This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Determine A so that the curve

$$y = 3x + 5$$

can be written in parametric form as

$$x(t) = t - 2, \quad y(t) = At - 1.$$

1. $A = 4$

2. $A = 5$

3. $A = -4$

4. $A = -3$

5. $A = 3$ **correct**

6. $A = -5$

Explanation:

We have to eliminate t from the parametric equations for x and y. Now from the equation for x it follows that $t = x + 2$. Thus

$$y = 3x + 5 = A(x + 2) - 1.$$

Consequently

$$\boxed{A = 3}.$$

002 10.0 points

Determine a Cartesian equation for the curve given in parametric form by

$$x(t) = 4\ln(4t), \quad y(t) = \sqrt{t}.$$

1. $y = \frac{1}{2}e^{x/8}$ **correct**

2. $y = \frac{1}{4}e^{x/4}$

3. $y = \frac{1}{2}e^{x/4}$

4. $y = \frac{1}{4}e^{4/x}$

5. $y = \frac{1}{2}e^{8/x}$

6. $y = \frac{1}{4}e^{x/2}$

Explanation:

We have to eliminate the parameter t from the equations for x and y. Now from the equation for x it follows that

$$t = \frac{1}{4}e^{x/4}.$$

But then

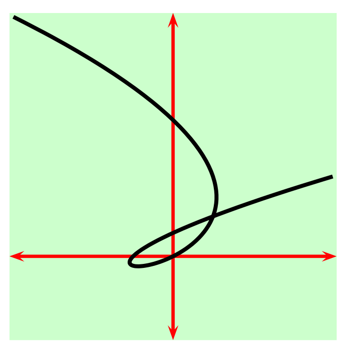
$$\boxed{y = \left(\frac{1}{4}e^{x/4}\right)^{1/2} = \frac{1}{2}e^{x/8}}.$$

003 10.0 points

Which one of the following could be the graph of the curve given parametrically by

$$x(t) = t + \sin 2t, \quad y(t) = t + \sin 3t\,?$$

1.



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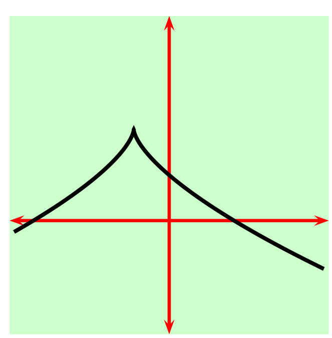

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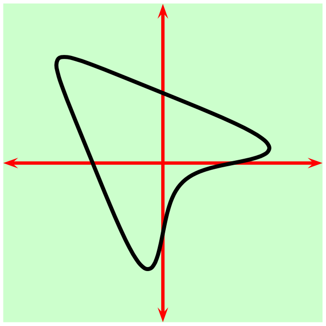

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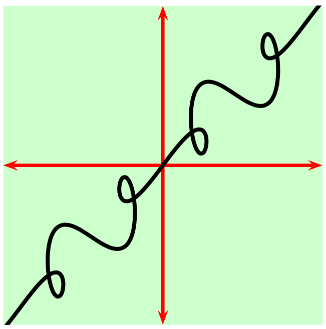
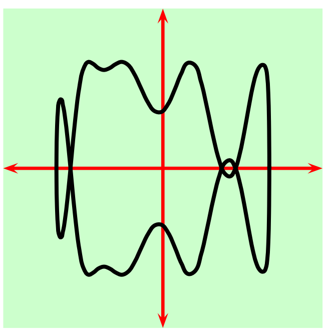


correct

5.



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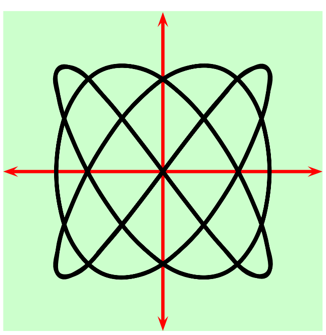


Explanation:

These examples illustrate the diversity of curves in 2-space. But simple properties such as

(i) behaviour as $t \to \infty$,

(ii) x and y-intercepts,

(iii) passing through the origin or not,

(iv) symmetry (even or oddness),

can often be used to determine which graph goes with which function. For instance, three of the graphs above lie within a square centered at the origin, suggesting that the other three are unbounded; on the other hand, only three pass through the origin; and some, but not all, have various symmetries such as evenness, oddness, or symmetry about a line.

