Q1.

$\frac{dz}{dt}$ for $z = \overset{f}{x} \ln(\overset{g}{x+11y})$

where $x = \sin t$ and $y = \cos t$

$f'g + g'f$

$\ln(x+11y) + \frac{1}{x+11y} \cdot x \to \ln(x+11y) + \frac{x}{x+11y}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial y}\left(\frac{dy}{dt}\right) + \frac{\partial z}{\partial x}\left(\frac{dx}{dt}\right)$$

$$= \frac{11x}{x+11y}(-\sin(t)) + \left(\ln(x+11y) + \frac{x}{x+11y}\right)\cos(t)$$

$$= \frac{-11x\sin(t)}{x+11y} + \cos(t)\ln(x+11y) + \frac{\cos(t)x}{x+11y}$$

$$= \cos(t)\ln(x+11y) + \frac{\cos(t)x - 11x\sin(t)}{x+11y}$$

Q2: Find $\frac{\partial z}{\partial u}$ when

$z = e^r\cos(\theta)$; $r = 6uv$, $\theta = \sqrt{u^2+v^2}$

$= (u^2+v^2)^{1/2} \Rightarrow \frac{1}{\cancel{2}} \cdot \frac{1}{(u^2+v^2)} \cdot \cancel{2}u$

$= \frac{u}{(u^2+v^2)}$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial u} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial u}$$

$$= (e^r\cos\theta)(6v) + (-e^r\sin\theta)\left(\frac{u}{(u^2+v^2)^{1/2}}\right)$$

$$= e^r\left(6v\cos\theta - \frac{u\sin\theta}{\sqrt{u^2+v^2}}\right)$$

Q3: Find $\frac{dw}{dt}$ when $w = xe^{\frac{y}{z}}$ and

Let $x = t^2$, $y = 4 - t$, $z = 4 + (t)$

$(xe^{y/z})[(yz^{-1})]'$

$-yz^{-2}$

$\frac{-y}{z^2}$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= (e^{y/z})(2t) + \left(\frac{xe^{y/z}}{z}\right)(-1) + \left(\frac{-xe^{y/z}y}{z^2}\right)$$

$$= e^{y/z}\left(2t - \frac{x}{z} - \frac{xy}{z^2}\right)$$

Q4: Find $\frac{\partial z}{\partial s}$ when $z = x^2 - 3xy + y^2$ and

$x = 2s + 3t$, $y = st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= (2x - 3y)(2) + (-3x + 2y)(t)$$

$$= 4x - 6y - 3x + [t]\,2yt$$

Q6: The radius of a right circular cylinder is increasing at a rate of 4 inches per minute while the height is decreasing at a rate of 7 inches per minute. Determine the rate of change of the volume when $r = 3$ and $h = 4$

Let $V = \pi r^2 h$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$2rh\frac{dr}{dt} + r^2\frac{dh}{dt}$$

$$(2)(3)(4)(4) + (3)^2(-7)$$

$$16(6) + (9)(-7)$$

$$96 - 63 = (33\pi)\ \text{cu. in./min}$$

Q7: If $z = f(x,y)$ and $x = r\cos 3\theta$, $y = 3r\sin\theta$, express $\frac{\partial z}{\partial r}$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{\partial z}{\partial x}\cos 3\theta + \frac{\partial z}{\partial y}(3\sin\theta)$$

Q8: If $z = f(x,y)$ and $f_x(4,3) = 4$, $f_y(4,3) = -2$

find $\frac{dz}{dt}$ at $t = 5$ when $x = g(t)$, $y = h(t)$ and

$g(5) = 4$ $g'(5) = 5$

$h(5) = 3$ $h'(5) = 2$.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$(4)(5) + (-2)(2)$$

$$20 - 4 = \boxed{16}$$