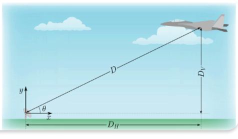
SOH → $\sin\theta = \frac{opp}{hyp}$
CAH → $\cos\theta = \frac{adj}{hyp}$
TOA → $\tan\theta = \frac{opp}{adj}$

Problem 1: (5% of Assignment Value)
A plane flies towards a ground-based radar dish. Radar locates the plane at a distance $D = 25.3$ km from the dish, at an angle $\theta = 42.7°$ above horizontal.



a) What is the plane's horizontal distance, in meters, from the radar dish?

If $\cos\theta = \frac{adj}{hyp}$, then

$\cos(42.7°) = \frac{D_H}{D}$, where $D = 25.3$ km

$D\cos(42.7°) = D_H$

$\therefore 18.5933 \text{ km} = D_H$

Since the problem wants it in meters (1 km = 1000 m),

$\therefore D_H \cdot 1000 = \boxed{18593.33 \text{ m}} = D_H$

b) What is the plane's vertical distance, in meters, above the radar dish?

If $\sin\theta = \frac{D_V}{D}$, then

$(25.3)\sin(42.7°) = \frac{D_V}{25.3 \text{ km}}(25.3)$

$(25.3)\sin(42.7°) = D_V$

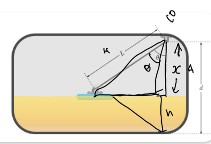
$\therefore 17.157 \text{ km} = D_V$, but we need to convert it into meters, thus 1 km = 1000 m

$\therefore D_V \cdot 1000 = 17157.93 \text{ m} = D_V$

c) Write an expression for the position vector, $\vec{r}$, in rectangular Cartesian unit-vector form using the coordinates axis provided in the drawing.

$D\cos\theta\,\hat{i} + D\sin\theta\,\hat{j} = \vec{B}$

Problem 2: (5% of Assignment Value)
The fuel tank on a car has a depth of $d = 0.432$ m. The fuel level in the tank is detected by a sensor at the end of an arm with length $L = 0.783$ m. The arm is free to rotate about a pivot at an upper corner of the fuel tank. The angle between the arm and the vertical is θ, as indicated in the drawing.



$\cos\theta = \frac{x}{L}$
$x = L\cos\theta$
$h = d - x$
$h = d - L\cos\theta$

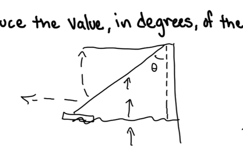
a) Derive an expression for the sensor height, h, above the horizontal tank bottom as a function of L, d, and θ.

SOH CAH TOA

$(L)\cos\theta = \frac{x}{L}(L)$ $\quad d = x + h$
$L\cos\theta = x$ $\quad d - x = h$

$\boxed{d - L\cos\theta = h}$

b) Use logic to deduce the value, in degrees, of the angle θ when tank is full. No computation needed



$d - L\cos\theta = h$, where $h = d$
$d - L\cos\theta = d$
$-L\cos\theta = 0$
$\cos\theta = 0$
$\theta = \cos^{-1}(0)$

c) Calculate the angle, in degrees, associated with a half full tank.

$d - L\cos\theta = h$, where $h = \frac{1}{2}d$

$0.432 - 0.783\cos\theta = \frac{0.432}{2}$

$0.432 - 0.783\cos\theta = 0.216$

$-0.783\cos\theta = -0.216$

$\arccos(\cos\theta) = 0.3072$

$\theta = \arccos(0.3072)$

$\theta = 72.1098°$

d) Note that the arm is longer than the tank is deep, $L > d$. What angle, in degrees, is associated with an empty tank.

Problem 3: (5% of Assignment Value)
The time-dependent position of a particle is given by

$$\vec{r} = (5.0 \text{ m/s}^2)t^2\,\hat{i} + (4.0 \text{ m/s}^2)t^2\,\hat{j}$$

a) What is the magnitude, in meters, of the particle's distance from the origin at $t = 0.0$s?