In the case of the curve given parametrically by

$$x(t) = t + \sin 2t, \quad y(t) = t + \sin 3t$$

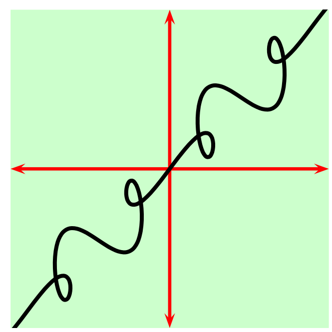
we see that

$$\lim_{t \to \infty} x(t) = \lim_{t \to \infty} y(t) = \infty,$$

while $x(0) = y(0) = 0$. But only two of the graphs have these properties. On the other hand,

$$x(-t) = -x(t), \qquad y(-t) = -y(t),$$

i.e., the function is ODD. Thus the graph will be symmetric about the origin, leaving



as the only possible graph.

keywords: 2D-graph, parametric function, limit at infinity, symmetry, periodicity

004 10.0 points

Determine a Cartesian equation for the curve given in parametric form by

$$x(t) = 4e^t, \quad y(t) = 3e^{-2t}.$$

1. $xy^2 = 12$

2. $\dfrac{x}{y^2} = 12$

3. $\dfrac{x^2}{y} = 36$

4. $x^2 y = 48$ **correct**

5. $\dfrac{x^2}{y} = 48$

6. $x^2 y = 36$

Explanation:

We have to eliminate the parameter t from the equations for x and y. Now from the equation for x it follows that

$$e^t = \frac{x}{4},$$

from which in turn it follows that

$$y = 3\left(\frac{4}{x}\right)^2.$$

Consequently,

$$\boxed{x^2 y = 48}.$$

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005 10.0 points

Find a Cartesian equation for the curve given in parametric form by

$$x(t) = 2\cos 4t, \quad y(t) = 5\sin 4t.$$

1. $25x^2 - 4y^2 = 100$

2. $4x^2 + 25y^2 = 100$

3. $\dfrac{x^2}{25} - \dfrac{y^2}{4} = \dfrac{1}{100}$

4. $\dfrac{x^2}{4} - \dfrac{y^2}{25} = \dfrac{1}{100}$

5. $25x^2 + 4y^2 = 100$ **correct**

6. $\dfrac{x^2}{25} + \dfrac{y^2}{4} = \dfrac{1}{100}$

Explanation:

We have to eliminate the parameter t from the equations for x and y. Now

$$\cos^2\theta + \sin^2\theta = 1.$$

Thus

$$\frac{x^2}{4} + \frac{y^2}{25} = 1.$$

But then after simplification, the curve has Cartesian form

$$\boxed{25x^2 + 4y^2 = 100}.$$

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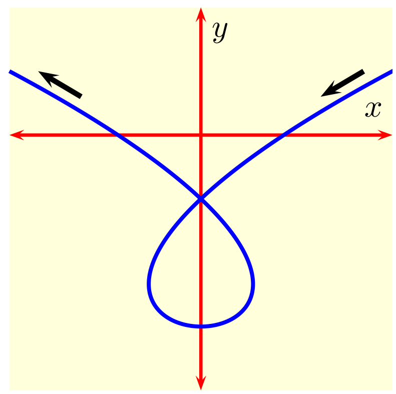
006 10.0 points

Which one of the following could be the graph of the curve given parametrically by

$$x(t) = t^2 - 3, \qquad y(t) = t^3 - 2t,$$

where the arrows indicate the direction of increasing t?

1.



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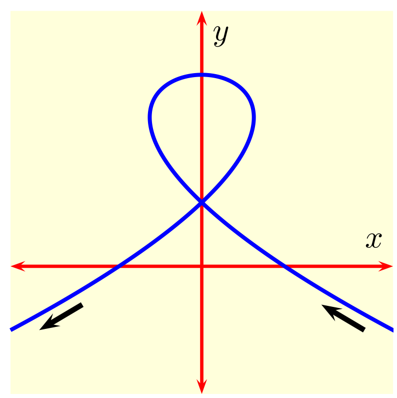

