Question #1: If $y = y_0(x)$ is the solution of the differential equation:

$$y\frac{dy}{dx} = 4x(64+y^2)$$

$$y\frac{dy}{dx} = 256x + 4y^2x$$

$$x(256 + 4y^2)$$

$$\cancel{dx}\ \frac{y}{64+y^2}\frac{dy}{\cancel{dx}} = 4x\ dx$$

let $u = y^2 + 64$ $\int \frac{y}{64+y^2}dy = \int 4x\,dx$

$\frac{1}{2}du = \cancel{2}y\,dy$ $2\cdot\frac{1}{2}\ln|y^2+64| + C = 2x^2 + C \cdot 2$

$\frac{1}{2}\int\frac{1}{u}du$ $\ln|y^2+64| + C = 4x^2 + 2C$

$-C \qquad -1C$

$$\ln|y^2+64| = 4x^2 + C$$

$$e^{\ln|y^2+64|} = e^{4x^2+C}$$

$$y^2 + 64 = e^{4x^2+C}$$

$$-64 \qquad -64$$

$$\sqrt{y^2} = \sqrt{e^{4x^2} + C - 64}$$

$y = \sqrt{e^{4x^2+C} - 64}$

$y = (e^{4x^2+C} - 64)^{1/2}$

$y = (e^C \cdot e^{4x^2} - 64)^{1/2}$

$y = (Ce^{4x^2} - 64)^{1/2}$

$0 = (Ce^{4(0)^2} - 64)^{1/2}$

$0 = \sqrt{C - 64}$; Assuming $C = 64$

$\cancel{(0)^2} = 0 = (\sqrt{C-64})^2$

$0 = C - 64$

$+64$

$64 = C$

$y_0 = \sqrt{64(e^{4x^2} - 1)}$

$y_0(x) = 8(e^{4x^2} - 1)^{1/2}$; Let $x = 1$

$\boxed{y_0(1) = 8(e^4 - 1)^{1/2}}$

Question #2: If y_0 satisfies the equations

$$\frac{(x^2+9)}{y}\frac{dy}{dx} = \frac{xy}{y};\ y(0) = 5$$

$$\frac{1}{\cancel{(x^2+9)}}\cdot\frac{\cancel{(x^2+9)}}{y}\frac{dy}{dx} = x\cdot\frac{1}{(x^2+9)}$$

$$\cancel{dx}\ \frac{1}{y}\cdot\frac{dy}{\cancel{dx}} = \frac{x}{x^2+9}\ dx$$

Let $u = y$ $\quad \int\frac{1}{y}dy = \int\frac{x}{x^2+9}dx$ $\quad$ Let $u = x^2 + 9$

$du = dy$ $\quad du = 2x\,dx$

$\ln|u| = \ln|y|$ $\frac{1}{u}$ $\quad \frac{1}{2}\ln|x^2+9|$ $\quad \frac{1}{2}du = x\,dx$

$$\ln|y| + C = \frac{1}{2}\ln|x^2+9| + C$$

$$-C \qquad -C$$

$$\ln|y| = \frac{1}{2}\ln|x^2+9| + C$$

$$e^{\ln|y|} = e^{\ln|\sqrt{x^2+9}| + C}$$

$y = \sqrt{x^2+9}\cdot \cancel{e}^C$

$y = C\sqrt{x^2+9}$; let $x = 0$; $y = 5$

$5 = C\sqrt{0^2+9}$

$\frac{5}{3} = \frac{3C}{3} \rightarrow \frac{5}{3} = C$

$y_0 = \frac{5}{3}\sqrt{x^2+9}$; let $x = 6$

$\frac{5}{3}\sqrt{36+9}$

$\frac{5}{3}\sqrt{45} = y_0$

$\frac{5}{3}\sqrt{5\cdot 9} = y_0$

$\frac{5}{3}\sqrt{5}\cdot\sqrt{9}$

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$3\cdot\frac{5}{3} = \frac{15}{3} = 5\cdot\sqrt{5}$

$\boxed{5\cdot(5)^{1/2} = y_0(6)}$

Question #3: If y_0 satisfies the equations

$$4\frac{dy}{dx} + \frac{2}{xy^3} = 0,\ y(1) = 2$$

for $\{x, y\} > 0$, find the value of $y_0(e)$.