$\vec{r}(0.0) = (5.0)(0.0)^2\hat{i} + (4.0)(0.0)^2\hat{j}$

$\vec{r}(0.0) = 0$

b) What is the magnitude, in meters, of the particle's distance from the origin at $t = 2.0$?

$\vec{r} = (5.0)(2.0)^2\hat{i} + (4.0)(2.0)^2\hat{j}$

$\vec{r}(2.0) = 20\hat{i} + 16\hat{j}$

$|\vec{r}(2.0)| = \sqrt{(20)^2 + (16)^2} = \sqrt{400 + 256}$

$|\vec{r}(2.0)| = 25.6125$

c) Provide an expression in Cartesian unit-vector notation for the velocity vector of the particle. Do not include the units, but assume that units of length are meters, and units of time are meters.

$\vec{r} = (5.0)t^2\hat{i} + (4.0)t^2\hat{j}$

$\frac{d}{dt}\vec{r} = \vec{v}(t)$

$\vec{v}(t) = (10)t\,\hat{i} + (8.0)t\,\hat{j}$

d) What is the speed, in meters per second, of the particle when $t = 0.0$s?

$\vec{v}(0.0) = (10)(0.0)\hat{i} + (8.0)(0.0)\hat{j}$

$0 + 0$

$|\vec{v}(0.0)| = 0$

e) What is the speed, in meters per second, of the particle when $t = 2.0$s?

$\vec{v}(2.0) = 10(2.0)\hat{i} + 8(2.0)\hat{j}$

$\vec{v}(2.0) = 20\hat{i} + 16\hat{j}$

$|\vec{v}(2.0)| = \sqrt{(20)^2 + (16)^2} = \sqrt{400 + 256}$

$|\vec{v}(2.0)| = 25.6125$ m/s

Problem 4: (5% of Assignment Value)
A particle has a constant acceleration given by

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

and initially, at $t = 0$, the particle is at rest at the origin.

a) What is the particle's position in Cartesian unit-vector notation as a function of time?

$\int\vec{a} = \int a_x\hat{i} + a_y\hat{j}$

$\vec{v} = a_x t\,\hat{i} + a_y t\,\hat{j}$

$\int\vec{v} = \vec{r}(t)$

$= \int a_x t\,\hat{i} + a_y t\,\hat{j}$

$\vec{r} = a_x\left(\frac{1}{2}t^2\right)\hat{i} + a_y\left(\frac{1}{2}t^2\right)\hat{j}$

b) What is the particle's position in Cartesian unit-vector notation as a function of time?

$\int\vec{a} = \vec{v}(t)$

$\int\vec{a} = \int a_x\hat{i} + a_y\hat{j}$

$\vec{v}(t) = a_x t\,\hat{i} + a_y t\,\hat{j}$

c) What is the particle's path, expressing the y coordinate as a function of x? Your expression will be independent of time.

$x(t) = \frac{1}{2}a_x t^2 \longrightarrow \frac{2x(t)}{a_x} = \frac{a_x t^2}{a_x}$

$\frac{2x}{a_x} = t^2$

Plug in t^2

$y(t) = \frac{1}{2}a_y t^2 \longrightarrow \frac{1}{2}a_y\left(\frac{2x}{a_x}\right)$

$y(x) = \frac{a_y}{a_x}x = \boxed{\frac{a_y}{a_x}x}$

Problem 5: (5% of Assignment Value)
Taking north to be the positive y direction and east to be the positive x direction, a particle's position is given by

$$r(t) = (31.7 \text{ m/s})t\,\hat{i} + (6.79 \text{ m/s}^2)t^2\,\hat{j}$$

a) In what direction is the particle travelling at $t = 0.0$?

$\vec{r} = 31.7t\,\hat{i} + 6.79t^2\,\hat{j}$

$\vec{v} = \vec{r}' = 31.7\hat{i} + 13.58t\,\hat{j}$

$\vec{v}(0.0) = 31.7\hat{i} + (13.58)(0)\hat{j}$

b) At what time, in seconds, is the particle moving exactly northeast?

