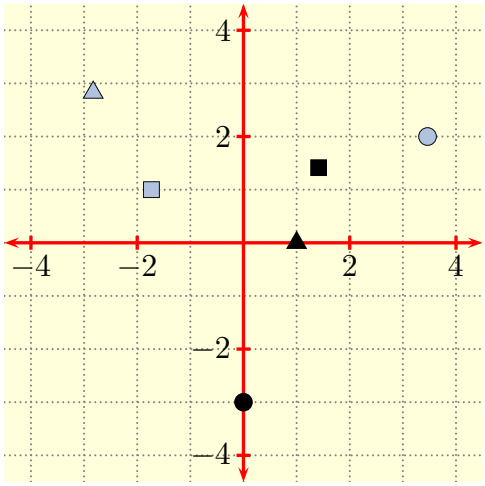
This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Locate the points given in polar coordinates by

$$P\left(4,\, \frac{3}{4}\pi\right), \quad Q\left(-3,\, \frac{1}{2}\pi\right) \quad R\left(4,\, \frac{1}{6}\pi\right),$$

among



1. P : ● Q : ○ R : △

2. P : ● Q : △ R : ○

3. P : △ Q : ○ R : ●

4. P : ○ Q : ● R : △

5. P : △ Q : ● R : ○ **correct**

6. P : ○ Q : △ R : ●

Explanation:

To convert from polar coordinates to Cartesian coordinates we use

$$x = r\cos\theta\,, \qquad y = r\sin\theta\,.$$

For then the points

$$P\left(4,\, \frac{3}{4}\pi\right), \quad Q\left(-3,\, \frac{1}{2}\pi\right) \quad R\left(4,\, \frac{1}{6}\pi\right),$$

correspond to

$$P : \triangle \qquad Q : \bullet \qquad R : \circ$$

in Cartesian coordinates.

keywords: polar coordinates, Cartesian coordinates, change of coordinates,

002 10.0 points

Which, if any, of

A. $(4,\, 7\pi/3)\,,$

B. $(4,\, \pi/3)\,,$

C. $(-4,\, 7\pi/6)\,,$

are polar coordinates for the point given in Cartesian coordinates by $P(2,\, 2\sqrt{3})$?

1. A and C only

2. C only

3. B only

4. none of them

5. A only

6. B and C only

7. all of them

8. A and B only **correct**

Explanation:

To convert from Cartesian coordinates to polar coordinates we use the relations:

$$x = r\cos\theta\,, \qquad y = r\sin\theta\,,$$

so that

$$r^2 = x^2 + y^2\,, \quad \tan\theta = \frac{y}{x}\,.$$

For the point $P(2,\, 2\sqrt{3})$ in Cartesian coordinates, therefore, one choice of r and θ is

$r = 4$ and $\theta = \pi/3$, but there are equivalent solutions for $r < 0$ as well as values of θ differing by integer multiples of π. For the given choices we thus see that

A. TRUE: differs from $\pi/3$ by 2π.

B. TRUE: solution noted already.

C. FALSE: θ incorrect.

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003 10.0 points

Find the Cartesian coordinates, (a, b), of the point given in polar coordinates by $P(2, \pi/3)$.

1. $(a, b) = (-1, \sqrt{3})$

2. $(a, b) = (2, 2\sqrt{3})$

3. $(a, b) = (\sqrt{3}, -1)$

4. $(a, b) = (1, -2)$

5. $(a, b) = (1, \sqrt{3})$ **correct**

6. $(a, b) = (\sqrt{3}, 1)$

7. $(a, b) = (-2, \sqrt{3})$

8. $(a, b) = (2\sqrt{3}, 2)$

Explanation:

Since the relationship between Cartesian coordinates and polar coordinates is

$$x = r\cos\theta\,, \qquad y = r\sin\theta\,,$$

the point $P(2, \pi/3)$ is given in Cartesian coordinates by

$$\boxed{P(2,\ \pi/3) = \Big(2\cos\frac{\pi}{3},\ 2\sin\frac{\pi}{3}\Big) = (1,\ \sqrt{3})}\,.$$

