This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find the domain of the function

$$f(x,y) \;=\; \ln(7 - x^2 - 8y^2)\,.$$

1. $\left\{(x,\,y) : \frac{1}{8}x^2 + \frac{1}{7}y^2 \;>\; 1\right\}$

2. $\left\{(x,\,y) : \frac{1}{8}x^2 + \frac{1}{7}y^2 \;<\; 1\right\}$

3. $\left\{(x,\,y) : \frac{1}{7}x^2 + \frac{8}{7}y^2 \;\leq\; 1\right\}$

4. $\left\{(x,\,y) : \frac{1}{8}x^2 + \frac{1}{7}y^2 \;\leq\; 1\right\}$

5. $\left\{(x,\,y) : \frac{1}{7}x^2 + \frac{8}{7}y^2 \;>\; 1\right\}$

6. $\left\{(x,\,y) : \frac{1}{7}x^2 + \frac{8}{7}y^2 \;<\; 1\right\}$ **correct**

Explanation:
Since $\ln x$ is defined only for $x > 0$, the function

$$f(x,y) \;=\; \ln(7 - x^2 - 8y^2)$$

is defined only for

$$\boxed{\left\{(x,\,y) : \frac{1}{7}x^2 + \frac{8}{7}y^2 \;<\; 1\right\}}\,.$$

keywords: function several variables, ln function, domain

002 10.0 points

Find the domain of the function

$$f(x,y) \;=\; \sqrt{x^2 + 4y^2 - 6}\,.$$

1. $\left\{(x,\,y) : \frac{1}{6}x^2 + \frac{2}{3}y^2 \;<\; 1\right\}$

2. $\left\{(x,\,y) : \frac{1}{4}x^2 + \frac{1}{6}y^2 \;>\; 1\right\}$

3. $\left\{(x,\,y) : \frac{1}{6}x^2 + \frac{2}{3}y^2 \;\geq\; 1\right\}$ **correct**

4. $\left\{(x,\,y) : \frac{1}{6}x^2 + \frac{2}{3}y^2 \;>\; 1\right\}$

5. $\left\{(x,\,y) : \frac{1}{4}x^2 + \frac{1}{6}y^2 \;<\; 1\right\}$

6. $\left\{(x,\,y) : \frac{1}{4}x^2 + \frac{1}{6}y^2 \;\geq\; 1\right\}$

Explanation:
Since $\sqrt{x}$ is defined only for $x \geq 0$, the function

$$f(x,y) \;=\; \sqrt{x^2 + 4y^2 - 6}$$

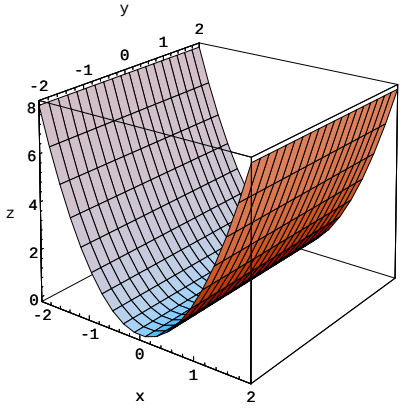
is defined only for

$$\boxed{\left\{(x,\,y) : \frac{1}{6}x^2 + \frac{2}{3}y^2 \;\geq\; 1\right\}}\,.$$

keywords: function several variables, square root function, domain

003 10.0 points

Which one of the following functions has



as its graph.

1. $f(x,y) = y^2 - x^2$

2. $f(x,y) = \frac{1}{2}(x^2 + y^2)$

3. $f(x,y) = 2x^2$ **correct**

4. $f(x,y) = \dfrac{1}{2}(8 - x^2 - y^2)$

5. $f(x,y) = 2y^2$

Explanation:

The cross-section of the graph by the plane $y = 0$ gives a parabola opening upwards with vertex at the point O(0,0,0). Similarly, the cross-section of the graph by the plane $x = 0$ gives a straight line parallel to the x-y plane, indicating that the function does not have any Hence

$$\boxed{f(x,y) = 2x^2\,.}$$

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keywords:

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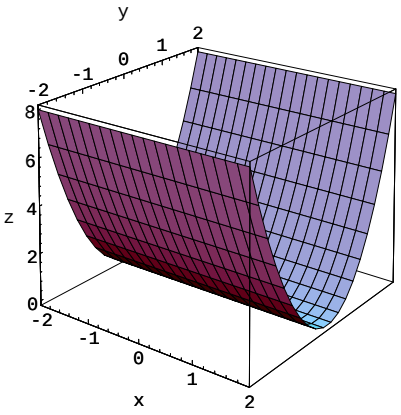
004 10.0 points