If $\vec{v} = 31.7\hat{i} + 13.58t\,\hat{j}$, then for $\sin(\alpha)$

$\frac{31.7}{13.58} = t$

$2.3343 = t$ for $\sin(\alpha)$

Problem 6: (5% of Assignment Value)
An airplane starts at rest and accelerates at 6.7 m/s² at an angle of 31° south of west.

a) After 9s, how far, in meters, in the westerly direction has the airplane traveled?

$x(t) = x_0 + v_0 t + \frac{1}{2}a_x t^2$

$= \frac{(6.7\cos(31°))(9)^2}{2}$

$x(9) = 232.592$

b) After 9s, how far, in meters, in the southerly direction has the airplane traveled?

$y(t) = \frac{1}{2}a_y t^2$, where $a_y = r\sin\theta$

$a_y = 6.7\sin(31°)$

$= \frac{(6.7\sin(31°))(9)^2}{2}$

$y(9) = 139.755$

Problem 7: (5% of Assignment Value)
A boat leaves the dock at $t = 0.00$ s, and, starting from rest, maintains a constant acceleration of (0.477 m/s²) $\hat{i}$ relative to the water. Due to currents, however, the water itself is moving with a velocity of (0.408 m/s) $\hat{i}$ + (2.43 m/s) $\hat{j}$.

a) How fast, in meters per second, is moving at $t = 5.81$s?

$\vec{a}_{on\,water} = 0.477 \text{ m/s}^2\,\hat{i}$

$\vec{v}(t) = \vec{v}_0 + \vec{a}t$

$= 0.477t\,\hat{i}$

$\vec{v}(5.81) = 0.477(5.81)\hat{i}$

$= 2.7714\hat{i}$

$\vec{V}_{B\to g} = 2.77\hat{i} + 0.408\hat{i} + 2.43\hat{j}$

$= 3.178\hat{i} + 2.43\hat{j}$

$|\vec{V}_{B\to G}| = \sqrt{(3.178)^2 + (2.43)^2}$

$|\vec{V}_{B\to G}| = 4.0017$

b) How far, in meters, is the boat from the dock at $t = 5.81$s?

$\vec{r} = \vec{r}_0 + \vec{v}_0 + (t)\frac{1}{2}\vec{a}t^2$

$\vec{r}(5.81) = \frac{\vec{a}(5.81)^2}{2}$

$\vec{r}(5.81) = 8.0508\hat{i}$

Displacement of water

$\vec{v}_{w/g} = 0.408\hat{i} + 2.43\hat{i}$

$\vec{r}_{w/g} \equiv \vec{v}_{w/g}t$

$\vec{r}_{w/g} = 0.408\hat{i} + (t)\,2.43\hat{j} + $; Let $t = 5.81$

$0.408(5.81)\hat{i} + 2.43(5.81)\hat{j}$

$= 2.3705\hat{i} + 14.1183$

Total disp.

$\vec{r}_{b/g} = \vec{r}_{b/w} + \vec{r}_{w/g}$

$8.0508\hat{i} + 2.3705\hat{i} + 14.1183\hat{j}$

$\vec{r}_{b/g} = 10.4213\hat{i} + 14.1183\hat{j}$

$|\vec{r}_{b/g}| = \sqrt{(10.4213)^2 + (14.1183)^2}$

$\boxed{\vec{r}_{b/g}} = 17.5979$

Problem 8: (5% of Assignment Value)
A spaceship is traveling at a velocity of

$$\vec{v}_0 = (21.9 \text{ m/s})\,\hat{i}$$

when its rockets fire, giving it an acceleration of

$$\vec{a} = (2.42 \text{ m/s}^2)\,\hat{i} + (5.45 \text{ m/s}^2)\,\hat{j}$$

How fast, in meters per second, is the rocket moving 3.79s after the rockets fire?