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keywords: polar coordinates, Cartesian coordinates

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004 10.0 points

Find a polar equation for the curve given by the Cartesian equation

$$3y^2 = x\,.$$

1. $r = 3\csc\theta\cot\theta$

2. $3r = \csc\theta\cot\theta$ **correct**

3. $3r = \sec\theta\cot\theta$

4. $3r = \sec\theta\tan\theta$

5. $r = 3\sec\theta\tan\theta$

6. $r = 3\csc\theta\tan\theta$

Explanation:

We have to substitute for x, y in

$$3y^2 = x$$

using the relations

$$x = r\cos\theta\,, \quad y = r\sin\theta\,.$$

In this case the Cartesian equation becomes

$$3r^2\sin^2\theta = r\cos\theta\,.$$

Consequently, the polar form of the equation is

$$\boxed{3r = \csc\theta\cot\theta}\,.$$

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005 10.0 points

Find a Cartesian equation for the curve given by the polar equation

$$r + 6\cos\theta = 0\,.$$

1. $(x-3)^2 + y^2 = 9$

2. $(x-3)^2 + y^2 + 9 = 0$

3. $x^2 + (y+3)^2 = 9$

4. $(x+3)^2 + y^2 + 9 = 0$

5. $(x+3)^2 + y^2 = 9$ **correct**

6. $x^2 + (y-3)^2 \;=\; 9$

7. $x^2 + (y-3)^2 + 9 \;=\; 0$

8. $x^2 + (y+3)^2 + 9 \;=\; 0$

Explanation:
We have to replace r and θ in the polar equation

$$r + 6\cos\theta \;=\; 0$$

using the relations

$$x \;=\; r\cos\theta\,, \qquad y \;=\; r\sin\theta\,.$$

As a first simplification, notice that

$$r^2 + 6r\cos\theta \;=\; 0\,.$$

But then

$$x^2 + y^2 + 6x \;=\; r^2 + 6r\cos\theta \;=\; 0\,.$$

Consequently, by completing the square we get the Cartesian equation

$$\boxed{(x+3)^2 + y^2 \;=\; 9}\,.$$

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006 10.0 points

Find a polar representation for the curve whose Cartesian equation is

$$(x+1)^2 + y^2 \;=\; 1\,.$$

1. $r \;=\; \sin\theta$

2. $r + 2\sin\theta \;=\; 0$

3. $r \;=\; \cos\theta$

4. $r \;=\; 2\cos\theta$

5. $r + 1\sin\theta \;=\; 0$

6. $r + 2\cos\theta \;=\; 0$ **correct**

7. $r \;=\; 2\sin\theta$

8. $r + 1\cos\theta \;=\; 0$

Explanation:
We have to substitute for x, y in

$$(x+1)^2 + y^2 \;=\; 1$$

using the relations

$$x \;=\; r\cos\theta\,, \qquad y \;=\; r\sin\theta\,.$$

But after expansion the Cartesian equation becomes

$$x^2 + 2x + 1 + y^2 \;=\; 1\,.$$

Now $x^2 + y^2 = r^2$, so

$$r^2 + 2r\cos\theta \;=\; 0\,,$$

which after cancellation gives the polar representation

$$\boxed{r + 2\cos\theta \;=\; 0}\,.$$

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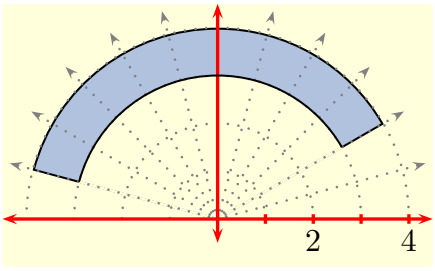
007 10.0 points

Which one of the following shaded-regions in the plane consists of all points whose polar coordinates satisfy the inequalities

$$0 \;\le\; r \;<\; 3\,, \qquad \frac{1}{12}\pi \;\le\; \theta \;\le\; \frac{5}{6}\pi\,?$$

1.



