This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find all nonzero values of k for which the function $y = A\sin kt + B\cos kt$ satisfies the differential equation

$$y'' + 9y = 0$$

for all values of A and B.

1. $k = -9$

2. $k = 3$

3. $k = 9$

4. $k = -3$

5. $k = 3,\ -3$ **correct**

6. $k = 9,\ -9$

Explanation:

We will begin by solving for y''.

$$\begin{aligned} y &= A\sin kt + B\cos kt \\ y' &= Ak\cos kt - Bk\sin kt \\ y'' &= -Ak^2\sin kt - Bk^2\cos kt \\ &= -k^2(A\sin kt + B\cos kt) \\ &= -k^2 y. \end{aligned}$$

We compute

$$\begin{aligned} y'' + 9y &= -k^2 y + 9y \\ &= (9 - k^2)y. \end{aligned}$$

Hence $y'' + 9y = 0$ if and only if $k^2 = 9$, i.e., if and only if $k = \pm 3$. Hence,

$$\boxed{k = 3,\ -3}.$$

002 10.0 points

The family of solutions to the differential equation $y' = -10xy$ is $y = Ce^{-5x^2}$.

Find the solution that satisfies the initial condition $y(0) = 1$.

1. $y = e^{5x^2}$

2. $y = e^{-5x^2} + 1$

3. $y = e^{-5(x^2+1)}$

4. $y = e^{-5x^2}$ **correct**

5. $y = e^{-5(x+1)^2}$

Explanation:

This equation contains only one unknown constant, so we will substitute the indicated values of x and y into the equation to solve for C as follows:

$$\begin{aligned} y(0) &= Ce^{-5(0)^2} \\ &= C. \end{aligned}$$

This shows that $y(0) = 1$ if and only if $C = 1$. Hence,

$$\boxed{y(x) = e^{-5x^2}}.$$

003 10.0 points

Which of the following answers lists all constant solutions to the equation

$$\frac{dy}{dt} = y^4 - 5y^3 + 6y^2\ ?$$

1. $y = -5,\ 6$

2. $y = 2,\ 3$

3. $y = -5,\ 0,\ 6$

4. $y = 0,\ 2,\ 3$ **correct**

5. $y = 0$

Explanation:

A constant solution to a differential equation is a solution where y is constant, hence where $y' = 0$. So in this case

$$\begin{aligned} y' &= y^4 - 5y^3 + 6y^2 \\ &= y^2\left(y^2 - 5y + 6\right) \\ &= y^2(y-2)(y-3). \end{aligned}$$

Hence $y' = 0$ if and only if

$$\boxed{y = 0,\ 2,\ 3}.$$

---

004 10.0 points

Find all values of r for which the function $y = e^{rt}$ satisfies the differential equation

$$y'' - 4y' - 12y = 0.$$

1. $r = -2,\ 6$ **correct**

2. $r = -2$

3. $r = 12$

4. $r = -6,\ 2$

5. $r = -12,\ -4$

6. $r = 4,\ 12$

Explanation:

We will begin by solving for y' and y''.

$$\begin{aligned} y &= e^{rt} \\ y' &= re^{rt} \\ y'' &= r^2e^{rt}. \end{aligned}$$

We compute

$$\begin{aligned} y'' - 4y' - 12y &= r^2e^{rt} - 4re^{rt} - 12e^{rt} \\ &= (r^2 - 4r - 12)e^{rt} \\ &= (r+2)(r-6)e^{rt}. \end{aligned}$$

Hence $y'' - 4y' - 12y = 0$ if and only if $k = -2,\ 6$. Thus,

$$\boxed{k = -2,\ 6}.$$

---

005 10.0 points

Find all values of k that don't result in a zero function for which the function $y = \sin kt$ satisfies the differential equation

$$y'' + 36y = 0$$

1. $k = -36$

2. $k = 6,\ -6$ **correct**

3. $k = -6$

4. $k = 6$

5. $k = 36$

6. $k = 36,\ -36$

Explanation:

We will begin by solving for y''.

$$\begin{aligned} y &= \sin kt \\ y' &= k\cos kt \\ y'' &= -k^2 \sin kt. \end{aligned}$$

We compute

$$\begin{aligned} y'' + 36y &= -k^2 \sin kt + 36 \sin kt \\ &= (36 - k^2)\sin kt. \end{aligned}$$

Hence $y'' + 36y = 0$ if and only if $k^2 = 36$, i.e., if and only if $k = \pm 6$. Thus,

$$\boxed{k = 6,\ -6}.$$