This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

**001 10.0 points**

Determine $\dfrac{dz}{dt}$ when

$$z = x\ln(x+11y)$$

and

$$x = \sin t\,, \qquad y = \cos t\,.$$

**1.** $\dfrac{dz}{dt} = \ln(x+11y)\cos t + \dfrac{x\cos t - 11x\sin t}{x+11y}$ **correct**

**2.** $\dfrac{dz}{dt} = \ln(x+11y)\cos t - \dfrac{11x\sin t}{x+11y}$

**3.** $\dfrac{dz}{dt} = \dfrac{\ln(x+11y)\sin t - 11x\cos t}{x+11y}$

**4.** $\dfrac{dz}{dt} = \ln(x+11y)\sin t + \dfrac{x\sin t - 11x\cos t}{x+11y}$

**5.** $\dfrac{dz}{dt} = \ln(x+11y)\cos t + \dfrac{x(\sin t - \cos t)}{x+11y}$

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\,.$$

Here, we have that

$$\frac{\partial z}{\partial x} = \frac{x}{x+11y} + \ln(x+11y), \qquad \frac{dx}{dt} = \cos t$$

and

$$\frac{\partial z}{\partial y} = \frac{11x}{x+11y}, \qquad \frac{dy}{dt} = -\sin t\,.$$

It follows that

$$\boxed{\frac{dz}{dt} = \ln(x+11y)\cos t + \frac{x\cos t - 11x\sin t}{x+11y}}\,.$$

keywords:

**002 10.0 points**

Use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ when

$$z = e^r\cos\theta$$

and

$$r = 6uv\,, \qquad \theta = \sqrt{u^2+v^2}\,.$$

**1.** $\dfrac{\partial z}{\partial u} = e^r\Big(6v\cos\theta - \dfrac{\sin\theta}{2\sqrt{u^2+v^2}}\Big)$

**2.** $\dfrac{\partial z}{\partial u} = ue^r\Big(6v\cos\theta + \dfrac{\sin\theta}{\sqrt{u^2+v^2}}\Big)$

**3.** $\dfrac{\partial z}{\partial u} = e^r\Big(6v\cos\theta + \dfrac{\sin\theta}{\sqrt{u^2+v^2}}\Big)$

**4.** $\dfrac{\partial z}{\partial u} = e^r\Big(6v\cos\theta + \dfrac{u\sin\theta}{2\sqrt{u^2+v^2}}\Big)$

**5.** $\dfrac{\partial z}{\partial u} = e^r\Big(6v\cos\theta - \dfrac{u\sin\theta}{\sqrt{u^2+v^2}}\Big)$ **correct**

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial u} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial u}\,.$$

But

$$\frac{\partial z}{\partial r} = e^r\cos\theta, \qquad \frac{\partial r}{\partial u} = 6v$$

while

$$\frac{\partial z}{\partial \theta} = -e^r\sin\theta, \qquad \frac{\partial \theta}{\partial u} = \frac{u}{\sqrt{u^2+v^2}}\,.$$

Thus

$$\boxed{\frac{\partial z}{\partial u} = e^r\Big(6v\cos\theta - \frac{u\sin\theta}{\sqrt{u^2+v^2}}\Big)}\,.$$

keywords: partial differentiation, Chain Rule,

**003 10.0 points**

Use the Chain Rule to find $\dfrac{dw}{dt}$ when

$$w = xe^{y/z}$$

and

$$x = t^2, \quad y = 4 - t, \quad z = 4 + t.$$

1. $\dfrac{dw}{dt} = \left(2t - \dfrac{x}{z} - \dfrac{xy}{z}\right)e^{y/z}$
2. $\dfrac{dw}{dt} = \left(t - \dfrac{x}{z} - \dfrac{4xy}{z}\right)e^{y/z}$
3. $\dfrac{dw}{dt} = \left(2t - \dfrac{x}{z} - \dfrac{xy}{z^2}\right)e^{y/z}$ **correct**
4. $\dfrac{dw}{dt} = \left(2t + \dfrac{x}{z} + \dfrac{xy}{z^2}\right)e^{y/z}$
5. $\dfrac{dw}{dt} = \left(t + \dfrac{x}{z} + \dfrac{4xy}{z^2}\right)e^{y/z}$
6. $\dfrac{dw}{dt} = \left(t + \dfrac{x}{z} + \dfrac{4xy}{z}\right)e^{y/z}$

