This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find $\frac{dy}{dx}$ when

$$x(t) = 4te^t, \quad y(t) = t - e^t.$$

1. $\frac{dy}{dx} = \frac{1+e^t}{4e^t(1-t)}$
2. $\frac{dy}{dx} = \frac{4e^t(1+t)}{1-e^t}$
3. $\frac{dy}{dx} = \frac{1-e^t}{4e^t(1+t)}$ **correct**
4. $\frac{dy}{dx} = \frac{1-e^t}{4e^t(1-t)}$
5. $\frac{dy}{dx} = \frac{4e^t(1-t)}{1+e^t}$
6. $\frac{dy}{dx} = \frac{4e^t(1+t)}{1+e^t}$

Explanation:

Differentiating with respect to t we see that

$$x'(t) = 4(e^t + te^t), \quad y'(t) = 1 - e^t.$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{1-e^t}{4e^t(1+t)}}.$$

002 10.0 points

Find $\frac{dy}{dx}$ when

$$x(t) = t\ln t, \quad y(t) = \sin^4 t.$$

1. $\frac{dy}{dx} = \frac{1+\ln t}{3\sin^3 t\cos t}$
2. $\frac{dy}{dx} = \frac{4\sin^3 t\cos t}{1+\ln t}$ **correct**
3. $\frac{dy}{dx} = \frac{3\sin^3 t\cos t}{1+\ln t}$
4. $\frac{dy}{dx} = \frac{4\cos^3 t\sin t}{1+\ln t}$
5. $\frac{dy}{dx} = \frac{1+\ln t}{4\sin^3 t\cos t}$
6. $\frac{dy}{dx} = \frac{1+\ln t}{4\cos^3 t\sin t}$
7. $\frac{dy}{dx} = \frac{1+\ln t}{3\cos^3 t\sin t}$
8. $\frac{dy}{dx} = \frac{3\cos^3 t\sin t}{1+\ln t}$

Explanation:

Differentiating with respect to t we see that

$$x'(t) = \frac{t}{t} + \ln t, \quad y'(t) = 4\sin^3 t\cos t.$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{4\sin^3 t\cos t}{1+\ln t}}.$$

003 10.0 points

Find $\frac{d^2y}{dx^2}$ for the curve given parametrically by

$$x(t) = 1 + 2t^2, \qquad y(t) = 2t^2 + t^3.$$

1. $\frac{d^2y}{dx^2} = \frac{3t}{16}$
2. $\frac{d^2y}{dx^2} = \frac{8t}{3}$
3. $\frac{d^2y}{dx^2} = \frac{3}{16t}$ **correct**
4. $\frac{d^2y}{dx^2} = \frac{3}{4t}$
5. $\frac{d^2y}{dx^2} = \frac{10}{3t}$

6. $\dfrac{d^2y}{dx^2} = \dfrac{10t}{3}$

Explanation:
Differentiating with respect to t we see that

$$x'(t) = 4t\,, \qquad y'(t) = 4t + 3t^2\,.$$

Thus

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{4t+3t^2}{4t} = 1 + \frac{3}{4}t\,.$$

On the other hand, by the Chain Rule,

$$\frac{d}{dt}\Big(\frac{dy}{dx}\Big) = \frac{dx}{dt}\Big\{\frac{d}{dx}\Big(\frac{dy}{dx}\Big)\Big\} = \Big(\frac{dx}{dt}\Big)\frac{d^2y}{dx^2}\,.$$

Consequently,

$$\boxed{\frac{d^2y}{dx^2} = \frac{d}{dt}\Big(\frac{dy}{dx}\Big)\Big/\frac{dx}{dt} = \frac{3}{16t}}\,.$$

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004 10.0 points

Find an equation for the tangent line to the curve given parametrically by

$$x(t) = e^{2t}, \qquad y(t) = 2t^2 + 4t - 4$$

at the point $P(1, -4)$.

1. $y = 2x - 6$ **correct**

2. $y = -2x - 2$

3. $y = 4x - 6$

4. $y = 4x - 2$

5. $y = 2x - 2$

6. $y = -2x - 6$

Explanation:
Notice first that $P(1, -4)$ is the point corresponding to the choice $t = 0$. We can thus use the point slope formula with $t = 0$ to find an equation for the tangent line at P.

