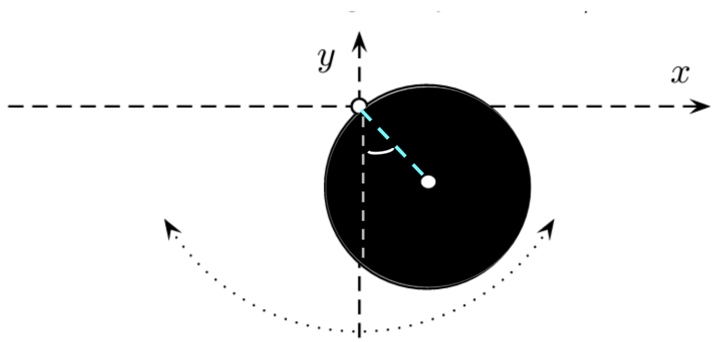
PHY 303K - Waves and Oscillations Practice Problems

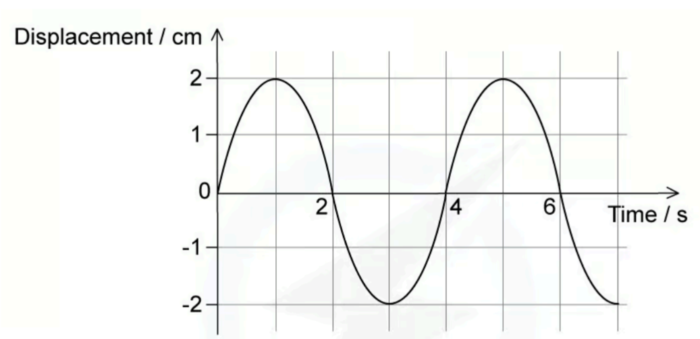
1) A student sets up the harmonic oscillator in which a mass is attached to a spring with constant k that is attached to a wall. The student pulls the mass a distance x from equilibrium and releases it from rest. The student then measures the maximum speed of the mass during its oscillation and calculates the total mechanical energy, E_total, of the system. Assume no energy is lost to friction.
   a) If the student changes the initial displacement from x to 3x, in terms of E_total, what will the new mechanical energy of the system be?
   b) If the mass is quadrupled, how will the period change?

2) A vertical spring attached to the ceiling has a force constant of 850 N/m. A block of mass 1 kg is attached to the spring and oscillates freely with an amplitude of 10 cm. What is the speed of the block at the halfway point?

3) Find the angular frequency of small oscillations for a disk of mass m = 11 kg and radius R = 0.7 m, suspended from a pivot as shown below.



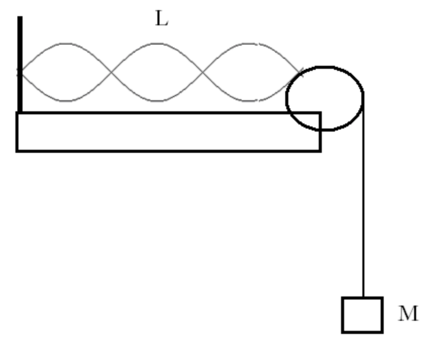
4) The graph shows how the displacement of a single molecule of water evolves as a wave passes through.



a) Which statement is correct?
   i) The wave has an amplitude of 2 cm and could be either transverse or longitudinal.
   ii) The wave has an amplitude of 2 cm and has a time period of 6 s.
   iii) The wave has an amplitude of 4 cm and has a time period of 4 s.
   iv) The wave has an amplitude of 4 cm and must be transverse.

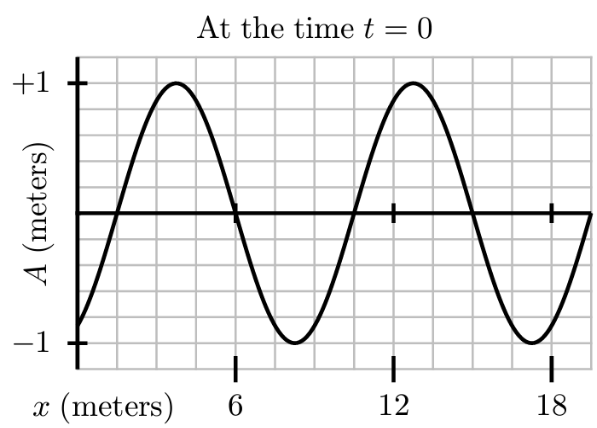
b) Now assume the wave is due to a whale's call (a sound wave which acts longitudinally) and that a positive amplitude corresponds to a positive displacement to the right. Graph the displacement of the particles immediately to the left and right of the particle described in the figure above at time t = 1s. Assume the wave started at time t = 0s.

