Q_1: Determine the value of the double integral

$$I = \iint_R (6-x)\,dx\,dy$$

Let $R = \{(x,y): 1 \le x \le 6,\ 0 \le y \le 4\}$

$$I = \underbrace{\int_0^4\int_1^6 (6-x)\,dx\,dy}$$

$$6x - \frac{x^2}{2}\Big]_1^6 = \left(6(6) - \frac{36}{2}\right) - \left(6 - \frac{1}{2}\right)$$

$$\int_0^4 18 - \frac{11}{2}\,dy \qquad (36-18) \qquad -\frac{11}{2}$$

$$18 - \frac{11}{2}$$

$$18y - \frac{11}{2}y\Big]_0^4 \rightarrow 72 - 22 = \boxed{50}$$

Q_2: Evaluate the integral

$$I = \int_0^1\int_1^2 (2x + x^2y)\,dy\,dx$$

$$I = \underbrace{\int_0^1\int_1^2 (2x + x^2y)\,dy}\,dx$$

$$2xy + \frac{x^2y^2}{2}\Big]_1^2 = (4x + 2x^2) - \left(2x + \frac{x^2}{2}\right)$$

$$4x + 2x^2 - 2x - \frac{x^2}{2}$$

$$= 2x + 2x^2 - \frac{x^2}{2}$$

$$\int_0^1 2x + 2x^2 - \frac{x^2}{2}$$

$$x^2 + \frac{2}{3}x^3 - \frac{1}{6}x^3\Big]_0^1 = 1 + \frac{2}{3} - \frac{1}{6}$$

$$= \frac{6}{6} + \frac{4}{6} - \frac{1}{6}$$

$$= \frac{9}{6} \rightarrow \frac{3}{2}$$

Q_3: Evaluate the integral

$$I = \int_2^3\int_0^2 e^{x-y}$$

$$= \underbrace{\int_2^3\int_0^2 e^x e^{-y}\,dx}\,dy$$

$$e^x e^{-y}\Big]_0^2 \rightarrow e^{x-y}\Big]_0^2 = \left(e^{2-y}\right) - \left(e^{0-y}\right)$$

$$= e^{2-y} - e^{0-y}$$

$$\int_2^3 e^{2-y} - e^{-y} \qquad = e^{2-y} - e^{-y}$$

$$-e^{2-y} + e^{y}\Big]_2^3 = \left(-e^{2-3} + e^3\right) - \left(-e^{2-2} + e^2\right)$$

$$-e^{-1} + e^3 + e^0 - e^2$$

$$I = e^3 - e^2 - e^1 + 1$$

Q_4: Determine the value of the double integral

$$I = \iint_A \frac{3xy^2}{9+x^2}\,dA$$

over the rectangle

$A = \{(x,y): 0 \le x \le 2,\ -1 \le y \le 1\}$

$$= \underbrace{\int_0^2\int_{-1}^1 \frac{3x}{9+x^2}\,y^2\,dy}\,dx$$

$$\frac{3x}{9+x^2}\left(\frac{1}{3}y^3\right)\Big]_{-1}^1 \rightarrow \frac{xy^3}{9+x^2}\Big]_{-1}^1 = \left(\frac{x}{9+x^2}\right) - \left(\frac{-x}{9+x^2}\right)$$

$$\frac{2x}{9+x^2}$$

$$= \int_0^2 \frac{2x}{9+x^2}\,dx \qquad \text{Let } u = 9+x^2,\ du = 2x\,dx$$

$$= \int \frac{1}{u}\,du$$

$$\ln(u) \rightarrow \ln(9+x^2)\Big]_0^2 = \left(\ln(9+(2)^2)\right) - \left(\ln(9+0)\right)$$

$$\ln(13) \qquad -\ln(9)$$

$$= \ln\left(\frac{13}{9}\right)$$

Q_5: Evaluate the iterated integral

$$I = \int_1^3\underbrace{\int_0^3 \frac{2}{(x+y)^2}\,dx}\,dy \qquad \text{Let } u = x+y,\ du = dx$$

$$\int \frac{2}{u^2}\,du \rightarrow 2u^{-2} \rightarrow -2u^{-1} \rightarrow -\frac{2}{u}$$

$$= -\frac{2}{x+y}\Big]_0^3 = \left(-\frac{2}{3+y}\right) - \left(-\frac{2}{0+y}\right)$$

