Q_1: $x^2+y^2=r^2$

$x = r\cos\theta$

$y = r\sin\theta$

$x = (e^\theta - 8)\cos\theta$

$y = (e^\theta - 8)\sin\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}(e^\theta-8)\sin\theta}{\frac{d}{d\theta}(e^\theta-8)\cos\theta}$

$f'g+g'f$

$\left.\frac{e^\theta\sin\theta+\cos\theta(e^\theta-8)}{e^\theta\cos\theta-\sin\theta(e^\theta-8)}\right|_{\theta=\frac{\pi}{4}} = \frac{e^{\frac{\pi}{4}}\sin(\frac{\pi}{4})+\cos(\frac{\pi}{4})(e^{\frac{\pi}{4}}-8)}{e^{\frac{\pi}{4}}\cos(\frac{\pi}{4})-\sin(\frac{\pi}{4})(e^{\pi/4}-8)} = \frac{\frac{\sqrt2}{2}(e^{\frac{\pi}{4}}+(e^{\frac{\pi}{4}}-8))}{\frac{\sqrt2}{2}(e^{\frac{\pi}{4}}-(e^{\pi/4}-8))} = \frac{e^{\frac{\pi}{4}}+e^{\frac{\pi}{4}}-8}{\cancel{e^{\frac{\pi}{4}}}-\cancel{e^{\pi/4}}+8} = \frac{2e^{\pi/4}-8}{8} = \frac{2e^{\pi/4}}{8}-\frac{8}{8}$

$\frac{1}{4}e^{\pi/4}-1$

Q_2: $x=(3+\sin\theta)\cos\theta$

$y=(3+\sin\theta)\sin\theta$

$f'g+g'f$

$\frac{\cos\theta\cos\theta-\sin\theta(3+\sin\theta)}{\cos\theta\sin\theta+\cos\theta(3+\sin\theta)}$

$\frac{\cos(\frac{\pi}{6})\cos(\frac{\pi}{6})-\sin(\frac{\pi}{6})(3+\sin(\frac{\pi}{6}))}{\cos(\frac{\pi}{6})\sin(\frac{\pi}{6})+\cos(\frac{\pi}{6})(3+\sin(\frac{\pi}{6}))}$

Flip, since you forgot to flip your equations.

$-\frac{3}{4}$

$\frac{8}{4}$ $\quad \frac{-6}{4}$ $\quad -\frac{3\cdot2}{4}$ $\quad \frac{2}{4}=\frac{1}{2}$

$\frac{\frac34-\frac12(3+\frac12)}{\frac{\sqrt3}{4}+\frac{\sqrt3}{2}(3+\frac12)} = \frac{\frac34-\frac32-\frac14}{\frac{\sqrt3}{4}+\frac{3\sqrt3}{2}+\frac{\sqrt3}{4}} = \frac{-1}{\frac{2\sqrt3}{4}+\frac{6\sqrt3}{4}} = \frac{-1}{\frac{8\sqrt3}{4}} = \frac{-1}{2\sqrt3}$

Q_3: $x=r\cos\theta$

$y=r\sin\theta$ where $r=4\cos\theta-3\sin\theta$

$\left(\frac{4\sqrt2}{2}-3\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2}$

$f'g+g'f$

$x=(4\cos\theta-3\sin\theta)\cos\theta \xrightarrow{x'} (-4\sin\theta-3\cos\theta)\cos\theta-\sin\theta(4\cos\theta-3\sin\theta)$

$y=(4\cos\theta-3\sin\theta)\sin\theta \xrightarrow{y'} (-4\sin\theta-3\cos\theta)\sin\theta+\cos\theta(4\cos\theta-3\sin\theta)$

$\Big|_{\theta=\frac{\pi}{4}} \quad \frac{\left(-\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)\frac{\sqrt2}{2}-\frac{\sqrt2}{2}\left(\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)}{\left(-\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)\frac{\sqrt2}{2}+\frac{\sqrt2}{2}\left(\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)} = \frac{-7-1}{-7+1} = \frac{-8}{-6} \stackrel{12}{=} \frac{4}{3}$

$\left(\frac{-7\sqrt2}{2}\right)$ $\quad \left(\frac{\sqrt2}{2}\right)$

$\frac{-14}{4}-\frac{2}{4}=\frac{-16}{4}=-4$

$\frac{-14}{4}+\frac{2}{4}=\frac{-12}{4}=-3$

$x=\left(\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)\frac{\sqrt2}{2}=\frac12$