5) A string with a length of 1.5 m resonates in three loops as shown here. The suspended mass is 1.2 kg. Take the wave speed to be 9.89 m/s.
   a) What is the wavelength?
   b) What is the string's linear density?
   c) If the suspended mass is increased, what will happen to the wave speed? What about the string's linear density?



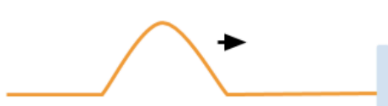
6) The musical note A above middle C has a frequency of 440 Hz.
   a) If the speed of sound is known to be 340 m/s in air, what is the wavelength of this note?
   b) What would the wavelength be on Mars where the speed of sound is 240 m/s due to its less dense atmosphere?

7) What is the function that describes the waveform below?

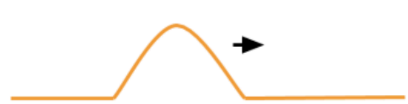


8) What does the reflected wave look like in the following two scenarios?

Fixed end

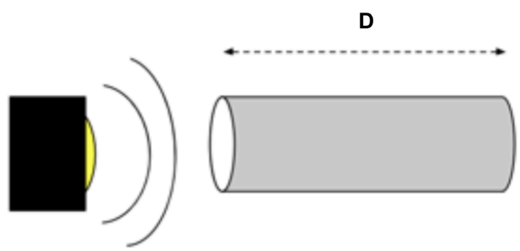


Free end



9) Consider a standing wave with a wavelength of 4 cm on a string of length 20 cm. How many antinodes does the wave have?

10) If D = 20 cm, what is the fundamental wavelength that can be set up for the tube below?



Solutions

1) Source: Khan Academy

a)

1 / 4 When released from rest, the mass will have zero kinetic energy, so the total energy of the system will be the initial potential energy stored in the spring, $E_{total} = U_k$. To find the new mechanical energy, $E_{totalnew}$, and compare it to E_{total}, we can use the potential energy equation, $U_k = \frac{1}{2}kx^2$, and plug in the new initial displacement.

2 / 4 Initially the mass was released from x, so the initial potential energy and therefore E_{total} was $U_k = \frac{1}{2}kx^2$

Let's plug in the new initial displacement $3x$ to find the new mechanical energy of the system.

$$U_k = \frac{1}{2}kx^2$$

$$= \frac{1}{2}k(3x)^2$$

$$= \frac{1}{2}k \cdot 9x^2$$

$$= \frac{9}{2}kx^2$$

3 / 4 If we compare the initial total energy of the system, $E_{total} = \frac{1}{2}kx^2$, to the new energy, $E_{totalnew} = \frac{9}{2}kx^2$, we can see that the new energy is nine times the original value.

4 / 4 **So, in terms of E_{total} the new mechanical energy of the system would be $9 \cdot E_{total}$.**

b) $T_m = 2\pi\sqrt{\frac{m}{k}}$ $\qquad$ $T_{4m} = 2\pi\sqrt{\frac{4m}{k}} = 2\,T_m$

2)

$$K_i + U_i = K_f + U_f$$

$$0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}k(\frac{A}{2})^2$$

$$v = \sqrt{\frac{3\,k\,A^2}{4\,m}} = 2.525\ \frac{m}{s}$$

3) $\omega = \sqrt{\frac{mgd}{I}}$

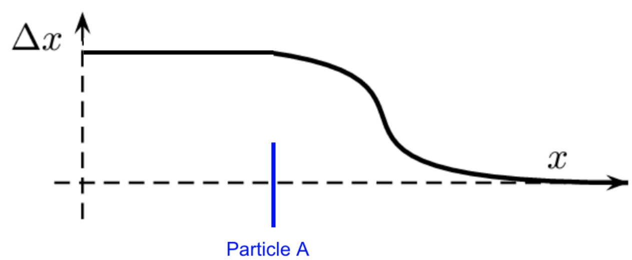
$$I = I_{disk}^{CM} + Md^2 \quad \rightarrow \quad I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$\omega = \sqrt{\frac{MgR}{\frac{3}{2}MR^2}} = \sqrt{\frac{2\,g}{3\,R}}$$