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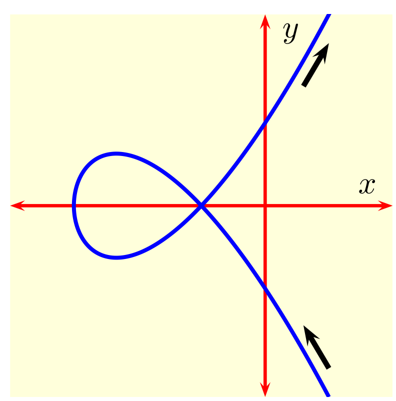
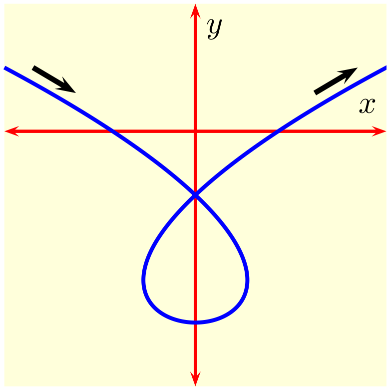


correct

4.



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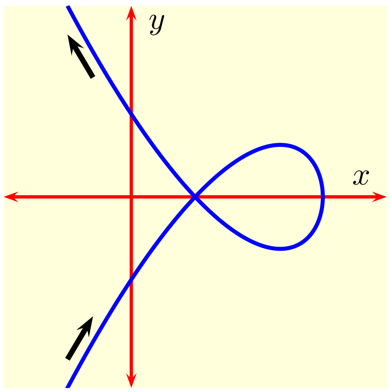

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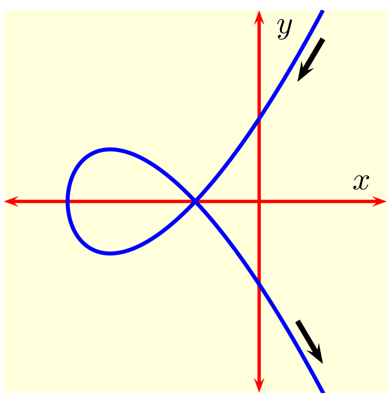


Explanation:

All the graphs are symmetric either about the y-axis or the x-axis. Let's check which it is for the graph of

$$(x(t),\, y(t)) \;=\; (t^2-3,\; t^3-2t)\,.$$

Now

$$x(-t) = (-t)^2 - 3 = t^2 - 3 = x(t)\,,$$

and

$$y(-t) = (-t)^3 - 2(-t) = -(t^3-2t) = -y(t)\,,$$

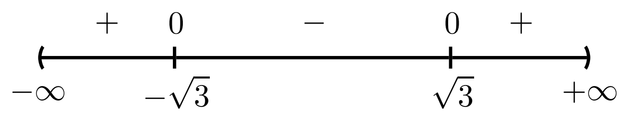
so

$$(x(-t),\, y(-t)) \;=\; (x(t),\, -y(t))\,.$$

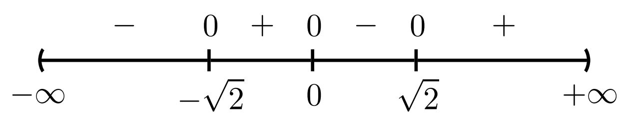
Thus the graph is symmetric about the x-axis, eliminating three choices.

To decide which one of the remaining three it is, we can check the path traced out as t ranges from $-\infty$ to $+\infty$ by looking at sign charts for $x(t)$ and $y(t)$ because this will tell us in which quadrant the graph lies:

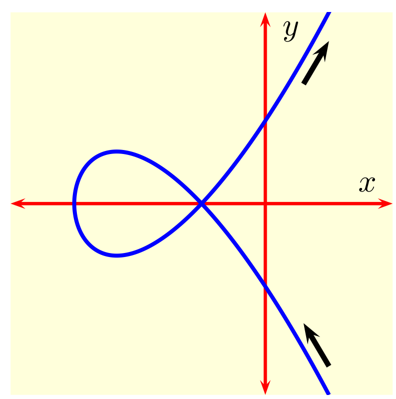
1. $x(t) = t^2 - 3$:



2. $y(t) = t(t^2 - 2)$:



Thus the graph starts in quadrant I, crosses the y-axis into quadrant II, then crosses the x-axis into quadrant IV, crosses back into quadrant II and so on. Consequently, the graph is



keywords: parametric curve, graph, direction,