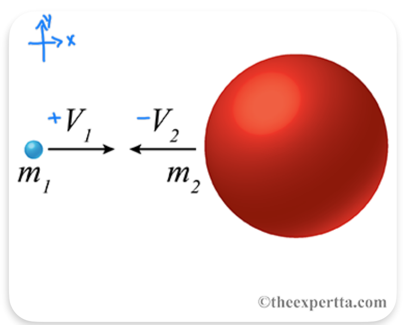
Problem #1: Two masses, m1 and m2, are traveling toward each other. The speed of m1 is v1 and oriented along the positive x-direction, while that m2 is v2 and oriented along the negative x-direction. The masses are such that m1 = (m2/10).

If the two masses stick together, in what direction do they travel after they impact?



Symbolic approach

We know that the momentum conservation equation is

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

The final velocity of the system is

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Now let $m_1 = \frac{m_2}{10}$

$$v_f = \frac{\frac{m_2}{10} v_1 - m_2 v_2}{\frac{m_2}{10} + m_2}$$

$$v_f = \frac{\frac{m_2}{10} v_1 - m_2 v_2}{\frac{11 m_2}{10}}$$

$$= v_f = \frac{\frac{1}{10} v_1 - v_2}{\frac{11}{10}} = \frac{v_1 - 10 v_2}{11}$$

∴ The direction of motion depends on the sign of v_f.

1. If $v_1 > 10 v_2$ then,

$v_f > 0$ (motion to the right) [m_1 direction]

2. If $v_1 = 10 v_2$ then,

$v_f = 0$ (System stays stationary)

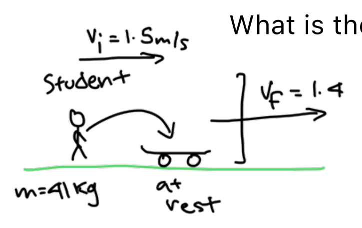
3. If $v_1 < 10 v_2$ them

$v_f < 0$ (Motion to the left) [m_2 direction]

∴ Since we do not have sufficent information about v_1, v_2, and v_f to determine the direction of v_f, then "not enough information" is correct.

Problem #2: A student of mass *m* = 41kg runs at a velocity *vi* = 1.5m/s before jumping on a skateboard that is initially at rest. After jumping on the board the student has a velocity *vf* = 1.4m/s.

What is the mass of the skateboard, in kilograms?



Velocity final after jumping is 1.4 m/s

before jumping is 1.5 m/s

We can use the equation of the conservation of momentum

$$m_1 v_i = (m_1 + m_b) v_f \implies \frac{m_1 v_i}{v_f} - m_1 = m_b$$

∴ by [CoM]

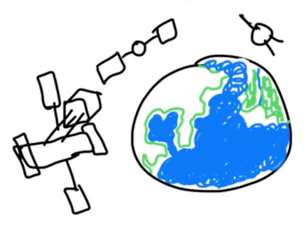
$$\frac{41(1.5)}{1.4} - 41 = m_b$$

$$\frac{61.5}{1.4} - 41 =$$

$$43.9285 - 41 = \boxed{2.92\text{ kg} = m_b}$$

∴ The mass of the skateboard is 2.92 kg via the conservation of momentum equation and theorem.

Problem #3: One hazard of space travel is the debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are greater numbers of very small objects, such as flakes of paint.



Calculate the magnitude of the force, in newtons, exerted by a 0.075mg chip of paint that strikes (and sticks to) a spacecraft window at a relative speed of 3.75 * 10^3 m/s, given the collision lasts 5.75 * 10^-8 m/s.

We need to convert from mg to kg to calculate in kg·m/s²

We know $m_p = 0.075\text{ mg} \to 7.5 \cdot 10^{-8}$ kg

To find the magnitude of the force we can use Impulse-momentum theory

$$F = \frac{\Delta p}{\Delta t} = \frac{mv}{\Delta t}$$

∴ [by IMT] $\frac{(7.5 \cdot 10^{-8})(3.75 \cdot 10^{3})}{5.75 \cdot 10^{-8}} = 4.8913 \cdot 10^{3}$

$= \boxed{4891.3\text{ N}}$

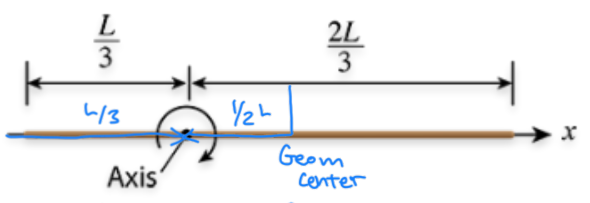
Therefore the magnitude of the force in respect to the collision is 4891.3 newtons.

Problem #5: The diagram shows a uniform rod of mass *M* and length *L*, where the axis of rotation is a distance of *L/3* from one end.

Determine an expression for the moment of inertia.

$\frac{L}{3} + \frac{2L}{3}$

$\frac{3L}{3}$



Then the distance from the center to the axis is: $d = \frac{L}{2} - \frac{L}{3} = \frac{3L}{6} - \frac{2L}{6} = \frac{L}{6}$

We know that the parallel axis theorem states that the rod can be split into segments.

∴ [By PAT] The moment of inertia of a uniform rod [center] is

$$I_{center} = \frac{1}{12} ML^2.$$

[Parallel Axis Theorem] Let $d = \frac{L}{6}$

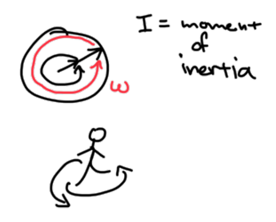
$$I = I_{center} + Md^2$$
$$= \frac{1}{12} ML^2 + Md^2$$

$$I = \frac{1}{12} ML^2 + M\left(\frac{L}{6}\right)^2$$
$$= \frac{1}{12} ML^2 + M\frac{L^2}{36}$$
$$= ML^2\left(\frac{1}{12} + \frac{1}{36}\right)$$
$$= ML^2\left(\frac{4}{36}\right) = \boxed{\frac{1}{9} ML^2} = I$$

Therefore the expression for the moment of inertia by the parallel axis theorem is $\frac{1}{9} ML^2$.