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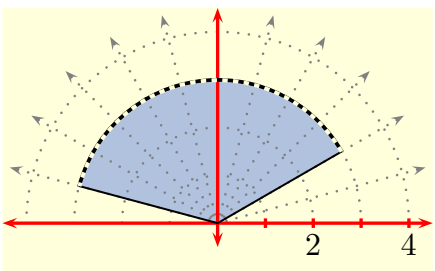
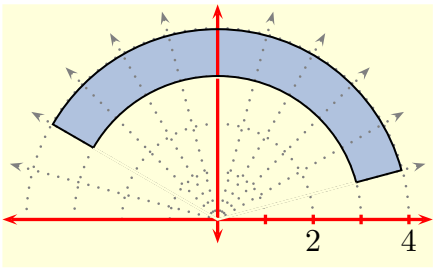


3.



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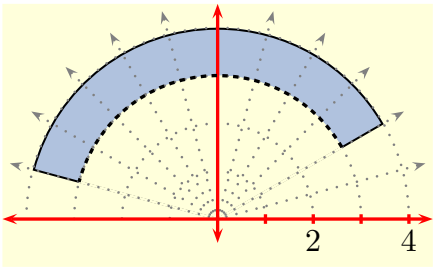
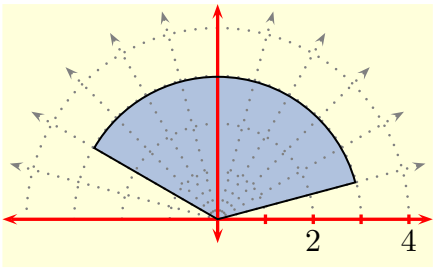
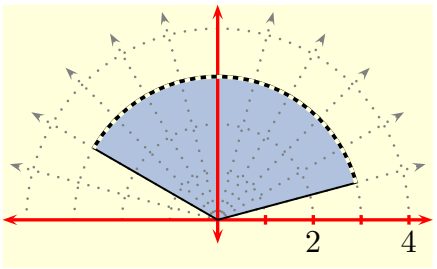


5.



6. **correct**

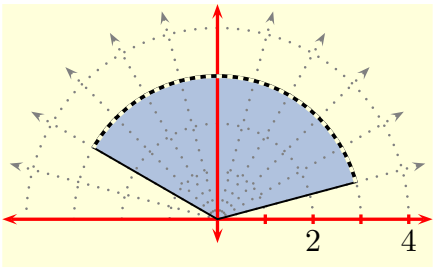


Explanation:

Using the definition of polar coordinates (r, θ), we see that the region defined by the inequalities

$$0 \le r < 3\,, \qquad \frac{1}{12}\pi \le \theta \le \frac{5}{6}\pi$$

is



keywords: polar coordinates, inequalities, polar graph,

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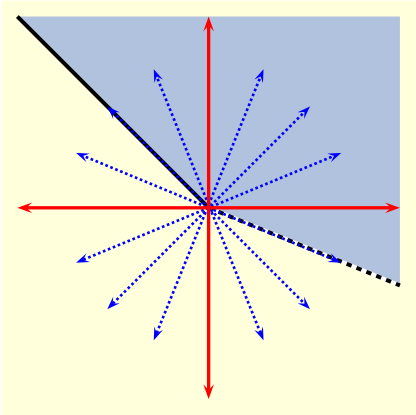
008 10.0 points

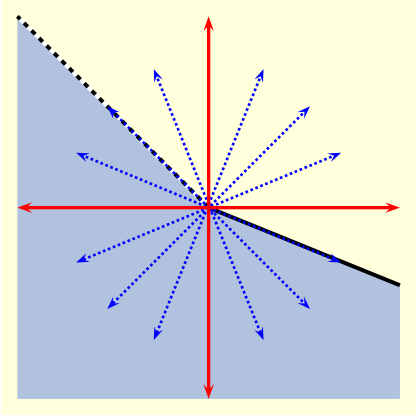
Which one of the following shaded regions consists only of points whose polar coordinates satisfy the condition

$$-\frac{\pi}{8} \le \theta < \frac{3\pi}{4}\,?$$

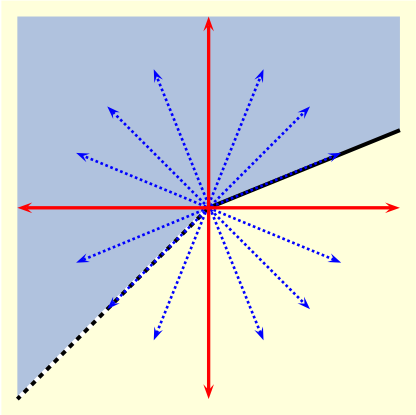
1.

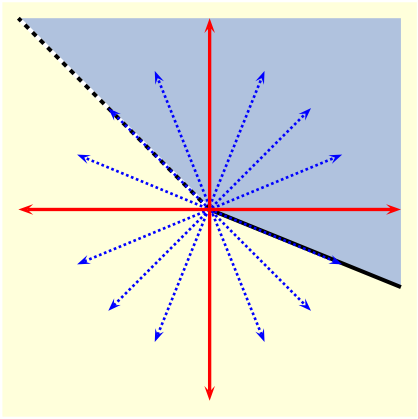


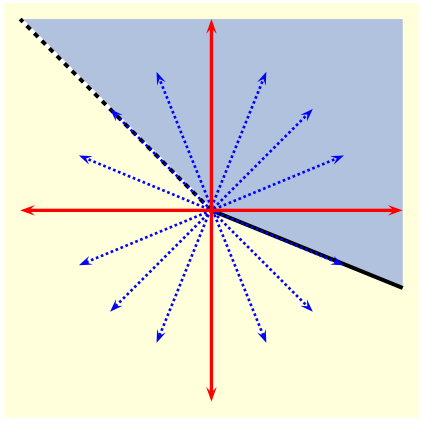
2.



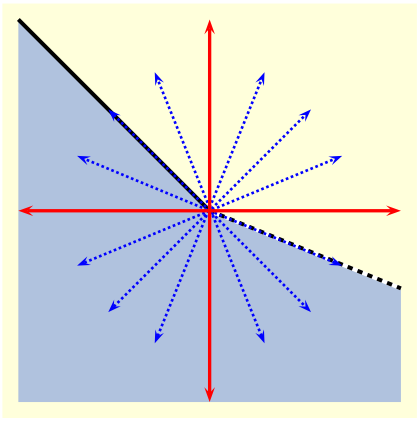
3.

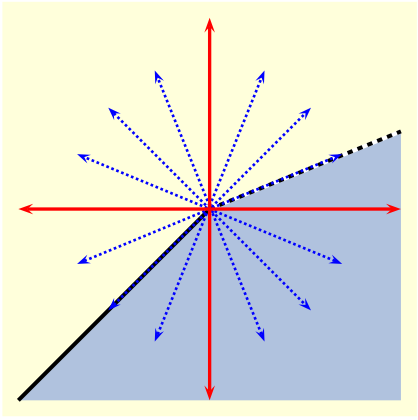


4.  **correct**



keywords:

5. 

6. 

Explanation:

Since $\theta > 0$ corresponds to rotating counter-clockwise around the origin, while $\theta < 0$ corresponds to rotating clockwise, we see that the region of the plane specified by

$$-\frac{\pi}{8} \le \theta < \frac{3\pi}{4}$$

is the shaded region shown in