4)

a) **i** - amplitude is defined as displacement from equilibrium, which is 2 cm in this case. We can't determine if it's transverse or longitudinal because the graph is displacement vs. time. If the graph was displacement vs. direction of wave travel we would know it was a transverse wave.

b)



5) Relevant equations: $v = \sqrt{\frac{F}{\mu}} \rightarrow \mu = \frac{mg}{v^2}$

a) 1 m
b) 0.12 kg/m
c) The wave speed would increase. Linear density of the string remains constant.

6) Relevant equations: $v = \frac{\lambda}{T} = \lambda f$

a) 0.7727 m
b) 0.5454 m

7) $y = sin[\frac{2\pi}{9}x - \omega t - \frac{\pi}{3}]$

8) Source: Khan Academy

A wave that travels down a rope gets reflected at the rope's end. If the end of the rope is free, then the wave returns right side up. If the end of the rope is fixed, then the wave will be inverted.

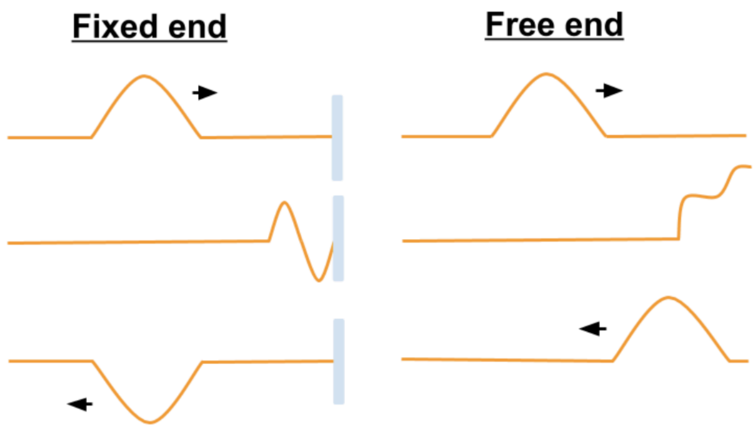
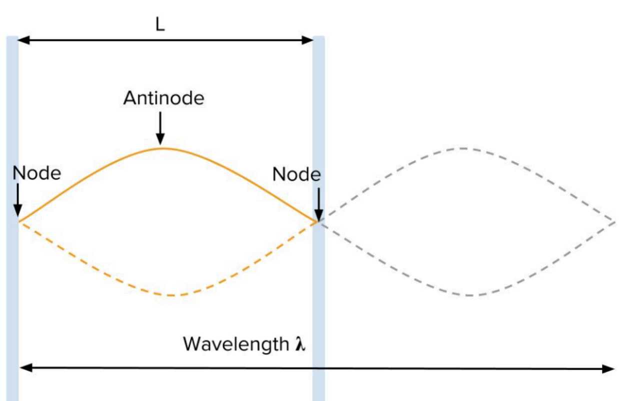


Figure 1: A wave pulse reflected from a free end returns right side up. A wave pulse reflecting from a fixed end is inverted.

For a rope with two fixed ends, another wave travelling down the rope will interfere with the reflected wave. At certain frequencies, this produces standing waves where the nodes and antinodes stay at the same places over time. For all standing wave frequencies, the nodes and antinodes alternate with equal spacing.

9) $L = n\left(\frac{\lambda}{2}\right) \quad \rightarrow \quad n = \frac{2L}{\lambda} = \frac{2 \cdot 20\ cm}{4\ cm} = 10$

Recall:



10) From the equation in the previous problem, setting $n = 1$ for the fundamental frequency, we find that $\lambda = 40\ cm$.