This print-out should have 12 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001   10.0 points

Determine the integral

$$I = \int \frac{x^2}{(4 - x^2)^{3/2}} \, dx \, .$$

1. $I = \dfrac{x}{\sqrt{4 - x^2}} + \sin^{-1}\left(\dfrac{x}{2}\right) + C$

2. $I = \dfrac{2x}{\sqrt{4 - x^2}} - \sin^{-1}\left(\dfrac{x^2}{4}\right) + C$

3. $I = \dfrac{2x^2}{\sqrt{4 - x^2}} + \sin^{-1}\left(\dfrac{x^2}{4}\right) + C$

4. $I = \dfrac{x^2}{\sqrt{4 - x^2}} + \sin^{-1}\left(\dfrac{x^2}{2}\right) + C$

5. $I = \dfrac{2x}{\sqrt{4 - x^2}} - \sin^{-1}\left(\dfrac{x}{4}\right) + C$

6. $I = \dfrac{x}{\sqrt{4 - x^2}} - \sin^{-1}\left(\dfrac{x}{2}\right) + C$

002   10.0 points

Evaluate the integral

$$I = \int_0^1 \frac{3x^2}{(2 - x^2)^{3/2}} \, dx \, .$$

1. $I = \sqrt{3} - \dfrac{\pi}{3}$

2. $I = 3\left(\sqrt{2} - \dfrac{\pi}{4}\right)$

3. $I = 3\left(1 + \dfrac{\pi}{4}\right)$

4. $I = \sqrt{3} + \dfrac{\pi}{3}$

5. $I = \sqrt{2} + \dfrac{\pi}{3}$

6. $I = 3\left(1 - \dfrac{\pi}{4}\right)$

003   10.0 points

Evaluate the integral

$$I = \int_0^2 \frac{1}{\sqrt{16 - x^2}} \, dx \, .$$

1. $I = \dfrac{1}{3}$

2. $I = \dfrac{1}{6}\pi$

3. $I = \dfrac{1}{6}$

4. $I = \dfrac{1}{4}$

5. $I = \dfrac{1}{4}\pi$

6. $I = \dfrac{1}{3}\pi$

004   10.0 points

Evaluate the integral

$$I = \int_{\sqrt{2}}^2 \frac{6}{x\sqrt{x^2 - 1}} \, dx \, .$$

1. $I = \dfrac{3}{4}$

2. $I = \dfrac{1}{2}$

3. $I = 1$

4. $I = \dfrac{3}{4}\pi$

5. $I = \pi$

6. $I = \dfrac{1}{2}\pi$

005   10.0 points

Evaluate the integral

$$I = \int_0^2 \frac{x^2 + 5}{4 + x^2} \, dx \, .$$

1. $I = \dfrac{1}{2}\left(4 + \dfrac{1}{8}\right)\pi$

2. $I = \dfrac{1}{4}\left(2 - \dfrac{1}{8}\pi\right)$

3. $I = 2 + \dfrac{1}{8}\pi$

4. $I = 2 - \dfrac{1}{8}\pi$

5. $I = 4 - \dfrac{1}{8}\pi$

006　10.0 points

To which of the following does the integral

$$I = \int \frac{x^5}{\sqrt{1 - x^2}}\, dx$$

reduce after an appropriate trig substitution?

1. $I = \displaystyle\int \sin^5(\theta)\, d\theta$

2. $I = \displaystyle\int \tan(\theta)\sec^5(\theta)\, d\theta$

3. $I = \displaystyle\int \sin^5(\theta)\sec^6(\theta)\, d\theta$

4. $I = \displaystyle\int \sin^5(\theta)\sec^5(\theta)\, d\theta$

5. $I = \displaystyle\int \sec^5(\theta)\sin^6(\theta)\, d\theta$

007　10.0 points

To which one of the following does the integral
$$I = \int \frac{x^2}{\sqrt{x^2 + 1}}\, dx$$
reduce after an appropriate trig substitution?

1. $I = \displaystyle\int \sec^3(\theta)\, d\theta$

2. $I = \displaystyle\int \tan^3(\theta)\, d\theta$

3. $I = \displaystyle\int \tan^2(\theta)\sec^3(\theta)\, d\theta$

4. $I = \displaystyle\int \sin^2(\theta)\sec^3(\theta)\, d\theta$

5. $I = \displaystyle\int \sin^3(\theta)\, d\theta$

6. $I = \displaystyle\int \sin^3(\theta)\sec^2(\theta)\, d\theta$

008　10.0 points

Evaluate the integral
$$I = \int_0^{1/4} \frac{3}{\sqrt{1 - 4x^2}}\, dx\,.$$

1. $I = \dfrac{3}{8}\pi$

2. $I = \dfrac{1}{2}\pi$

3. $I = \dfrac{1}{4}\pi$

4. $I = \dfrac{1}{2}$

5. $I = \dfrac{3}{8}$

6. $I = \dfrac{1}{4}$

009　10.0 points

Evaluate the integral
$$I = \int_0^1 \frac{1}{\sqrt{4 - 3x^2}}\, dx\,.$$

1. $I = \dfrac{1}{3}$

2. $I = \dfrac{\pi}{3\sqrt{3}}$

3. $I = 2$

4. $I = \dfrac{\frac{1}{2}\pi}{\sqrt{3}}$

5. $I = \dfrac{2\pi}{3\sqrt{3}}$

6. $I = \dfrac{1}{2}$

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010   10.0 points

Evaluate the integral

$$I = \int_0^1 \frac{3}{\sqrt{x^2+1}}\, dx \,.$$

1. $I = \sqrt{2}\,(\sqrt{2}-1)$

2. $I = 3\ln(1+\sqrt{2})$

3. $I = 3\,(\sqrt{2}-1)$

4. $I = \sqrt{2}\,(1+\sqrt{2})$

5. $I = 3\ln(\sqrt{2}-1)$

6. $I = \sqrt{2}\,\ln(1+\sqrt{2})$

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011   10.0 points

Evaluate the integral

$$I = \int_0^2 (6 - \sqrt{4-x^2})\, dx \,.$$

1. $I = 6+\pi$

2. $I = 6+2\pi$

3. $I = 12-\pi$

4. $I = 12-2\pi$

5. $I = 12+2\pi$

6. $I = 6-\pi$

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012   10.0 points

Evaluate the integral

$$I = \int_0^1 \frac{x^2}{1+x^2}\, dx \,.$$

1. $I = \dfrac{1}{4}(4-\pi)$

2. $I = \dfrac{1}{8}(4-\pi)$

3. $I = \dfrac{1}{4}(4+\pi)$

4. $I = \dfrac{1}{8}(\pi-2)$

5. $I = \dfrac{1}{8}(\pi+2)$

6. $I = \dfrac{1}{4}(\pi-2)$

Handwritten work:

$x = 4\sin x \qquad \arcsin\left(\frac{x}{4}\right) = \theta$

$dx = 4\cos x$

$\sqrt{4-(4\sin x)^2}$

$\sqrt{4-16\sin^2\theta}$

$6 - (2 - 4\sin\theta)$

$6 - 2 + 4\sin\theta$

$4 + 4\sin\theta$

$4\int 1 + \sin\theta$

$4\left(\theta + \cos\theta\right]_0^{\pi/6}$

$\left(\frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right)\right) - \left(\cos(0)\right)$

$4\left(\frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1\right)$

$\frac{4\pi}{6} + \frac{4\sqrt{3}}{2} - 4$