$\vec{v}(t) = \vec{v}_0 + \vec{a}t$

$(21.9 \text{ m/s})\hat{i} + (2.42)\hat{i}t + (5.45 \text{ m/s}^2)t\,\hat{j}$

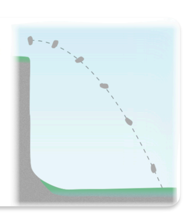
$(21.9)\hat{i} + (2.42)(3.79)\hat{i} + (5.45)(3.79)\hat{j}$

$|\vec{v}(3.79)| = 31.0718\hat{i} + 20.655\hat{j}$

$|\vec{v}(3.79)| = \sqrt{(31.0718)^2 + (20.655)^2}$

$|\vec{v}(3.79)| = 37.3109$

Problem 9: (5% of Assignment Value)
A stone is thrown off of a cliff with an initial velocity that is horizontal. There is no air resistance, and it follows the path shown which is typical for projectile motion.



What direction is the acceleration of the stone?

Since a in this case is gravity, this means

$a = -g$, where $g = 9.81$ m/s

∴ ↓ Graphically

Problem 10: (5% of Assignment Value)
A projectile is launched from ground level at an angle of 30 degrees above the horizontal. Neglect air resistance and consider the motion from just after the moment it is launched to just before the moment it lands on the ground.

a) When is the projectile's velocity equal to zero?

The projectile's velocity is composed of both horizontal and vertical. The horizontal component of velocity remains costant througтout the motion (since air resistance is neglected). The vertical component of velocity decrease at the projectile rises, reaching zero at the highest point of its trajectory.

$V_x = V_0\cos(30°)$ & $V_y = V_0\sin(30°) \rightarrow |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

This means total velocity of the projectile is never zero, because the horizontal component is always zero.

∴ The projectile's velocity is never zero.

b) When does the projectile have the smallest speed?

The speed of a projectile is the magnitude of its velocity which is composed of both horizontal and vertical components. The horizontal component of the velocity remains constant throughout the motion (since there's no horizontal acceleration and air resistance is neglected). The vertical component of the velocity decreases as the projectile rises, reaches zero at the highest point, and then increases again (in downward direction) as the projectile falls back down.

At the highest point, vertical component, $v_y = 0$, so the total speed is just the horizontal velocity, v_x, which is the smallest speed during the motion. The speed is not zero here, but it is at its minimum since only the horizontal component remains.

∴ At the highest point!

c) When does the projectile's speed equal its launch speed?

Launch speed refers to the magnitude of the velocity at the moment of launch; since the projectile is launched at an angle, its initial speed consists of both a horizontal component v_x and a vertical component v_y. During its motion, the projectile's speed changes due to the influence of gravity on the vertical component of its velocity. However, air resistance is neglected; which means that gravity only affects the vertical component.

At launch [projectile's speed comp]

$V_{0x} = V_0\cos\theta$ (horizontal compnt)

$V_{0y} = V_0\sin\theta$ (vertical compnt)

Total launch speed: $V_0 = \sqrt{V_{0y}^2 + V_{0x}^2}$

At the highest point, the vertical velocity component is $v_y = 0$; so the speed is only the horizontal component, which is less than the launch speed.

Just before landing, the projectile's horizontal velocity is the same as it was at launch because there is no horizontal acceleration. The vertical velocity just before landing is equal in magnitude (but opposite in direction) to the vertical velocity at launch, because the projectile follows a symmetric path under influence of gravity.

Before landing [total speed]

$V = \sqrt{v_x^2 + (-v_y)^2} = V_0$ ∴ the projectile's speed before landing is equal to launch speed

∴ The projectile's speed equals its launch speed just before landing on the ground.

d) After launch, when does the projectile's velocity equal its launch velocity?

The launch velocity vector is a vector quantity that has both magnitude (speed) and direction. At launch the velocity is composed of: a horizontal component, $V_{0x} = V_0\cos(\theta)$, which remains constant throughout the projectile's flight; a verticle component, $V_{0y} = V_0\sin(\theta)$, which changes due to acceleration

Key topics/points
- Symmetry of motion — Motion is symmetric
- At launch → Velocity vector point upward @ angle θ; Horiz. velocity stays constant. Vert velocity increases as projectile rises.
- At highest point → Vert compnt is equal to 0. Horz. compnt remains. ∴ Total velocity ≠ launch velocity @ this point.
- Just before landing → Vert compnt is same as launch but points downward; Horz. compnt. remains the same; the total velocity vector is equal in magnitude and direction to the launch velocity.