Now by the Chain Rule and Product Rule,

$$x'(t) = 2e^{2t}\,, \qquad y'(t) = 4t + 4\,,$$

so

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{2t+2}{e^{2t}}\,.$$

The tangent line at $P(1, -4)$, *i.e.*, when $t = 0$, thus has

$$\text{slope} = 2\,,$$

and by the point slope formula an equation for the tangent line at $P(1, -4)$ is

$$y + 4 = 2(x - 1)\,.$$

After simplification this becomes

$$\boxed{y = 2x - 6}\,.$$

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keywords:

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005 10.0 points

Determine all values of t for which the curve given parametrically by

$$x = t^3 - 3t^2 + 2t\,, \quad y = 3t^3 + t^2 - 2$$

has a horizontal tangent?

1. $t = -2$

2. $t = 0\,, \quad \dfrac{2}{9}$

3. $t = 0\,, \quad 2$

4. $t = -\dfrac{2}{9}$

5. $t = 0\,, \quad -\dfrac{2}{9}$ **correct**

6. $t = 2$

Explanation:
After differentiation with respect to t we see that

$$y'(t) = 9t^2 + 2t\,, \quad x'(t) = 3t^2 - 6t + 2\,.$$

Now

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{9t^2 + 2t}{3t^2 - 6t + 2},$$

so the tangent line to the curve will be horizontal at the solutions of

$$y'(t) = t(9t + 2) = 0,$$

hence at

$$\boxed{t = 0, \quad -\frac{2}{9}}.$$

006 10.0 points

Find $\frac{d^2y}{dx^2}$ when

$$x(t) = \sin 3\pi t, \quad y(t) = \cos 3\pi t.$$

1. $\frac{d^2y}{dx^2} = 3\pi \sec^2 3\pi t$
2. $\frac{d^2y}{dx^2} = -3 \sec^2 3\pi t$
3. $\frac{d^2y}{dx^2} = -3\pi \sec^3 3\pi t$
4. $\frac{d^2y}{dx^2} = -\sec^3 3\pi t$ **correct**
5. $\frac{d^2y}{dx^2} = \sec^3 3\pi t$
6. $\frac{d^2y}{dx^2} = 3 \sec^2 3\pi t$

Explanation:

Differentiating with respect to t we see that

$$x'(t) = 3\pi \cos 3\pi t, \quad y'(t) = -3\pi \sin 3\pi t.$$

Thus

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = -\tan 3\pi t.$$

But then

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\Big(\frac{dy}{dx}\Big)\Big/\frac{dx}{dt} = -\frac{3\pi \sec^2 3\pi t}{3\pi \cos 3\pi t}.$$

Consequently,

$$\boxed{\frac{d^2y}{dx^2} = -\sec^3 3\pi t}.$$

keywords: derivative, second derivative, parametric curve, trig functions,

007 10.0 points

Which one of the following integrals gives the length of the parametric curve

$$x(t) = t^2, \quad y(t) = 2t, \quad 0 \le t \le 4.$$

1. $I = \int_0^4 |t^2 + 1|\, dt$
2. $I = 2\int_0^4 \sqrt{t^2 + 1}\, dt$ **correct**
3. $I = \int_0^4 \sqrt{t^2 + 1}\, dt$
4. $I = \int_0^2 |t^2 + 1|\, dt$
5. $I = 2\int_0^2 |t^2 + 1|\, dt$
6. $I = 2\int_0^2 \sqrt{t^2 + 1}\, dt$

Explanation:

The arc length of the parametric curve

$$(x(t), y(t)), \qquad a \le t \le b$$

is given by the integral

$$I = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt.$$

But when

$$x(t) = t^2, \quad y = 2t,$$

we see that

$$x'(t) = 2t, \qquad y'(t) = 2.$$

Consequently, the curve has

$$\boxed{\text{arc length} = 2\int_0^4 \sqrt{t^2 + 1}\, dt}.$$

008 10.0 points

Find the length of the curve defined by

$$x(t) = \frac{1}{3}(2t+3)^{3/2}$$

$$y(t) = t + \frac{t^2}{2}$$

for $0 \le t \le 1$.

1. $\frac{5}{2}$ **correct**
2. $\frac{5}{3}$
3. $\frac{2}{5}$
4. $\frac{3}{2}$
5. $\frac{2}{3}$

Explanation:

$$x'(t) = \sqrt{2t+3}$$

$$h'(t) = 1 + t$$

Substuting these into the formula for length of a curve we have:

$$L = \int_0^1 \sqrt{(\sqrt{2t+3})^2 + (1+t)^2}\ dt$$

Which simplifies to:

$$L = \int_0^1 \sqrt{t^2+4t+4}\ dt$$

$$L = \int_0^1 \sqrt{(t+2)^2}\ dt$$

$$L = \int_0^1 (t+2)\ dt = \frac{5}{2}$$