This print-out should have 9 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find the value of f_x and f_y at $(1, -1)$ when

$$f(x, y) = \frac{2}{xy} + 4x^2 + y^2.$$

1. $f_x\Big|_{(1,-1)} = -6, \quad f_y\Big|_{(1,-1)} = 0$

2. $f_x\Big|_{(1,-1)} = 6, \quad f_y\Big|_{(1,-1)} = -3$

3. $f_x\Big|_{(1,-1)} = 10, \quad f_y\Big|_{(1,-1)} = 0$

4. $f_x\Big|_{(1,-1)} = 10, \quad f_y\Big|_{(1,-1)} = -4$ **correct**

5. $f_x\Big|_{(1,-1)} = -6, \quad f_y\Big|_{(1,-1)} = -4$

Explanation:

Differentiating f first with respect to x and then with respect to y we obtain

$$\frac{\partial f}{\partial x} = -\frac{2}{x^2 y} + 8x, \quad \frac{\partial f}{\partial y} = -\frac{2}{xy^2} + 2y.$$

Thus at $(1, -1)$,

$$\boxed{f_x\Big|_{(1,-1)} = 10, \quad f_y\Big|_{(1,-1)} = -4}.$$

002 10.0 points

Determine $f_{xx} + f_{yy}$ when

$$f(x, y) = (x - 5)(y + 1)(x + y + 3).$$

1. $f_{xx} + f_{yy} = 2(x + y + 6)$

2. $f_{xx} + f_{yy} = 2(x + y - 4)$ **correct**

3. $f_{xx} + f_{yy} = x + y - 4$

4. $f_{xx} + f_{yy} = x + y + 6$

5. $f_{xx} + f_{yy} = 2(x + y - 6)$

Explanation:

Using the product rule to differentiate

$$f(x, y) = (x - 5)(y + 1)(x + y + 3)$$

first with respect to x and then with respect to y we obtain

$$f_x = (y + 1)(x + y + 3) + (x - 5)(y + 1)$$

and

$$f_y = (x - 5)(x + y + 3) + (x - 5)(y + 1).$$

Repeating this we next obtain

$$f_{xx} = 2(y + 1), \quad f_{yy} = 2(x - 5).$$

Consequently,

$$\boxed{f_{xx} + f_{yy} = 2(x + y - 4)}.$$

003 10.0 points

Determine $f_{xx}f_{yy} - (f_{xy})^2$ when

$$f(x, y) = \frac{2}{3}x^3 + 2y^2 + 6x + 2y + 2xy.$$

1. $f_{xx}f_{yy} - (f_{xy})^2 = 16x + 4$

2. $f_{xx}f_{yy} - (f_{xy})^2 = 8x + 4$

3. $f_{xx}f_{yy} - (f_{xy})^2 = 16x - 4$ **correct**

4. $f_{xx}f_{yy} - (f_{xy})^2 = 16x - 2$

5. $f_{xx}f_{yy} - (f_{xy})^2 = 8x - 4$

Explanation:

After differentiation once

$$f_x = 2x^2 + 6 + 2y, \quad f_y = 4y + 2 + 2x.$$

After differentiating a second time therefore, we see that

$$f_{xx} = 4x, \quad f_{xy} = 2, \quad f_{yy} = 4.$$

Consequently,

$$\boxed{f_{xx}f_{yy} - (f_{xy})^2 = 16x - 4.}$$

004 10.0 points

Determine $f_x - f_y$ when

$$f(x, y) = 3x^2 + xy - 2y^2 - x + 3y.$$

1. $f_x - f_y = 7x - 3y - 4$
2. $f_x - f_y = 7x - 3y + 2$
3. $f_x - f_y = 5x + 5y + 2$
4. $f_x - f_y = 5x - 3y - 4$
5. $f_x - f_y = 7x + 5y + 2$
6. $f_x - f_y = 5x + 5y - 4$ **correct**

Explanation:

After differentiation we see that

$$f_x = 6x + y - 1, \quad f_y = x - 4y + 3.$$

Consequently,

$$\boxed{f_x - f_y = 5x + 5y - 4}.$$

005 10.0 points

Determine f_x when

$$f(x, y) = (2x + y)\, e^{x/y}.$$

1. $f_x = \left(\frac{x}{y} - 1\right) e^{x/y}$
2. $f_x = \left(\frac{2x}{y} + 3\right) e^{x/y}$ **correct**
3. $f_x = \left(\frac{2x}{y} + 1\right) e^{x/y}$
4. $f_x = \left(\frac{x}{y} + 1\right) e^{x/y}$
5. $f_x = \left(\frac{x}{y} - 3\right) e^{x/y}$
6. $f_x = \left(\frac{2x}{y} - 3\right) e^{x/y}$