Problem #6: An ice skater is spinning with her arms out and is not being acted upon by an external torque

a. When she pulls her arms close to her body what happens to her angular momentum?
c. What happens to her angular speed when she pulls her arms in?



We know that by angular momentum conservation

$$L = I\omega$$

Note! If no external torque acts on the system then angular momentum is conserved.

$$L_{initial} = L_{final}$$

We also know that we can represent moment of inertia in relation to the axis of rotation:

$$I = \sum mr^2$$

Part A. Via conservation of angular momentum [$L_{initial} = L_{final}$],

If we decrease in moment of inertia; then by increase prop. in ω [angular] to keep L constant.

∴ angular momentum remains unchanged.

Part B: Using the equation of angular momentum and since

L is conserved: $I_{ini}\omega_{ini} = I_{final}\omega_{final}$

[Rearrange to find ω_{final}] → $\omega_{final} = \frac{I_{ini}}{I_{final}} \omega_{ini}$

Since $I_{final} < I_{initial}$ (moment of inertia decrease as the arms are pulled):

$$\omega_{final} > \omega_{initial}$$

∴ The skater's angular speed, ω, increases.

Problem #7: An object of mass *m* is released from rest a distance *R* above the surface of a planet of mass *M* and radius *R*.

A. Derive an expression for the speed with which it hits the planet's surface *v*.
B. Calculate this speed in m/s, assuming *M* = 21 * 10^23 kg and *R* = 11*10^3 km

We know that by Gravitational and Energy laws, that the total mechanical energy is conserved:

$$E_{initial} = E_{final}$$

We also know that orbital gravitational energy is:

$$E_{initial} = E_{orbit} = -\frac{GMm}{2R} \text{ where}$$

[Gravitational Potential Energy] is $-\frac{GMm}{r}$

While the E_{final} is the sum of gravitational surface energy and kinetic energy.

$$E_{final} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

∴ By $E_{initial} = E_{final}$ [conserved],

$$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$2\left(\frac{GMm}{R} - \frac{GMm}{2R}\right) = \frac{1}{2}mv^2 \cdot 2$$

$$\frac{1}{m} \cdot \frac{GMm}{R} = \frac{mv^2}{m}$$

$$\sqrt{\frac{GM}{R}} = \sqrt{v^2}$$

$$\sqrt{\frac{GM}{R}} = v$$

Part A. An expression for the speed which an object hits the planet's surface is

$$\boxed{v = \sqrt{\frac{GM}{R}}}$$

Part B. Now using the equation from part A. Let G = the gravitational costant, M equals the planet's mass, and R equals the planet's radius.

$$v = \sqrt{\frac{G\left(\frac{21 \cdot 10^{23}}{1000}\right)}{11 \cdot 10^{3}}} = \boxed{3.5695 \cdot 10^{3}\text{ m/s}} = 3569.5\text{ m/s}$$

Problem 8b: The Sun has a mass of 1.99 * 10^30 kg and a radius of 6.96 * 10^8 m.
By what factor would your weight increase if you could stand on the Sun?

We know that g_{earth} = the gravitational acceleration of earth. In the previous question we were asked to solve for the Sun's gravitational acceleration

Let $M = 1.99 \cdot 10^{30}$ kg

$R = 6.96 \cdot 108$ m

Thus, let $g = 9.81 = g_{earth}$

and let $g_{sun} = \frac{GM}{R^2}$

$$\therefore g_{sun} = \frac{G(1.99 \cdot 10^{30}\text{ kg})^2}{(6.96 \cdot 10^{8}\text{ m})^2} = \boxed{274.182\text{ m/s}^2}$$

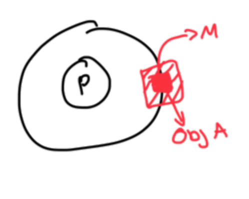
To calculate the ratio of $\frac{g_{sun}}{g_{earth}}$ to know the factor:

Let $\frac{g_{sun}}{g_{earth}} = g_{factor}$

$$\therefore \frac{274.182\text{ m/s}^2}{9.81\text{ m/s}^2} = \boxed{27.9492 = g_{factor}}$$

Thus the factor would be ~28 for my weight to increase if I could stand on the sun!

Problem 11: Consider an object moving in a circular orbit around a planet, due to the force of gravity.
If an object's mass is increased, will it orbit faster or slower if the radius of its orbit does not change?



We know circular orbital velocity is found by balancing gravitational force and centripetal force; the gravitational force is

$$F_{gravity} = \frac{GMm}{r^2}, \text{ in the other hand,}$$

the centripetal force needed to keep the obj. in orbit is:

$$F_c = \frac{mv^2}{r}$$

∴ Circular motion, the forces equalize:

From here we solve for circular orbital velocity

1 $\cancel{r} \cdot \frac{GMm}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}} \cdot \cancel{r}$

2 $\frac{1}{\cancel{m}} \cdot \frac{GM\cancel{m}}{r} = \frac{\cancel{m}v^2}{\cancel{m}}$

3 $\sqrt{\frac{GM}{r}} = \sqrt{v^2}$

4 $\boxed{\sqrt{\frac{GM}{r}} = v}$

We can see that the orbital velocity is not dependent on the object's weight as shown in the cancellation of m (step 3) and not appearing in the final formula (step 4).

∴ The object's orbital velocity stays the SAME