∴ Projectile's velocity just before it lands will have same magnitude as launch velocity but will point to opposite direction

Velocity components at launch and just before landing:

At launch:

$\vec{V}_{launch} = V_0\cos(\theta)\hat{i} + V_0\sin(\theta)\hat{j}$

Just before launch:

$\vec{V}_{landing} = V_0\cos(\theta)\hat{i} - V_0\sin(\theta)\hat{j}$

Both velocities have the same magnitude, meaning the projectile's velocity just before landing equals its launch velocity

∴ The projectile's velocity to its launch velocity just before landing on the ground.

$R = \frac{V_0^2\sin(2\theta)}{g}$

$\frac{(30)^2\sin(2(45))}{9.8} = 91.83673$

$\frac{40^2\sin(2(45))}{9.8} = 163.2653$

$\frac{50^2\sin(2(45))}{9.8} = 255.1020$

$V = a \cdot t = 2.97 \cdot 0.98$

$= 2.9106$

$a_c = \frac{v^2}{r} = \frac{(2.9106)^2}{2.51}$

$a_c = 3.3751$

$a_T = \sqrt{(a_c)^2 + (a_t)^2} = 4.4958$

$a_t = \frac{v_f - v_i}{2.75} = -4.2545$

$a_c = \frac{v^2}{r} = \frac{48^2}{294} = 7.8367$

$a_T = \sqrt{(7.8367)^2 + (-4.2545)^2}$

$= 8.9171$

$a_c = \frac{v^2}{r} = \frac{(26.7)^2}{11.9} = a_c$ (centripetal acceleration)

$= \frac{712.89}{11.9} = 59.9067$

$V_{0x} = V_0\cos(\theta)$

$V_x^2 = V_0^x t + x_1$

$t = \frac{V_0^x}{g}$

$x_{max} = 46.7\cos(36.5)\cdot(3.83)$

$x_{max} = 143.8020$

$V_{0y} = V_0\sin(\theta)$

$V_y^2 = V_{0y}^2 + 2ay$; $y = y_{max}$ when $v_y^2 = 0$

$0 = v_{0y}^2 + 2ay$

$\frac{-v_{0y}^2}{2a} = y_{max}$; $a = g = -9.8 \text{ m/s}^2$

$-\frac{(46.7\sin(36.5))^2}{2(-9.8)} = \frac{-771.6297}{-19.6} = \boxed{39.368864 = y_{max}}$

$x_f = x_i + 22\cos(57)t$

$x_f = x_i + 22\cos(57)t$, where $t = 3.54$

$x_i + 22\cos(57)(3.54)$

$x_f = x_i + \boxed{42.4164}$

$y_f = y_i + V_0 t + \frac{1}{2}(-9.8)t^2$

$y_f = (22\sin(57))t + \frac{9.8}{2}t^2$

$3.9 = 18.45t + 4.9t^2$

-3.9

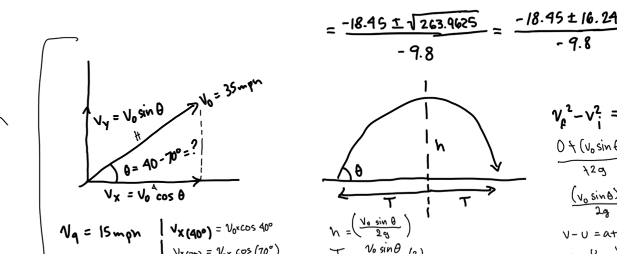
$18.45t + 4.9t^2 - 3.9$ (B, A, C)