Explanation:

Differentiating with respect to x keeping y fixed, we see that

$$f_x = 2e^{x/y} + \left(\frac{2x + y}{y}\right) e^{x/y}.$$

Consequently,

$$\boxed{f_x = \left(\frac{2x}{y} + 3\right) e^{x/y}}.$$

006 10.0 points

Find u_t when

$$u = xe^{-2t} \sin\theta.$$

1. $u_t = -2e^{-2t} \sin\theta$
2. $u_t = e^{-2t} \sin\theta$
3. $u_t = -2xe^{-2t} \sin\theta$ **correct**
4. $u_t = 2xe^{-2t} \sin\theta$
5. $u_t = xe^{-2t} \cos\theta$

Explanation:

Differentiating

$$u = xe^{-2t} \sin\theta$$

with respect to t keeping x and θ fixed, we see that

$$\boxed{u_t = -2xe^{-2t} \sin\theta}.$$

007 10.0 points

Determine the second partial f_{xy} of f when

$$f(x, y) = \frac{3x^2}{y} + \frac{y^2}{16x}.$$

1. $f_{xy} = \dfrac{6x}{y^2} + \dfrac{y}{8x^2}$

2. $f_{xy} = 6x - y$

3. $f_{xy} = -\dfrac{6x}{y^2} - \dfrac{y}{8x^2}$ **correct**

4. $f_{xy} = \dfrac{6x}{y^2} - \dfrac{y}{8x^2}$

5. $f_{xy} = 6x + y$

Explanation:

Differentiating with respect to x, we obtain

$$f_x = \frac{6x}{y} - \frac{y^2}{16x^2},$$

and so after differentiation with respect to y we see that

$$\boxed{f_{xy} = -\frac{6x}{y^2} - \frac{y}{8x^2}.}$$

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008 10.0 points

Determine

$$\frac{\partial z}{\partial y}$$

when $z = 3e^{-x/y}$.

1. $\dfrac{\partial z}{\partial y} = \dfrac{3}{y^2}\, e^{-x/y}$

2. $\dfrac{\partial z}{\partial y} = \dfrac{3x}{y^2}\, e^{-x/y}$ **correct**

3. $\dfrac{\partial z}{\partial y} = -\dfrac{3}{y^2}\, e^{-x/y}$

4. $\dfrac{\partial z}{\partial y} = -\dfrac{3x}{y}\, e^{-x/y}$

5. $\dfrac{\partial z}{\partial y} = -\dfrac{3x}{y^2}\, e^{-x/y}$

6. $\dfrac{\partial z}{\partial y} = \dfrac{3x}{y}\, e^{-x/y}$

Explanation:

Differentiating z with respect to y keeping x fixed we see that

$$\frac{\partial z}{\partial y} = 3e^{-x/y} \cdot \frac{\partial(-x/y)}{\partial y}.$$

Consequently,

$$\boxed{\frac{\partial z}{\partial y} = \frac{3x}{y^2}\, e^{-x/y}}.$$

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009 10.0 points

Determine f_{xy} when

$$f(x,\, y) = 2y \tan^{-1}\Big(\frac{y}{x}\Big).$$

1. $f_{xy} = \dfrac{xy^2}{x^2 + y^2}$

2. $f_{xy} = \dfrac{xy}{x^2 + y^2}$

3. $f_{xy} = \dfrac{4x^2 y}{(x^2 + y^2)^2}$

4. $f_{xy} = -\dfrac{4x^2 y}{(x^2 + y^2)^2}$ **correct**

5. $f_{xy} = -\dfrac{4xy}{x^2 + y^2}$

6. $f_{xy} = -\dfrac{x^2 y}{(x^2 + y^2)^2}$

Explanation:

By the Chain Rule,

$$\frac{\partial}{\partial x}\Big(\tan^{-1}\frac{y}{x}\Big) = -\frac{y}{x^2}\Big(\frac{1}{1 + (y/x)^2}\Big)$$
$$= -\frac{y}{x^2 + y^2}.$$

Since we can choose whether to differentiate with respect to x or y first, for simplicity we will choose to first differentiate with respect to x. But then

$$f_x = \frac{-2y^2}{x^2 + y^2}.$$

Differentiating partially now with respect to y we see that

$$f_{xy} = \frac{(x^2 + y^2)(-4y) + 2y^2(2y)}{(x^2 + y^2)^2}.$$

Consequently,

$$\boxed{f_{xy} = -\frac{4x^2 y}{(x^2 + y^2)^2}}.$$

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keywords: partial differentiation, mixed partial derivative, Chain Rule, inverse tangent, PartialDiffMV, PartialDiffMVExam,