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}.$$

When

$$w = xe^{y/z}$$

and

$$x = t^2, \quad y = 4 - t, \quad z = 4 + t,$$

therefore,

$$\frac{dw}{dt} = 2te^{y/z} - \frac{x}{z}e^{y/z} - \frac{xy}{z^2}e^{y/z}.$$

Consequently,

$$\boxed{\frac{dw}{dt} = \left(2t - \frac{x}{z} - \frac{xy}{z^2}\right)e^{y/z}}.$$

**004 10.0 points**

Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ when

$$z = x^2 - 3xy + y^2,$$

and

$$x = 2s + 3t, \qquad y = st.$$

1. $\dfrac{\partial z}{\partial s} = 4x - 6y - 3xs + 2ys$
2. $\dfrac{\partial z}{\partial s} = 6x - 6y - 3xs + 2ys$
3. $\dfrac{\partial z}{\partial s} = 4x - 6y - 3xt + 2yt$ **correct**
4. $\dfrac{\partial z}{\partial s} = 6x - 9y - 3xs + 2ys$
5. $\dfrac{\partial z}{\partial s} = 6x - 9y - 3xt + 2yt$
6. $\dfrac{\partial z}{\partial s} = 4x - 9y - 3xt + 2yt$

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}.$$

Now

$$\frac{\partial z}{\partial x} = 2x - 3y, \quad \frac{\partial x}{\partial s} = 2$$

while

$$\frac{\partial z}{\partial y} = -3x + 2y, \quad \frac{\partial y}{\partial s} = t.$$

Thus

$$\frac{\partial z}{\partial s} = 2(2x - 3y) + t(-3x + 2y).$$

Consequently,

$$\boxed{\frac{\partial z}{\partial s} = 4x - 6y - 3xt + 2yt}.$$

**005 10.0 points**

Use the Chain Rule to find $\dfrac{\partial u}{\partial p}$ for

$$u = \frac{x+y}{y+z}$$

when

$$x = p + 8r + 9t, \qquad y = p - 8r + 9t,$$

and

$$z = p + 8r - 9t.$$

1. $\dfrac{\partial u}{\partial p} = -\dfrac{9t}{p^2}$ **correct**
2. $\dfrac{\partial u}{\partial p} = \dfrac{9t^2}{p^3}$
3. $\dfrac{\partial u}{\partial p} = \dfrac{9}{p^2}$
4. $\dfrac{\partial u}{\partial p} = -\dfrac{9}{p^2}$
5. $\dfrac{\partial u}{\partial p} = \dfrac{9t}{p^2}$

**Explanation:**

By the Chain Rule,

$$\frac{\partial u}{\partial p} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial p} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial p} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial p}.$$

But

$$\frac{\partial u}{\partial x} = \frac{1}{y+z}, \qquad \frac{\partial x}{\partial p} = 1$$

while

$$\frac{\partial u}{\partial y} = \frac{z-x}{(y+z)^2}, \qquad \frac{\partial y}{\partial p} = 1$$

and

$$\frac{\partial u}{\partial z} = \frac{-x-y}{(y+z)^2}, \qquad \frac{\partial z}{\partial p} = 1.$$

Consequently,

$$\frac{\partial u}{\partial p} = \frac{1}{y+z} + \frac{z-x}{(y+z)^2} + \frac{-x-y}{(y+z)^2}.$$

$$= \frac{2(z-x)}{(y+z)^2} = \boxed{-\frac{9t}{p^2}}.$$

keywords:

**006 10.0 points**

The radius of a right circular cylinder is increasing at a rate of 4 inches per minute while the height is decreasing at a rate of 7 inches per minute. Determine the rate of change of the volume when $r = 3$ and $h = 4$.