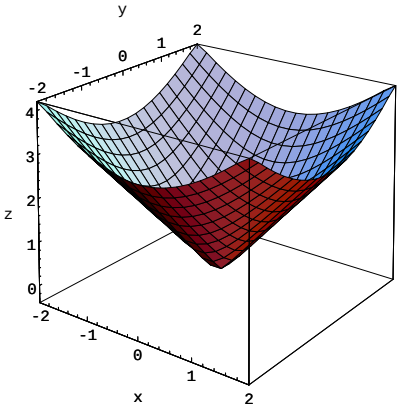
Which one of the following surfaces is the graph of

$$f(x,y) = y^2 - x^2\ ?$$

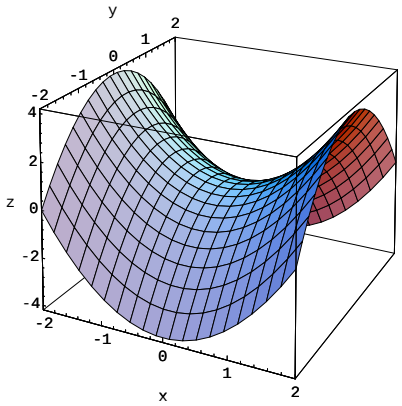
1.



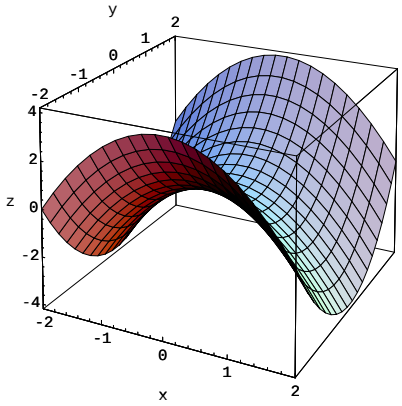
2.



3.

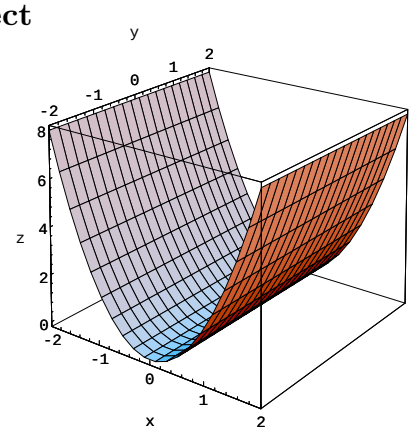


4.



correct

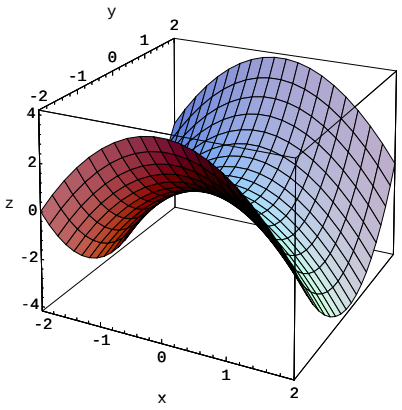
5.



Explanation:

When $y = 0$ the function becomes $z = -x^2$. Its graph is a parabola in the x-z plane opening downwards , i.e., a parabola opening downwards in the plane $y = 0$. Thus the cross-section of the graph of f by the plane $y = 0$ should give a parabola opening downwards . Similarly, the cross-section of the graph by the plane $x = 0$ gives a parabola opening upwards.

This is exactly the case with



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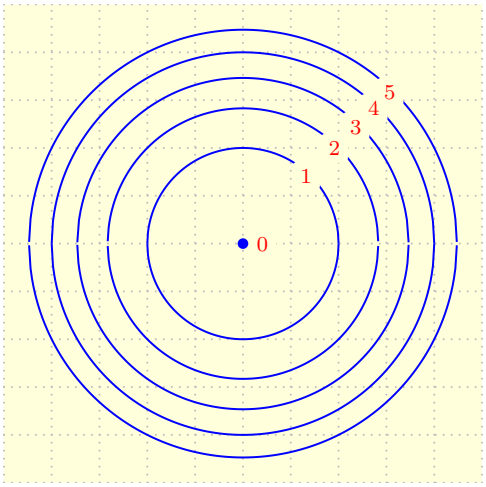
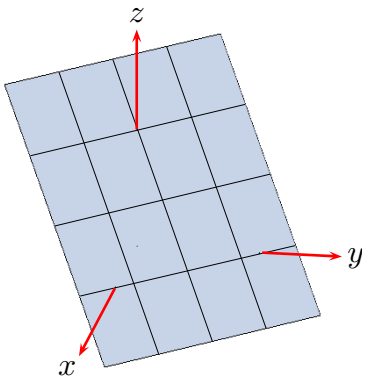
005 10.0 points