$Q_{form} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-18.45 \pm \sqrt{(18.45)^2 - 4(4.9)(3.9)}}{2(4.9)} = \frac{-18.45 \pm \sqrt{340 - 76.44}}{9.8}$

$= \frac{-18.45 \pm \sqrt{263.9625}}{-9.8} = \frac{-18.45 \pm 16.2469}{-9.8} = 3.6405$ or 0.2248

Question 13b



$V_0 = 15$ mph

$V_x(40°) = V_0\cos 40°$

$V_x(70°) = V_{0x}\cos(70°)$

CAH

$\cos(\theta) = \frac{A}{H} = \frac{V_x}{V_0} = \frac{V_{0y}\sin\theta}{V_{0x}\cos\theta}$

$\cos^{-1}(\cos\theta) = \left(\frac{15}{35}\right)\cos^{-1}$

$\boxed{\theta = \cos^{-1}\left(\frac{15}{35}\right) = 64.623}$

$h = \left(\frac{V_0\sin\theta}{2g}\right)$

$T = \frac{V_0\sin\theta}{g}(2)$

$R = \Delta x = V_x \times T$

$\frac{V_0\cos\theta \cdot 2V_0\sin\theta}{g}$

$\frac{V_0\sin 2\theta}{g} = R$

$v_f^2 - v_i^2 = 2a\,\Delta x$ ← $v_f^2 = v_i^2 + 2a\,\Delta x$ [original equation found in the sheet.]

$0 + (V_0\sin\theta)^2 = -2gH$

$\frac{(V_0\sin\theta)^2}{2g} = h$

$V - U = at$

$t = \frac{U}{g} = \frac{V_0\sin\theta}{g}$

Question 11e

Total time $= 2t$

$\theta = 36.5$
$V_0 = 46.7$
$g = -9.8$

$R = x_{max} = V_x \cdot T$

$d = v_0 t + \frac{1}{2}at^2$

$t = \frac{V_0\sin(\theta)}{g}$ $\quad x_{max} =$

$t = \frac{46.7\sin(36.5)}{9.8} = 2.8345 = \frac{V_0\sin\theta}{-g}$

$V = V_0 + at$

$0 = V_0\sin\theta - gt$

$2t = T = 5.66$

$((V_0\cos\theta)t)[t]\,0 = \left(V_0\cos\theta \cdot \frac{V_0\sin\theta}{g}\right)2$

$R = \frac{2V_0^2\cos\theta\sin\theta}{g} = 212.8158$

Use first initial velocity → "just as it begins to reduce its speed"

$\Delta x = d$
$\Delta y = h$

$x_1 - x_0 = V_0 t + \frac{1}{2}at^2$
Δx

$\vec{V}_w = [0.203\hat{i} + 1.01\hat{j} \text{ m/s}]$

$\vec{a}_b = 0.02\hat{i}$

$|\vec{v}| = S$

Velocity = speed and direction

$|\Delta x|$ = distance

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$

$= 1.43$ m/s

$\vec{V}_b(t = 4.01) = ?$

$\vec{V}_b = \vec{a}\cdot t$

$\vec{V}_b = 0.0202\hat{i}\cdot t$

$\vec{V}_b = 0.0202\hat{i}\,t$

$\vec{r} = r_x\hat{i} + r_y\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

$\Delta x = V_{i,x}t\,(+)\,\frac{1}{2}at^2$

$x = (0.203)t$

$\Delta y = V_{i,y}t = 1.01 + 4.01$

$|\vec{V}_b|t\,\hat{i}$

$\frac{\Delta x}{t} = \frac{1}{2}(V_f + V_i)$ → $\bar{V}$

$\frac{\Delta x}{\Delta t} = \frac{1}{2}(V_f + V_i)$

$x = (0.203) + (t)\frac{1}{2}\,0.202\,t^2$

$\vec{r} = \left[(0.203)(4.01) + \frac{1}{2}\,0.202\,(4.01)^2\right]\hat{i} + (1.01 \cdot 4.01)\hat{j}$

$\vec{r} = \sqrt{(x_{comp})^2 + (y_{comp})^2}$