$$4\frac{dy}{dx} + \frac{2}{xy^3} = 0;\ y(1) = 2$$

$$(y^3)\,4\frac{dy}{dx} = -\frac{2}{xy^3}\,(y^3)$$

$$\cancel{dx}\ 4y^3\frac{dy}{\cancel{dx}} = -\frac{2}{x}\,dx$$

$$\int 4y^3\,dy = \int\frac{-2}{x}dx$$

$$4\int y^3\,dy$$

$$y^4 + C = -2\ln|x|$$

$$y^4 + C = \ln|x^{-2}| + C$$

$$-C \qquad -C$$

$$\sqrt[4]{y^4} = \sqrt[4]{\ln|x^{-2}| + C}$$

$y = \sqrt[4]{\ln|x^{-2}| + C}$; Let $x = 1$; $y = 2$

$^4(2) = (\sqrt[4]{\ln|1^{-2}| + C})^4$

$16 = \cancel{\ln|1|}^{0} + C$

$16 = C$

$y_0(e) = \sqrt[4]{\ln|e^{-2}| + 16}$

$\sqrt[4]{-2\ln|e| + 16}$

$-2 + 16$

$y_0(e) = \sqrt[4]{14}$

$\boxed{y_0(e) = (14)^{1/4}}$

Question #4: If $y = y_0(x)$ is the solution of the differential equation

$$\sqrt{9-x^2}\frac{dy}{dx} + 3xy = 0$$

$$-3xy \qquad -3xy$$

$$\frac{1}{\sqrt{9-x^2}}\cdot\frac{\sqrt{9-x^2}}{y}\frac{dy}{dx} = \frac{-3xy}{y}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -3x\cdot\frac{1}{\sqrt{9-x^2}}$$

$$\cancel{dx}\ \frac{1}{y}\cdot\frac{dy}{\cancel{dx}} = \frac{-3x}{\sqrt{9-x^2}}\cdot dx$$

$\int\frac{1}{y}dy = \int\frac{-3x}{\sqrt{9-x^2}}dx$ $\quad$ Let $u = 9 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\ln|y| + C = 3\sqrt{9-x^2} + C$ $\quad \frac{1}{\sqrt{u}} \quad \frac{1}{-\frac{1}{2}+1}u^{-\frac{1}{2}+1}$

$-C \qquad -C$

$\ln|y| = 3\sqrt{9-x^2} + C$ $\quad \frac{3}{2}\cdot 2\sqrt{u}$

$e^{\ln|y|} = e^{3\sqrt{9-x^2}}\ \cancel{e}^C$

$y = Ce^{3\sqrt{9-x^2}}$; Let $y = 6$; $x = 3$ $\quad \cancel{3\sqrt{9-(3)^2}}^{0}$

$6 = Ce^{3\sqrt{9-(3)^2}}$

$\boxed{6 = C}$

$y_0 = 6e^{3\sqrt{9-x^2}}$

$y_0(0) = 6e^{3\cancel{\sqrt{9-(0)^2}}^{3}}$

$\boxed{y_0(0) = 6e^9}$

Question #5: Find the amount A in an account after 5 years when:

$$\frac{dA}{dt} = \frac{0.07A}{A},\ A(0) = \$100$$

$$\cancel{dt}\ \frac{1}{A}\cdot\frac{dA}{\cancel{dt}} = 0.07\,dt$$

$$\int\frac{1}{A}\cdot dA = \int 0.07\,dt$$

$\ln|A| + C = 0.07t + (+)C$

$-C \qquad -C$

$\ln|A| = 0.07t + (+)C$

$e^{\ln|A|} = e^{0.07t + (+)C}$

$A = e^{0.07t}\ \cancel{e}^C$

$A = Ce^{0.07t}$

$100 = Ce^{0.07t}$; Let $t = 0$ and $A = 100$

$100 = C\cancel{e^{0.07(0)}}^{1}$

$100 = C$

$A_0 = 100e^{0.07t}$; Let $t = 5$

$A(5) = 100e^{0.07(5)}$

$\boxed{A(5) = 100e^{0.35}}$