$y=\left(\frac{4\sqrt2}{2}-\frac{3\sqrt2}{2}\right)\frac{\sqrt2}{2}=\frac12$

$\frac{\sqrt2}{2}$

$y-y_1=\frac34(x-x_1)$

$y-\frac12=\frac34(x-\frac12)$

$y-\cancel{\frac12}=\frac43x-\frac46+\frac12 \quad \frac36$

$+\frac12$

$\frac34x-\frac38+\frac48$

Q_4: Find the y-intercept of the tangent line to the graph of

$r=3e^{-\theta}-4$

at the point P corresponding to $\theta=0$

$f'g+g'f$

$y=r\sin\theta \longrightarrow (3e^{-\theta}-4)\sin\theta$ $[y']$: $\cancel{-3e^{-\theta}\sin\theta}+\cos\theta(3e^{-\theta}-4)$

$x=r\cos\theta \longrightarrow (3e^{-\theta}-4)\cos\theta$ $[x']$: $-3e^{-\theta}\cos\theta-\cancel{\sin\theta(3e^{-\theta}-4)}$

$= \frac{-1}{-3} = \frac{1}{3}$

$y-y_1=\frac13(x-x_1)$

$y=\frac13(x+1)$

$\frac13x+\boxed{\frac13}$ ✓

$y=\cancel{(3e^{-\theta}-4)\sin\theta}\ 0$

$x=(3e^{-\theta}-4)\cos\theta=-1$

Q_5: Find the length of the curve defined by:

$r=\sin\theta$ for $0\le\theta\le\pi$.

$f'g+g'f$

$x=(\sin\theta)\cos\theta$ $[x']$: $\cos^2\theta-\sin^2\theta \stackrel{T}{=} \cos2\theta$

$y=\sin\theta\sin\theta$ $[y']$: $\cos\theta\sin\theta+\cos\theta\sin\theta$

$\cos\theta\sin\theta(1+1)$

$2\cos\theta\sin\theta$

$\sin2\theta$

$\int_0^\pi \sqrt{(\cos2\theta)^2+(\sin2\theta)^2}\,d\theta$

Let $u=2\theta$

$du=2\,d\theta$

$\frac12\int\sqrt{\cos^2(u)+\sin^2(u)}\,du$

$\sqrt1$

$\frac12\int1\,du$

$\frac12u \to \frac12\cdot2\theta = \theta\Big|_0^\pi \to \pi-0=\boxed{\pi}$

Q_6: $r=1+2\cos\theta \to r^2$:

$r'=-2\sin\theta \to [(r')^2]$: $4\sin^2\theta$

$1+2\cos\theta$

$1 \quad 1 \quad 2\cos\theta$

$2\cos\theta \quad 2\cos\theta \quad 4\cos^2\theta$

$1+4\cos\theta+4\cos^2\theta$

$\int\sqrt{1+4\cos\theta+\underbrace{4\cos^2\theta+4\sin^2\theta}_{4}}\,d\theta$

$\sqrt{1+4+4\cos\theta}\,d\theta \to \sqrt{5+4\cos\theta}\,d\theta$

$\int\sqrt{5+4\cos\theta}\,d\theta$

Q_7: $r=1-\cos\theta$ $[r^2]$: $1-2\cos\theta+\cos^2\theta$

$r'=\sin\theta \to [(r')^2]$: $\sin^2\theta$

$1-\cos\theta$

$1 \quad 1 \quad -\cos\theta$

$\frac{\pi}{2}$ to π $\quad -\cos\theta$

$-\cos\theta \quad \cos^2\theta$

$\int\sqrt{1-2\cos\theta+\cos^2\theta+\sin^2\theta}\,d\theta$

$\int\sqrt{2-2\cos\theta}\,d\theta$

$\int\sqrt{2(1-\cos\theta)}\,d\theta$

$\sqrt2\int_{\frac{\pi}{2}}^{\pi}\sqrt{1-\cos\theta}\,d\theta$

$2\sin^2\left(\frac{\theta}{2}\right)$

$\sqrt2\int_{\frac{\pi}{2}}^{\pi}\sqrt{2\sin^2\left(\frac{\theta}{2}\right)}$

$2\int_0^\pi\sin\left(\frac{\theta}{2}\right)$

$\frac{f'g-g'f}{g^2}=\frac24=\frac12$

Let $u=\frac{\theta}{2}$ $\quad f \quad g$

$2\,du=\cancel{\frac12}\,d\theta$ ✗

$4\int\sin(u)$

$4(-\cos(u))$: $-4\cos\left(\frac{\theta}{2}\right)\Big|_0^\pi$

$\boxed{I=4}$

$-4(0-(1))$

$-4(-1)$

$\boxed{I=4}$ ✓