This print-out should have 10 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Determine whether the series

$$\sum_{n=0}^{\infty} 4\left(\frac{2}{3}\right)^n$$

is convergent or divergent, and if convergent, find its sum.

1. convergent, sum = $\frac{12}{5}$

2. convergent, sum = 12 **correct**

3. divergent

4. convergent, sum = -13

5. convergent, sum = 13

Explanation:

The given series is an infinite geometric series

$$\sum_{n=0}^{\infty} a\,r^n$$

with $a = 4$ and $r = \frac{2}{3}$. But the sum of such a series is

(i) convergent with sum $\frac{a}{1-r}$ when $|r| < 1$,

(ii) divergent when $|r| \geq 1$.

Consequently, the given series is

$$\boxed{\text{convergent, sum} = 12}.$$

002 10.0 points

Determine whether the infinite series

$$4 - 3 + \frac{9}{4} - \frac{27}{16} + \frac{81}{64} \cdots$$

is convergent or divergent, and if convergent, find its sum.

1. convergent with sum $\frac{7}{16}$

2. divergent

3. convergent with sum $\frac{16}{7}$ **correct**

4. convergent with sum 16

5. convergent with sum $\frac{1}{16}$

Explanation:

The infinite series

$$4 - 3 + \frac{9}{4} - \frac{27}{16} + \frac{81}{64} \cdots = \sum_{n=1}^{\infty} a\,r^{n-1}$$

is an infinite geometric series with $a = 4$ and $r = -\frac{3}{4}$.

But a geometric series

$$\sum_{n=1}^{\infty} a\,r^{n-1}$$

(i) *converges with sum* $\frac{a}{1-r}$ *when* $|r| < 1$,

(ii) *and diverges when* $|r| \geq 1$.

Consequently, the given series is

$$\boxed{\text{convergent with sum } \frac{16}{7}}.$$

003 10.0 points

Determine whether the series

$$2 + 3 + \frac{9}{2} + \frac{27}{4} + \cdots$$

is convergent or divergent, and if convergent, find its sum.

1. convergent with sum = 9

2. convergent with sum $= \dfrac{1}{4}$

3. convergent with sum $= 4$

4. divergent **correct**

5. convergent with sum $= \dfrac{1}{9}$

Explanation:

The series

$$2 + 3 + \frac{9}{2} + \frac{27}{4} + \cdots = \sum_{n=1}^{\infty} a\, r^{n-1}$$

is an infinite geometric series in which $a = 2$ and $r = \dfrac{3}{2}$. But such a series is

(i) convergent with sum $\dfrac{a}{1-r}$ when $|r| < 1$,

(ii) divergent when $|r| \geq 1$.

Thus the given series is

$$\boxed{\text{divergent}}\,.$$

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004 10.0 points

Determine if the series

$$\sum_{n=1}^{\infty} \frac{1 + 2^n}{4^n}$$

converges or diverges, and if it converges, find its sum.

1. converges with sum $= \dfrac{3}{2}$

2. converges with sum $= \dfrac{11}{6}$

3. series diverges

4. converges with sum $= \dfrac{5}{3}$

5. converges with sum $= \dfrac{7}{6}$

6. converges with sum $= \dfrac{4}{3}$ **correct**

Explanation:

An infinite geometric series $\displaystyle\sum_{n=1}^{\infty} a\, r^{n-1}$

(i) converges when $|r| < 1$ and has

$$\text{sum} = \frac{a}{1-r},$$

while it

(ii) diverges when $|r| \geq 1$.

Now

$$\sum_{n=1}^{\infty} \frac{1}{4^n} = \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{1}{4}\right)^{n-1}$$

is a geometric series with $a = r = \dfrac{1}{4} < 1$. Thus it converges with

$$\text{sum} = \frac{1}{3},$$

while

$$\sum_{n=1}^{\infty} \frac{2^n}{4^n} = \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

is a geometric series with $a = r = \dfrac{1}{2} < 1$. Thus it too converges, and it has

$$\text{sum} = 1\,.$$

Consequently, being the sum of two convergent series, the given series

$$\boxed{\text{converges with sum} = \frac{1}{3} + 1 = \frac{4}{3}}\,.$$

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005 10.0 points

If the n^{th} partial sum of $\displaystyle\sum_{n=1}^{\infty} a_n$ is

$$S_n = \frac{3n-5}{n+1},$$

What is the sum of $\displaystyle\sum_{n=1}^{\infty} a_n$?