$$= -\frac{2}{3+y} + \frac{2}{y}$$

$$\int_1^3 -\frac{2}{3+y} + \frac{2}{y}\,dy$$

$$-\int_1^3 \frac{2}{3+y}\,dy + 2\int \frac{1}{y}\,dy$$

$$-2\int_1^3 \frac{1}{3+y}\,dy + 2\int_1^3 \frac{1}{y}\,dy$$

let $u = 3+y$, $du = dy$

$$2\ln(y)\Big]_1^3 = 2\ln(3) - \cancel{2\ln(1)}^{\,0}$$

$$= 2\ln(3)$$

$$\int \frac{1}{u} \rightarrow \ln(u)$$

$$-2\ln(3+y)\Big]_1^3 = \left(-2\ln(3+3)\right) - \left(-2\ln(3+1)\right)$$

$$= -2\ln(6) + 2\ln(4)$$

$$= 2\ln(4) - 2\ln(6)$$

$$= 2\ln\left(\frac{4}{6}\right)$$

$$2\ln\left(\frac{4}{6}\right) + 2\ln(3)$$

$$2\ln\left(\frac{4}{6}\cdot 3\right) = 2\ln(2)$$

Q_6: Evaluate the iterated integral

$$I = \int_1^3\underbrace{\int_1^3 \left(\frac{x}{y} + \frac{y}{x}\right)dy}\,dx$$

$$x\int_1^3 \frac{1}{y}\,dy + \frac{1}{x}\int_1^3 y\,dy$$

let $u = y$, $du = dy$

$$\frac{y^2}{2x}\Big]_1^3 = \frac{9}{2x} - \frac{1}{2x}$$

$$= \frac{8}{2x} = \frac{4}{x}$$

$$\int \frac{1}{u}\,du \rightarrow \ln(u)$$

$$x\ln(y)\Big]_1^3 = (x\ln(3)) - \cancel{(x\ln(1))}^{\,0}$$

$$\int_1^3 x\ln(3) + \int_1^3 \frac{4}{x}\,dx$$

dx

Let $u = x$, $du = dx$

$$\ln(3)\int_1^3 x + 4\int \frac{1}{x}\,dx \longrightarrow \int \frac{1}{u} = \ln(u)$$

$$\ln(3)\left(\frac{1}{2}x^2\right)\Big]_1^3 \qquad 4\ln(x)\Big]_1^3 = \left(4\ln(3)\right) - \cancel{(4\ln(1))}^{\,0}$$

$$\frac{\ln(3)}{2}x^2\Big]_1^3 = \frac{9\ln(3)}{2} - \frac{\ln(3)}{2} \qquad 4\ln(3)$$

$$= \ln(3)\left(\frac{9}{2} - \frac{1}{2}\right)$$

$$4\ln(3) + 4\ln(3)$$

$$\ln(3)(4+4) = 8\ln(3)$$

Q_7: Evaluate the double integral

$$I = \iint_A \frac{5+x^2}{1+y^2}\,dx\,dy$$

when $A = \{(x,y): 0 \le x \le 2,\ 0 \le y \le 1\}$

$$= \int_0^1\underbrace{\int_0^2 \frac{5+x^2}{1+y^2}\,dx}\,dy$$

$$\frac{1}{1+y^2}\int_0^2 5 + x^2\,dx$$

$$\frac{5x + \frac{1}{3}x^3}{1+y^2}\Bigg]_0^2 = \left(\frac{5(2) + \frac{1}{3}(2)^3}{1+y^2}\right) - \left(\cancel{5(0)+}^{\,0}\right)$$

$$10 + \frac{8}{3} = \frac{30}{3} + \frac{8}{3} = \frac{38}{3}$$

$$\frac{38}{3}\int_0^1 \frac{1}{1+y^2}\,dy$$

$$\rightarrow \frac{38}{3}\tan^{-1}(y)\Big]_0^1 = \left(\frac{38}{3}\left(\frac{\pi}{4}\right)\right) - \left(\cancel{\frac{38}{3}(0)}^{\,0}\right)$$

$$= \frac{38\pi}{12} = \frac{19}{6}\pi$$

$$2\overline{)38} = 19,\quad -2,\ 18$$

$$\frac{38 \div 2}{12 \div 2}\ \frac{19}{6}$$

Q_8: Evaluate the integral ★ Ask Martines

$$I = \iint_A 3xe^{2xy}\,dx\,dy$$

Over the rectangle $A = \{(x,y): 0 \le x \le 3,\ 0 \le y \le 2\}$

$$I = \int_0^2\underbrace{\int_0^3 3xe^{2xy}\,dx}\,dy \qquad uv - \int v\,du$$

let $u = 3x$, $dv = e^{2x}\,dx$

$du = 3\,dx$, $V = \frac{1}{2}e^{2x}$

$$\frac{3x}{2}e^{2x} - \int \frac{3}{2}e^{2x}\,dx = \frac{3x}{2}e^{2x} - \frac{3}{4}e^{2x}\Big]_0^3 \qquad \left(\frac{9}{2}e^6 - \frac{3}{4}e^6\right) - \left(\cancel{\frac{3}{2}}^{\,0} - \frac{3}{4}\right)$$

$$\frac{3}{2}\int e^{2x}\,dx$$

$$\frac{3}{2}\left(\frac{1}{2}e^{2x}\right)$$

$$\frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$

$$\frac{9}{2y}e^{6y} - \frac{3}{4y^2}e^{6y} + \frac{3}{4y^2}$$

$$3\overline{)39} = 13$$