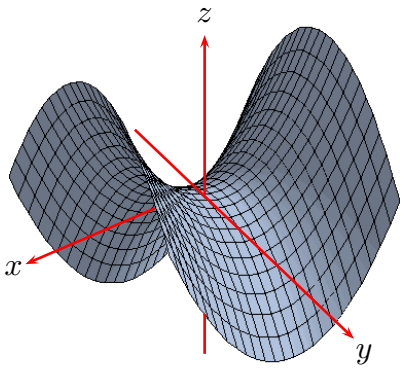
Which of the following surfaces could have contour map



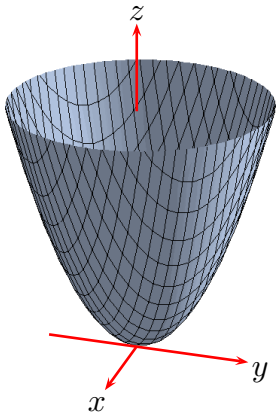
1.



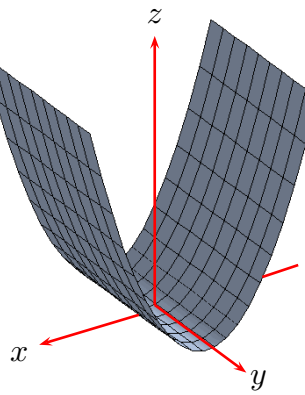
2.



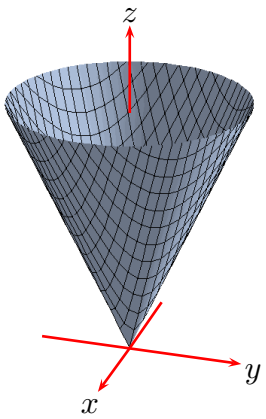
3. **correct**



4.



5.

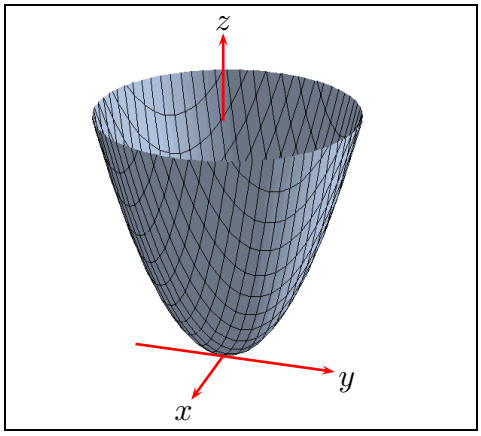


Explanation:

The graphs in the contour map show that the horizontal cross-sections of the surface are

all circles as in cones and paraboloids. On the other hand, the values of the contours and the grid show that these cross-sections grow increasingly fast. This occurs for a paraboloid, but not for a cone.

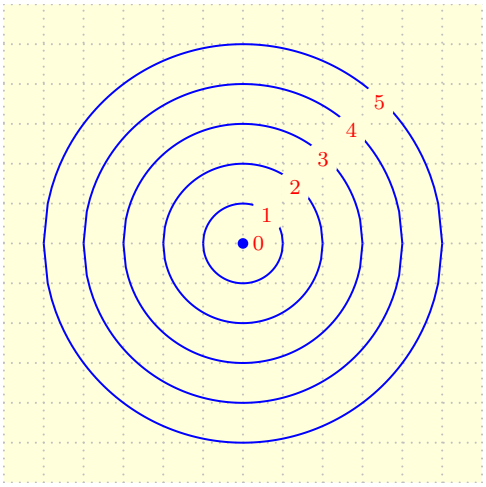
Consequently, of the surfaces shown, the only one having the given contour map is a



keywords:

006 10.0 points

Which of the following surfaces could have contour map



1. parabolic cylinder

2. plane

3. hyperbolic paraboloid

4. cone **correct**

5. paraboloid

Explanation:

The graphs in the contour map show that the horizontal cross-sections of the surface are all circles as in cones and paraboloids. On the other hand, the values of the contours and the grid show that these cross-sections increase at a constant rate. This occurs for a cone, but not for a paraboloid.

Consequently, of the surfaces listed, the only one having the given contour map is a

cone .

keywords: contour map, surface, quadric surface, plane,