1}}{a_n} = \frac{5(n+1)!(n+1)!(2n)!}{n!\,n!\,(2n+2)!}$$

$$= \frac{5(n+1)(n+1)}{(2n+1)(2n+2)} = \frac{5(n^2+2n+1)}{4n^2+6n+2}\,,$$

in which case

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{5}{4}\,.$$

Consequently, the Ratio Test ensures that the given series

$$\boxed{\text{diverges}}\,.$$

---

011 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty} \frac{3n+4}{(2n)!}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent

2. absolutely convergent **correct**

3. divergent

Explanation:

Because

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{a_{n+1}}{a_n}$$

$$= \frac{3n+7}{(2n+2)!}\cdot\frac{(2n)!}{3n+4}$$

$$= \frac{1}{(2n+1)(2n+2)}\cdot\frac{3n+7}{3n+4}$$

it follows that

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0\cdot 1$$

$$= 0$$

Consequently, by the Ratio Test, the given series is

$$\boxed{\text{absolutely convergent}}\,.$$

---

012 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty} (-1)^{n-1}\frac{2^n}{4n^2+5}$$

is absolutely convergent, conditionally convergent, or divergent.

1. absolutely convergent

2. conditionally convergent

3. divergent **correct**

Explanation:

The given series has the form

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n\,, \quad b_n = \frac{2^n}{4n^2+5}$$

of an alternating series. But the numerator is increasing very fast. Indeed

$$\lim_{n\to\infty} \frac{2^n}{4n^2+5} = \infty \neq 0\,.$$

But then

$$(-1)^{n-1}\frac{2^n}{4n^2+5} \not\to 0$$

as $n \to \infty$ since the presence of the term $(-1)^{n-1}$ will make the values oscillate more and more when $n \to \infty$.

Consequently, by the Divergence test, the given series is

$$\boxed{\text{divergent}}\,.$$

---

013 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$$

is absolutely convergent, conditionally convergent, or divergent.

1. absolutely convergent **correct**
2. conditionally convergent
3. divergent

Explanation:

The given series has the form

$$\sum_{n=1}^{\infty} a_n\,, \qquad a_n = \frac{(-3)^n}{n!}\,.$$

But then

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{3(n!)}{(n+1)!} = \frac{3}{n+1}\,,$$

in which case

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1\,.$$

Consequently, by the Ratio test, the given series is

$$\boxed{\text{absolutely convergent}}\,.$$

---

014 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty} 6^{-n}\, n!$$

is absolutely convergent, conditionally convergent, or divergent.

1. divergent **correct**
2. absolutely convergent
3. conditionally convergent

Explanation:

The given series can be written in the form

$$\sum_{n=1}^{\infty} 6^{-n}\, n! = \sum_{n=1}^{\infty} a_n$$

with

$$a_n = \frac{n!}{6^n}\,.$$

But

$$\frac{n!}{6^n} = \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{6\cdot 6\cdot \ldots \cdot 6} = \left(\frac{1}{6}\right)\left(\frac{2}{6}\right)\ldots\left(\frac{n}{6}\right)\,,$$

so

$$a_{n+1} > 2a_n$$

for all $n > 12$. Thus

$$\lim_{n\to\infty} a_n = \infty\,.$$

Conseqently, the given series

$$\boxed{\text{diverges}}\,.$$

---

015 10.0 points

Determine whether the following series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{5n^2+4}{2^n}$$

is absolutely convergent, conditionally convergent, or divergent.

1. conditionally convergent

2. divergent

3. absolutely convergent **correct**

Explanation:

The given series has the form

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n\,, \quad b_n = \frac{5n^2+4}{2^n}$$

of an alternating series. But the denominator is increasing very fast, so first let's check if the series is absolutely convergent rather than simply conditionally convergent. We use the Ratio test, for then

$$\left|\frac{(-1)^n b_{n+1}}{(-1)^{n-1} b_n}\right| = \frac{b_{n+1}}{b_n} = \frac{1}{2}\frac{5(n+1)^2+4}{5n^2+4}\,.$$

But

$$\frac{5(n+1)^2+4}{5n^2+4} = \frac{5n^2+10n+9}{5n^2+4} \longrightarrow 1$$

as $n \to \infty$. Thus

$$\lim_{n\to\infty} \left|\frac{(-1)^n b_{n+1}}{(-1)^{n-1} b_n}\right| = \frac{1}{2} < 1\,.$$

Consequently, by the Ratio test, the given series is

$$\boxed{\text{absolutely convergent}}\,.$$