1. rate $= 29\pi$ cu.in./min.
2. rate $= 37\pi$ cu.in./min.
3. rate $= 33\pi$ cu.in./min. **correct**
4. rate $= 25\pi$ cu.in./min.
5. rate $= 41\pi$ cu.in./min.

**Explanation:**

The volume cylinder of a cylinder of height $h$ and radius $r$ is given by

$$V(r, h) = \pi r^2 h.$$

When $h$ and $r$ are changing with $t$, then by the Chain Rule the rate of change of $V$ with respect to $t$ is given by

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= 2\pi r h\frac{dr}{dt} + \pi r^2\frac{dh}{dt}.$$

But

$$\frac{dr}{dt} = 4, \qquad \frac{dh}{dt} = -7,$$

in which case

$$\frac{dV}{dt} = 8\pi r h - 7\pi r^2.$$

Consequently, when $r = 3$ and $h = 4$,

$$\boxed{\left.\frac{dV}{dlt}\right|_{(r=3,\ h=4)} = 33\pi \text{ cu.in./min.}}.$$

**007 10.0 points**

If $z = f(x, y)$ and

$$x = r\cos 3\theta\,, \qquad y = 3r\sin\theta\,,$$

express $\dfrac{\partial z}{\partial r}$ in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

**1.** $\dfrac{\partial z}{\partial r} = 3\dfrac{\partial z}{\partial x}\cos\theta + \dfrac{\partial z}{\partial y}\sin 3\theta$

**2.** $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\sin 3\theta + 3\dfrac{\partial z}{\partial y}\cos\theta$

**3.** $\dfrac{\partial z}{\partial r} = 3r\Big(\dfrac{\partial z}{\partial y}\cos\theta - \dfrac{\partial z}{\partial x}\sin 3\theta\Big)$

**4.** $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\cos 3\theta + 3\dfrac{\partial z}{\partial y}\sin\theta$ **correct**

**5.** $\dfrac{\partial z}{\partial r} = 3r\Big(\dfrac{\partial z}{\partial y}\cos\theta + \dfrac{\partial z}{\partial x}\sin 3\theta\Big)$

**6.** $\dfrac{\partial z}{\partial r} = r\Big(3\dfrac{\partial z}{\partial y}\cos\theta - \dfrac{\partial z}{\partial x}\sin 3\theta\Big)$

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\,.$$

But when

$$x = r\cos 3\theta\,, \qquad y = 3r\sin\theta\,,$$

we see that

$$\frac{\partial x}{\partial r} = \cos 3\theta\,, \qquad \frac{\partial y}{\partial r} = 3\sin\theta\,.$$

In this case

$$\boxed{\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cos 3\theta + 3\frac{\partial z}{\partial y}\sin\theta}\,.$$

keywords: Chain Rule, partial differentiation, polar coordinates,

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**008 10.0 points**

If $z = f(x, y)$ and

$$f_x(4, 3) = 4, \qquad f_y(4, 3) = -2\,,$$

find $\dfrac{dz}{dt}$ at $t = 5$ when $x = g(t)$, $y = h(t)$ and

$$g(5) = 4\,, \qquad g'(5) = 5\,.$$

$$h(5) = 3\,, \qquad h'(5) = 2\,.$$

**1.** $\dfrac{dz}{dt} = 14$

**2.** $\dfrac{dz}{dt} = 18$

**3.** $\dfrac{dz}{dt} = 16$ **correct**

**4.** $\dfrac{dz}{dt} = 12$

**5.** $\dfrac{dz}{dt} = 20$

**Explanation:**

By the Chain Rule for Partial Differentiation,

$$\frac{dz}{dt} = f_x(g(t), h(t))g'(t) + f_y(g(t), h(t))h'(t)\,.$$

When $t = 5$, therefore,

$$\frac{dz}{dt}\Big|_{t=5} = f_x(g(5), h(5))g'(5) + f_y(g(5), h(5))h'(5) = f_x(4, 3)g'(5) + f_y(4, 3)h'(5)\,.$$

Consequently, given the values above,

$$\boxed{\frac{dz}{dt} = (4)(5) - (2)(2) = 16}\,.$$

keywords: