Begin Date: 8/27/2024 12:01:00 AM -- **Due Date:** 9/6/2024 11:59:00 PM **End Date:** 9/6/2024 11:59:00 PM

Problem 1:

When converting units, the presentation of a number is altered, but its value is unchanged. Conversion of units is achieved upon multiplication by a conversion factor which is a ratio equivalent to 1 but involving both types of units. For example, because

$$100 \text{ cm} = 1 \text{ m}$$

the following expressions all suggest candidate conversion factors:

$$1 = \frac{100 \text{ cm}}{1 \text{ m}} \qquad 1 = \frac{1 \text{ cm}}{10^{-2} \text{ m}} \qquad 1 = \frac{1 \text{ m}}{100 \text{ cm}} \qquad 1 = \frac{10^{-2} \text{ m}}{1 \text{ cm}}$$

Choose any valid conversion factor that cancels the old units while leaving the desired units in place.

Part (a) **Select *all* expressions that could be used to convert a distance $x = 1.07$ m to centimeters.**

The initial expression for x has meters in the numerator. The conversion factor should have meters in the denominator to cancel the original factor, and it should have centimeters in the numerator to provide the units for the resulting expression. Centimeters are small compared to meters, so it will take many to equal a single meter. Such an expression is given by

$$x = 1.07 \text{ m} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$$

Another perspective is that one centimeter is a fraction of a meter, and hence

$$x = 1.07 \text{ m} \times \left(\frac{1 \text{ cm}}{10^{-2} \text{ m}}\right)$$

$$\boxed{x = 1.07 \text{ m} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right) \quad \text{AND} \quad x = 1.07 \text{ m} \times \left(\frac{1 \text{ cm}}{10^{-2} \text{ m}}\right)}$$

Part (b) **Use an expression of the type determined in step (a) to convert the distance $x = 0.55$ m to centimeters.**

Use either

$$x = 0.55 \text{ m} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)$$

or

$$x = 0.55 \text{ m} \times \left(\frac{1 \text{ cm}}{10^{-2} \text{ m}}\right)$$

$$\boxed{x = 55.00 \text{ cm}}$$

Part (c) **Select *all* expressions that could be used to convert a length $L = 11.2$ cm to meters.**

The initial expression for L has centimeters in the numerator. The conversion factor should have centimeters in the denominator to cancel the original factor, and it should have meters in the numerator to provide the units for the resulting expression. Centimeters are small compared to meters, so it will take many to equal a single meter. Such an expression is given by

$$L = 11.2 \text{ cm} \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)$$

Another perspective is that one centimeter is a fraction of a meter, and hence

$$L = 11.2 \text{ cm} \times \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}}\right)$$

$$\boxed{L = 11.2 \text{ cm} \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \quad \text{AND} \quad L = 11.2 \text{ cm} \times \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}}\right)}$$

Part (d) **Use an expression of the type determined in step (c) to convert the length $L = 27.3$ cm to meters.**

Use either

$$L = 27.3 \text{ cm} \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)$$

Another perspective is that one centimeter is a fraction of a meter, and hence

$$L = 27.3 \text{ cm} \times \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}}\right)$$

$$\boxed{L = 0.2730 \text{ m}}$$

Problem 2:

When converting units, the presentation of a number is altered, but its value is unchanged. Conversion of units is achieved upon multiplication by a conversion factor which is a ratio equivalent to 1 but involving both types of units. The exponent on a conversion factor must match the exponent on the units. When converting more than one type of unit, include a separate conversion factor for each type.

Part (a) **Select *all* expressions that could be used to convert a density of $\rho = 0.82$ g/cm^3 to kilograms per cubed meter.**

A first conversion factor is required that involves both centimeters and meters. As a first step, let's re-express 1 as follows.

$$1 = \frac{1 \text{ cm}}{1 \text{ cm}}$$

It is necessary to have centimeters appear in the numerator in order to cancel the same units in the denominator of $\rho = 0.82$ g/cm^3. Looking at the definition of "centi", the "c" prefix is equivalent to a factor of 10^{-2}. Upon making this substitution,

$$1 = \frac{10^2 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ cm}}{10^{-2} \text{ m}}$$

Applying the basic properties of exponents, the fraction in the middle becomes the fraction on the right. The first ratio states that one hundred centimeters is the same as one meter. The second ratio states that one centimeter is a hundredth of a meter. These are reasonable sounding statements. The conversion factor must be applied to each factor of centimeters; since centimeters are squared, so is the conversion factor.

A second conversion factor is required that involves both grams and kilograms. As a first step, let's re-express 1 as follows.

$$1 = \frac{1 \text{ kg}}{1 \text{ kg}}$$

It is necessary to have grams appear in the denominator in order to cancel the same units in the numerator of $\rho = 0.82$ g/cm^3. Looking at the definition of "kilo", the "k" prefix is equivalent to a factor of 10^3. Upon making this substitution,

$$1 = \frac{1 \text{ kg}}{10^3 \text{ g}} = \frac{10^{-3} \text{ kg}}{1 \text{ g}}$$

Applying the basic properties of exponents, the fraction in the middle becomes the fraction on the right. The first ratio states that one thousand grams is the same as one kilogram. The second ratio states that one grams is a thousandth of a kilogram. These are reasonable sounding statements.

The correct expressions, incorporating three powers of the length conversion and a single instance of the mass conversion, from those listed are

$$\boxed{\rho = (0.82 \text{ g/cm}^3)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right) \text{ AND } \rho = (0.82 \text{ g/cm}^3)\left(\frac{1 \text{ cm}}{10^{-2} \text{ m}}\right)^3\left(\frac{10^{-3} \text{ kg}}{1 \text{ g}}\right)}$$

There are additional ways of formatting the conversion factor that are not listed.

Part (b) **Use an expression of the type determined in step (a) to convert the density of $\rho = 0.93$ g/cm^3 to kilograms per cubed meter.**

Using one of the correct choices from step (a),

$$\rho = (0.93 \text{ g/cm}^3)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)$$

$$= (0.93 \text{ g/cm}^3)\left(\frac{10^3 \text{ cm}^3 \cdot \text{kg}}{\text{m}^3 \cdot \text{g}}\right)$$

$\rho = 930.0 \text{ kg/m}^3$

Problem 3:

Convert the angle, specified in radians, to an angle specified in degrees.

Part (a) **Which angle is the same as an angle of $\frac{2\pi}{3}$ radians?**

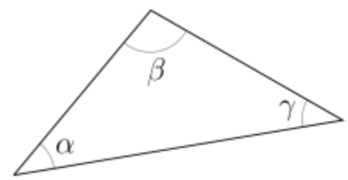
Noting that one-half of a circle is π radians, or equivalently 180°, a conversion factor may be applied.

$$\frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$$

120°

Problem 4:

The angles of a scalene triangle are labeled, as shown. The relative sizes of the angles may be different than those in the image.



Part (a) **Given that $\alpha = 35.3^\circ$ and $\beta = 52.4^\circ$, enter the value, in degrees, of the angle measure of γ.**

The sum of the angle measures of a triangle is always 180°, or π, in radians.

$$\alpha + \beta + \gamma = 180^\circ$$

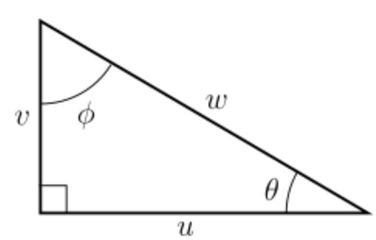
Solving for the one unknown,

$$\begin{aligned}\gamma &= 180^\circ - \alpha - \beta \\ &= 180^\circ - 35.3^\circ - 52.4^\circ\end{aligned}$$

$\gamma = 92.30^\circ$

Problem 5:

The sides and angles of a right-triangle are labeled in the drawing. While it is definitely a right triangle, do *not* assume that the side lengths and the angles are drawn to scale. Side lengths v and w are the only given parameters, and the rest are unknowns.



Part (a) **Which of the following provides a simple relationship between v, w, and the unknown u?**

The Pythagorean theorem relates three side lengths. Each trigonometric function relates an angle to the ratio of two side lengths. Since the third side length is requested, and the first two are provided, a relationship involving all three side lengths is required.

The Pythagorean theorem.

Part (b) **Enter a simple expression that uses the relationship chosen in the previous step to solve for the unknown u in terms of the parameters given in the problem statement.**

The Pythagorean theorem applies to the side lengths of a right triangle. The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. In the case at hand, the lengths of the legs are u and v while the length of the hypotenuse is w, and hence

$$u^2 + v^2 = w^2$$

(The most common error is a sign error caused by improperly assigning the legs and the hypotenuse.) Solving for u,

$$\boxed{u = \sqrt{w^2 - v^2}}$$

Since u is a length, it must be positive, and, hence, the negative root may be ignored.

Part (c) **Given $v = 12.2$ and $w = 25.2$, enter a numeric value for u.**

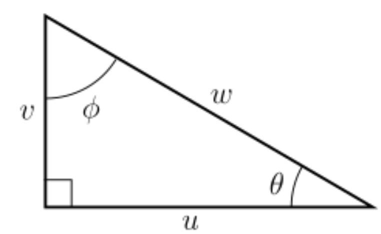
Beginning with the symbolic expression from the previous step, substitute numeric values, and simplify.

$$\begin{aligned} u &= \sqrt{w^2 - v^2} \\ &= \sqrt{(25.2)^2 - (12.2)^2} \end{aligned}$$

$$\boxed{u = 22.05}$$

Problem 6:

The sides and angles of a right-triangle are labeled in the drawing. While it is definitely a right triangle, do *not* assume that the side lengths and the angles are drawn to scale. Side lengths v and w are the only given parameters, and the rest are unknowns.



Part (a) **Which of the following provides a simple relationship between v, w, and the unknown ϕ?**

The Pythagorean theorem relates three side lengths. Each trigonometric function relates an angle to the ratio of two side lengths. Since an angle is involved, a trigonometric function is indicated. Since v is the length of the side adjacent to angle ϕ, and w is the length of the hypotenuse, the cosine function is indicated.

$\boxed{\text{The cosine function.}}$

Part (b) **Given $v = 12.2$ and $w = 25.2$, enter a numeric value, in degrees, for ϕ.**

The cosine function is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse.

$$\cos\phi = \frac{v}{w}$$

Solving for the angle and substituting numbers,

$$\begin{aligned} \phi &= \cos^{-1}\left(\frac{v}{w}\right) \\ &= \cos^{-1}\left(\frac{12.2}{25.2}\right) \end{aligned}$$

$\boxed{\theta = 61.04°}$

Problem 7:

In this problem, the symbols x and y are lengths, and n is an integer.

Part (a) **What should be the value of the exponent *n* so that the formula $\pi x^n y^1$ represents a volume?**

Volume has the dimensions of length cubed, so $n + 1 = 3$, so

$\boxed{n = 2.000}$

Part (b) **What should be the value of the exponent *n* so that the formula $4\pi x^n$ represents an area?**

Area has the dimensions of length squared, so

$\boxed{n = 2}$

Problem 8:

The units on each side of an equation should be the same, or the equation is wrong. You will determine which units belong on each side of an equation.

Quantity	Symbol	Unit	Abbreviation
mass	m	kilogram	kg
distance	x	meter	m
height	h	meter	m
time	t	second	s
velocity	v	meter/second	m/s
acceleration	a	meter/second²	m/s^2
acceleration due to gravity	g	meter/second²	m/s^2
Newton's gravitational constant	G	meter³/ kilogram*second²	$m^3/kg \cdot s^2$

Part (a) **What are the units on both sides of the equation: $v^2 = 2ax$?**

The given equation is $v^2 = 2ax$.

The unit on the left side is $(m/s)^2 = m^2/s^2$. The unit on the right side is $(m/s^2)(m) = m^2/s^2$. They are the same.

The answer is, therefore, ...

$\boxed{m^2/s^2}$

Part (b) **What units are on both sides of the equation: $x = vt + 1/2at^2$?**

The given equation is $x = vt + 1/2at^2$.

The unit on the left side is m. The unit of the first term on the right side is $(m/s)(s) = m$. The unit of the second term on the right side is $(m/s^2)(s)^2 = m$. They are all the same.

The answer is, therefore, ...

$\boxed{m}$

Part (c) **What are the units on both sides of the equation $mv^2 = mgh$?**

The given equation is $mv^2 = mgh$.

The unit on the left side is $(kg)(m/s)^2 = kg \cdot m^2/s^2$. The unit on the right side is $(kg)(m/s^2)(m) = kg \cdot m^2/s^2$. They are the same.

The answer is, therefore, ...

