This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 (part 1 of 3) 10.0 points

For the differential equation

$$\frac{dy}{dx} + 2y = 8e^{3\,x},$$

(i) first find its general solution.

1. $y = \frac{8}{5}e^{-3x} + Ce^{-2x}$
2. $y = \frac{1}{5}e^{3x} + Ce^{-2x}$
3. $y = \frac{8}{5}e^{-3x} + Ce^{2x}$
4. $y = \frac{8}{5}e^{3x} + Ce^{2x}$
5. $y = \frac{8}{5}e^{3x} + C\,e^{-2x}$ **correct**

Explanation:

The integrating factor for the first order differential equation

$$\frac{dy}{dx} + 2y = 8e^{3\,x}$$

is

$$\mu = e^{\int 2\,dx} = e^{2x}.$$

Thus after multiplying both sides by e^{2x} we can rewrite it as

$$\frac{d}{dx}\Big(y\,e^{2x}\Big) = 8\,e^{5x}.$$

Consequently, its general solution is given by

$$y\,e^{2x} = \int e^{5x}\,dx = \frac{8}{5}e^{5x} + C$$

where C is an arbitrary constant, so

$$\boxed{y = \frac{8}{5}e^{3x} + C\,e^{-2x}}$$

with C an arbitrary constant.

002 (part 2 of 3) 10.0 points

(ii) Then find the particular solution y_0 such that $y_0(0) = 8$.

1. $y_0 = \frac{8}{5}e^{3x} + \frac{32}{5}e^{-2x}$ **correct**
2. $y_0 = \frac{8}{5}e^{-3x} + \frac{32}{5}e^{2x}$
3. $y_0 = -\frac{8}{5}e^{3x} + \frac{32}{5}e^{-2x}$
4. $y_0 = \frac{8}{5}e^{3x} - \frac{32}{5}e^{-2x}$
5. $y_0 = \frac{1}{5}e^{3x} + \frac{32}{5}e^{-2x}$

Explanation:

For the particular solution y_0 the value of C is determined by the condition $y(0) = 8$ since

$$y(0) = 8 \quad\Longrightarrow\quad 8 = \frac{8}{5} + C.$$

Consequently,

$$\boxed{y_0 = \frac{8}{5}e^{3x} + \frac{32}{5}e^{-2x}.}$$

003 (part 3 of 3) 10.0 points

(iii) For the particular solution y_0 in (ii), determine the value of $y_0(1)$.

1. $\frac{8}{5}e^{3} - \frac{32}{5}e^{-2}$
2. $\frac{1}{5}e^{3} - \frac{32}{5}e^{-2}$
3. $\frac{8}{5}e^{-3} + \frac{32}{5}e^{2}$
4. $\frac{8}{5}e^{3} + \frac{32}{5}e^{-2}$ **correct**
5. $\frac{1}{5}e^{3} + \frac{32}{5}e^{-2}$

Explanation:

At $x = 1$, therefore,

$$\boxed{y_0(1) \;=\; \frac{8}{5}e^3 + \frac{32}{5}e^{-2}.}$$

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004 10.0 points

If y_0 is the solution of the equations

$$xy' + 2y \;=\; 4x, \quad y(1) = 6,$$

determine the value of $y_0(2)$.

1. $y_0(2) \;=\; \dfrac{11}{3}$

2. $y_0(2) \;=\; \dfrac{43}{12}$

3. $y_0(2) \;=\; \dfrac{7}{2}$

4. $y_0(2) \;=\; \dfrac{15}{4}$

5. $y_0(2) \;=\; \dfrac{23}{6}$ **correct**

Explanation:

After dividing we see that the given differential equation becomes

$$(\ddagger) \qquad y' + \frac{2}{x}y \;=\; 4,$$

which is a first order linear equation having integrating factor

$$\mu(x) \;=\; \exp\Big(\int \frac{2}{x}\,dx\Big) \;=\; x^2.$$

With this (‡) becomes

$$x^2y' + 2xy \;=\; 4x^2.$$

Thus

$$x^2y \;=\; \int 4x^2\,dx \;=\; \frac{4x^3}{3} + C,$$

where C is an arbitrary constant. For y_0 the value of C is determined by the condition $y(1) = 6$. Consequently,

$$y_0(x) \;=\; \frac{1}{3}\Big(4x + \frac{6(3)-4}{x^2}\Big).$$

At $x = 2$, therefore,

$$\boxed{y_0(2) \;=\; \frac{23}{6}.}$$

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005 10.0 points

If y_1 is the particular solution of the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} \;=\; 6x^2 \; - 6$$

which satisfies $y(1) = 6$, determine the value of $y_1(2)$.

1. $y_1(2) \;=\; 35$

2. $y_1(2) \;=\; 34$

3. $y_1(2) \;=\; 32$

4. $y_1(2) \;=\; 33$

5. $y_1(2) \;=\; 36$ **correct**

Explanation:

The integrating factor needed for the first order differential equation (‡) is

$$\mu = e^{-\int \frac{2}{x}\,dx} = e^{-2\ln x} = \frac{1}{x^2}.$$

After multiplying both sides of (‡) by $1/x^2$ we can thus rewrite the equation as

$$\frac{d}{dx}\Big(\frac{y}{x^2}\Big) = 6 - \frac{6}{x^2}.$$

Consequently, the general solution of (‡) is given by

$$\frac{y}{x^2} = 6x + \frac{6}{x} + C$$

where C is an arbitrary constant. For the particular solution y_1 the value of C is determined by the condition $y(1) = 6$ since

$$y(1) = 6 \quad \Longrightarrow \quad 6 = 12 + C,$$

and so

$$y_1(x) = x^2\Big(6x + \frac{6}{x} - 6\Big).$$

At $x = 2$, therefore,

$$\boxed{y_1(2) = 36.}$$

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006 10.0 points

Solve the differential equation $y' + 2y = 2e^x$.

1. $y = \frac{2}{3}e^{-x} + Ce^{-2x}$

2. $y = \frac{2}{3}e^{x} + Ce^{-2x}$ **correct**

3. $y = -\frac{2}{3}e^{x} + Ce^{2x}$

4. $y = -\frac{2}{3}e^{x} + Ce^{-2x}$

5. $y = \frac{2}{3}e^{x} + Ce^{2x}$

Explanation:

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007 10.0 points

Solve the differential equation

$$(5 + t)\frac{du}{dt} + u = 5 + t, \quad t > 0\,.$$

1. $u = \dfrac{t^2 + 5t}{2(t+5)} + C$

2. $u = \dfrac{t^2 + 10t + 2C}{2(t+5)}$ **correct**

3. $u = \dfrac{t^2 + 5t + 2C}{2(t+5)}$

4. $u = \dfrac{t^2 + 5t + C}{t+5}$

5. $u = \dfrac{t^2 + 5t}{t+5} + C$

Explanation:

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008 10.0 points

Solve the initial-value problem

$$t\frac{dy}{dt} + 2y = t^5, \quad t > 0, y\,(1) = 0\,.$$

1. $y = \dfrac{t^6}{7} - \dfrac{1}{7t^2}$

2. $y = \dfrac{t^5}{7} - \dfrac{1}{7t^2}$ **correct**

3. $y = \dfrac{t^5}{7} - \dfrac{1}{7t^3}$

4. $y = \dfrac{t^5}{7} + \dfrac{1}{7t^2}$

5. $y = \dfrac{t^5}{7} - \dfrac{1}{7t^4}$

Explanation: