This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = \frac{n-4}{4n+1},$$

and if it converges, find the limit.

1. converges with limit $= 0$
2. converges with limit $= -4$
3. converges with limit $= \frac{1}{4}$ **correct**
4. diverges
5. converges with limit $= -\frac{3}{5}$

Explanation:

Since

$$a_n = \frac{n-4}{4n+1} = \frac{1-\frac{4}{n}}{4+\frac{1}{n}},$$

while

$$\lim_{n\to\infty} \frac{4}{n} = \lim_{n\to\infty} \frac{1}{n} = 0,$$

properties of limits ensure that $\lim_{n\to\infty} a_n$ exists and that

$$\lim_{n\to\infty} a_n = \frac{1}{4}.$$

Consequently, the sequence $\{a_n\}$

$$\boxed{\text{converges with limit} = \frac{1}{4}}.$$

002 10.0 points

Determine if the sequence $\{a_n\}$ converges, when

$$a_n = \frac{7n^4 - 2n^3 + 4}{2n^4 + 3n^2 + 3},$$

and if it does, find its limit.

1. limit $= -\frac{2}{3}$
2. the sequence diverges
3. limit $= 0$
4. limit $= \frac{4}{3}$
5. limit $= \frac{7}{2}$ **correct**

Explanation:

After division by n^4 we see that

$$a_n = \frac{7 - \frac{2}{n} + \frac{4}{n^4}}{2 + \frac{3}{n^2} + \frac{3}{n^4}} \longrightarrow \frac{7}{2}$$

as $n \to \infty$. Thus $\{a_n\}_n$ converges and has

$$\boxed{\text{limit} = \frac{7}{2}}.$$

003 10.0 points

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = n(n-3),$$

and if it converges, find the limit.

1. diverges **correct**
2. converges with limit $= 0$
3. converges with limit $= 1$
4. converges with limit $= 9$
5. converges with limit $= 3$

Explanation:
Since

$$a_n = n(n-3) \implies a_n \longrightarrow \infty$$

as $n \to \infty$, the given sequence

$$\boxed{\text{diverges}}.$$

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004 10.0 points

Find a formula for the general term a_n of the sequence

$$\{a_n\}_{n=1}^{\infty} = \left\{1,\ 5,\ 9,\ 13,\ \ldots\right\},$$

assuming that the pattern of the first few terms continues.

1. $a_n = n+4$

2. $a_n = 5n-4$

3. $a_n = n+3$

4. $a_n = 3n-2$

5. $a_n = 4n-3$ **correct**

Explanation:
By inspection, consecutive terms a_{n-1} and a_n in the sequence

$$\{a_n\}_{n=1}^{\infty} = \left\{1,\ 5,\ 9,\ 13,\ \ldots\right\}$$

have the property that

$$a_n - a_{n-1} = d = 4.$$

Thus

$$a_n = a_{n-1} + d = a_{n-2} + 2d = \ldots$$
$$= a_1 + (n-1)d = 1 + 4(n-1).$$

Consequently,

$$\boxed{a_n = 4n-3}.$$

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keywords:

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005 10.0 points

Determine whether the sequence $\{a_n\}$ converges or diverges when

$$a_n = n^2 e^{-5n},$$

and if it converges, find the limit.

1. converges with limit $= 5$

2. sequence diverges

3. converges with limit $= \dfrac{2}{25}$

4. converges with limit $= -5$

5. converges with limit $= 0$ **correct**

Explanation:
The term a_n can be written as

$$a_n = n^2 e^{-5n} = \frac{n^2}{e^{5n}}.$$

To determine the limit we thus apply L'Hospital's Rule twice to

$$\lim_{x\to\infty} \frac{x^2}{e^{5x}}.$$

For then

$$\lim_{x\to\infty} \frac{x^2}{e^{5x}} = \lim_{x\to\infty} \frac{2x}{5e^{5x}}$$
$$= \lim_{x\to\infty} \frac{2}{25e^{5x}} = 0.$$

Consequently, the given sequence converges and has

$$\boxed{\text{limit} = 0}.$$

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keywords: sequence, exponential, limit, L'Hospital's rule

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006 10.0 points

Find a formula for the general term a_n of the sequence

$$\{a_n\}_{n=1}^{\infty} = \left\{1,\ -\frac{2}{5},\ \frac{4}{25},\ -\frac{8}{125},\ \ldots\right\},$$

assuming that the pattern of the first few terms continues.

1. $a_n = -\left(\frac{1}{2}\right)^n$
2. $a_n = \left(-\frac{5}{2}\right)^{n-1}$
3. $a_n = -\left(\frac{2}{5}\right)^n$
4. $a_n = \left(-\frac{2}{5}\right)^{n-1}$ **correct**
5. $a_n = -\left(\frac{5}{2}\right)^n$
6. $a_n = \left(-\frac{1}{2}\right)^{n-1}$

Explanation:

By inspection, consecutive terms a_{n-1} and a_n in the sequence

$$\{a_n\}_{n=1}^{\infty} = \left\{1,\ -\frac{2}{5},\ \frac{4}{25},\ -\frac{8}{125},\ \ldots\right\}$$

have the property that

$$a_n = ra_{n-1} = \left(-\frac{2}{5}\right)a_{n-1}.$$

Thus

$$a_n = ra_{n-1} = r^2 a_{n-2} = \ldots$$
$$= r^{n-1}a_1 = \left(-\frac{2}{5}\right)^{n-1} a_1 .$$

Consequently,

$$\boxed{a_n = \left(-\frac{2}{5}\right)^{n-1}}$$

since $a_1 = 1$.

keywords: sequence, common ratio

007 10.0 points

Determine if the sequence $\{a_n\}$ converges when

$$a_n = \frac{(2n+1)!}{(2n-1)!},$$

and if it converges, find the limit.

1. converges with limit $= 4$
2. converges with limit $= \frac{1}{4}$
3. converges with limit $= 1$
4. does not converge **correct**
5. converges with limit $= 0$

Explanation:

By definition, $m!$ is the product

$$m! = 1.2.3. \ldots .m$$

of the first m positive integers. When $m = 2n-1$, therefore,

$$(2n-1)! = 1.2.3. \ldots (2n-1),$$

while

$$(2n+1)! = 1.2.3. \ldots .(2n-1)2n(2n+1).$$

when $m = 2n+1$. But then,

$$\frac{(2n+1)!}{(2n-1)!} = 2n(2n+1).$$

Consequently, the given sequence

$$\boxed{\text{does not converge}}.$$

008 10.0 points

Which of the following sequences converge?

A. $\left\{\frac{e^n+5}{3n+4}\right\}$

B. $\left\{\frac{5e^n}{2+e^n}\right\}$

1. A only

2. neither of them

3. B only **correct**

4. both A and B

Explanation:

A. To test for convergence of the sequence

$$\left\{\frac{e^n+5}{3n+4}\right\}$$

we use L'Hospital's Rule with

$$f(x) = e^x + 5\,, \qquad g(x) = 3x+4$$

since

$$f(x) \longrightarrow \infty, \quad g(x) \longrightarrow \infty$$

as $x \to \infty$. Thus

$$\lim_{n\to\infty} \frac{e^n}{3n+4} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}\,.$$

But

$$\frac{f'(x)}{g'(x)} = \frac{e^x}{3} \longrightarrow \infty$$

as $n \to \infty$.

B. After simplification,

$$\frac{5e^n}{2+e^n} = \frac{5}{2e^{-n}+1}\,.$$

Thus

$$\frac{5e^n}{2+e^n} \longrightarrow 5$$

as $n \to \infty$ since $e^{-x} \to 0$ as $x \to \infty$.

Consequently,

$$\boxed{\text{only} B \text{ converges}}\,.$$

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009 10.0 points

Determine the least upper bound (LUB) and the greatest lower bound (GLB) for the sequence:

$$\left\{\frac{1}{2n}\right\}_{n=1}^{\infty}$$

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1. LUB $= 0$, GLB $= 1$

2. LUB $= 1$, GLB $= \frac{1}{2}$

3. LUB $= 1$, GLB $= 0$

4. LUB $= \frac{1}{2}$, GLB $= 0$ **correct**

5. LUB $= 0$, GLB $= \frac{1}{2}$

Explanation:

Evaluate the sequence at n=1 to find LUB $= \frac{1}{2}$.

Then evaluate the limit of the sequence as n approaches ∞ to find the GLB is 0

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010 10.0 points

Determine the least upper bound (LUB) and the greatest lower bound (GLB) for the sequence:

$$\left\{\frac{3n+1}{2n}\right\}_{n=1}^{\infty}$$

1. LUB $= 1$, GLB $= 0$

2. LUB $= 2$, GLB $= \frac{3}{2}$ **correct**

3. LUB $= 0$, GLB $= \frac{3}{2}$

4. LUB $= \frac{3}{2}$, GLB $= 0$

5. LUB $= \frac{3}{2}$, GLB $= 2$

Explanation:

Evaluate the sequence at n=1 to find LUB $= 2$.

Then evaluate the limit of the sequence as n approaches ∞ to find the GLB is $\frac{3}{2}$

011 10.0 points

Determine the least upper bound (LUB) and the greatest lower bound (GLB) for the sequence:

$$\left\{ \frac{n^2+1}{n} \right\}_{n=1}^{\infty}$$

1. LUB $= 0$, GLB $= 1$

2. LUB = none, GLB $= 2$ **correct**

3. LUB $= 2$, GLB = none

4. LUB $= 1$, GLB $= 0$

Explanation:

Evaluate the sequence at n=1 to find GLB $= 2$.

Then evaluate the limit of the sequence as n approaches ∞ which gives you ∞ to find that here is no LUB