1. sum $= 3$ **correct**

2. sum $= 4$

3. sum $= 1$

4. sum $= 0$

5. sum $= 2$

Explanation:

By definition

$$\text{sum} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{3n-5}{n+1}\right).$$

Thus

$$\boxed{\text{sum} = 3}.$$

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006 10.0 points

If the n^{th} partial sum S_n of an infinite series

$$\sum_{n=1}^{\infty} a_n$$

is given by

$$S_n = 7 - \frac{n}{6^n},$$

find a_n for $n > 1$.

1. $a_n = 7\left(\frac{7n-6}{6^n}\right)$

2. $a_n = 7\left(\frac{n-6}{6^{n-1}}\right)$

3. $a_n = \frac{7n-6}{6^n}$

4. $a_n = \frac{n-6}{6^{n-1}}$

5. $a_n = 7\left(\frac{5n-6}{6^n}\right)$

6. $a_n = \frac{5n-6}{6^n}$ **correct**

Explanation:

By definition, the n^{th} partial sum of

$$\sum_{n=1}^{\infty} a_n$$

is given by

$$S_n = a_1 + a_2 + \cdots + a_n.$$

In particular,

$$a_n = \begin{cases} S_n - S_{n-1}, & n > 1, \\ S_n, & n = 1. \end{cases}$$

Thus

$$a_n = S_n - S_{n-1} = \frac{n-1}{6^{n-1}} - \frac{n}{6^n} = \frac{6(n-1)}{6^n} - \frac{n}{6^n}$$

when $n > 1$. Consequently,

$$\boxed{a_n = \frac{5n-6}{6^n}}$$

for $n > 1$.

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007 10.0 points

Find the general term for the sequence of partial sums for the series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

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1. $\frac{n}{n-1}$

2. $\frac{2^n-1}{2^n}$ **correct**

3. $\frac{1}{2^n}$

4. 1

5. $\frac{n}{n+1}$

Explanation:

$s_1 = \frac{1}{2}$, $s_2 = \frac{3}{4}$, $s_3 = \frac{7}{8}$, $s_4 = \frac{15}{16}$
Therefore we can write
$s_n = \frac{2^n - 1}{2^n}$

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008 10.0 points

Find the general term for the sequence of partial sums for

$$\sum_{n=1}^{\infty} \frac{1}{3^n}$$

1. $\frac{\frac{3^n - 1}{2}}{3^n}$ **correct**

2. $\frac{1}{2}$

3. $\frac{3^n - 1}{3^n}$

4. 1

5. $\frac{1}{3^n}$

Explanation:

$s_1 = \frac{1}{3}$, $s_2 = \frac{4}{9}$, $s_3 = \frac{13}{27}$.
Therefore we can write
$s_n = \frac{\frac{3^n - 1}{2}}{3^n}$

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009 10.0 points

Find the general term for the sequence of partial sums for the series:

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots$$

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1. $\frac{n}{n+1}$ **correct**

2. $\frac{1}{n(n+1)}$

3. 1

4. $\frac{1}{n+1}$

5. $\frac{1}{2^n}$

Explanation:

$s_1 = \frac{1}{2}$, $s_2 = \frac{2}{3}$, $s_3 = \frac{3}{4}$, $s_4 = \frac{4}{5}$
Therefore we can write
$s_n = \frac{n}{n+1}$

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010 10.0 points

Determine whether the series is convergent or divergen by expressing s_n as a telescoping sum. If it is convergent find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

1. convergent with a sum of 0

2. divergent

3. convergent with a sum of 2

4. convergent with a sum of 1 **correct**

5. convergent with a sum of -1

Explanation:

Using partial fractions we can rewrite the general term as

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

We can then write out the sequence of partial sums

$$s_1 = 1 - \frac{1}{2}$$

$$s_2 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$s_3 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$$

Then we can see that

$$s_n = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$s_n = 1 - \frac{1}{n+1}$$

Then we can evaluate the limit

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} 1 - \frac{1}{n+1} = 1$$