$\boxed{\text{kg} \cdot \text{m}^2/\text{s}^2}$

Part (d) **What are the units on each side of the equation $GMm/x^2 = mv^2/x$? (M and m are two different masses.)**

The given equation is $GMm/x^2 = mv^2/x$.

The unit on the left side is $(\text{m}^3/\text{kg}\cdot\text{s}^2)(\text{kg})(\text{kg})/(\text{m})^2 = \text{kg}\cdot\text{m/s}^2$. The unit on the right side is $(\text{kg})(\text{m/s})^2/(\text{m}) = \text{kg}\cdot\text{m/s}^2$. They are the same.

The answer is, therefore, ...

$\boxed{\text{kg} \cdot \text{m/s}^2}$

Problem 9:

The density of a particular alloy is ***2.1*** g/cm^3.

Part (a) **What is this density in kg/m^3?**

Here we need to do a unit conversion that involves two conversions. There are 100 centimeters in one meter and 1000 grams in one kilogram.

$$\rho = \frac{2.1\text{ g}}{1\text{ cm}^3\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3}\left(\frac{1\text{ kg}}{1000\text{ g}}\right)$$

$$\boxed{\rho = 2100.\text{ kg/m}^3}$$

Problem 10:

A light-nanosecond is the distance light travels in 1 ns. A tomato plant has a height of ***4.05*** feet.

Part (a) **What is this height in light-nanoseconds? (Light travels at a speed of about 3.00×10^8 m/s.)**

Our goal here is to find how many light nanoseconds is equal to 4.05 feet. To begin, let's find how far a light nanosecond is in meters. We are told that light travels at 3.00×10^8 m/s and we can look up how the prefix "nano" affects seconds to find that one nanosecond is equal to 10^{-9} seconds. The distance light travels in one nanosecond is therefore:

$$\left(\frac{3.00\times10^8\text{ m}}{1\text{ s}}\right)\left(10^{-9}\text{ s}\right) = 0.3\text{ m}$$

Now that we have the distance of a light nanosecond in meters, we can convert this to feet.

$$(0.3\text{ m})\left(\frac{3.28084\text{ ft}}{1\text{ m}}\right) = 0.984252\text{ ft}$$

To find the solution, we can convert 4.05 feet to nanoseconds.

$$(4.05\text{ ft})\left(\frac{1\text{ light-nanoseconds}}{0.984252\text{ ft}}\right) = 4.115\text{ light-nanoseconds}$$

The correct answer is therefore:

$\boxed{4.115\text{ light-nanoseconds}}$

Problem 11:

There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3 kg/m^3. The nucleus of an atom has a radius about 10^{-5} times that of the entire atom, and contains nearly all the mass of the atom.

Part (a) **What is the approximate density, in kilograms per cubic meter, of a nucleus?**

The volume of an object with an average density is

$$V = \frac{m}{\rho_0} \text{ m}^3$$

where m is the mass in kg and ρ_0 is the average density in kg/m^3. Therefore,

$$\rho_0 = \frac{m}{V}$$

where V is the volume in m^3. The volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius in m. If

$$r_1 = 10^{-5} \cdot r$$

then

$$\rho = \frac{m}{V_1} = \frac{m}{\left(\frac{4}{3}\pi\left(r \cdot 10^{-5}\right)^3\right)} = 10^{15} \cdot \frac{m}{\left(\frac{4}{3}\pi r^3\right)} = 10^{15} \cdot \frac{m}{V} = 10^{15} \cdot \rho = 10^{15} \cdot 10^3 \text{ kg/m}^3$$

$$\boxed{\rho = 1.000 \times 10^{18} \text{ kg/m}^3}$$

Part (b) **One possible remnant of a supernova, called a neutron star, can have the density of a nucleus, while being the size of a small city. What would be the radius, in kilometers, of a neutron star with a mass 10 times that of the Sun? The radius of the Sun is 7 × 10^8 m and its mass is 1.99 × 10^{30} kg.**

The volume of an object with an average density is

$$V = \frac{m}{\rho_0} \text{ m}^3$$

where m is the mass in kg and ρ_0 is the average density in kg/m^3. Therefore,

$$\rho_0 = \frac{m}{V}$$

where V is the volume in m^3. The volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius in m. Solving for the radius,

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

$$r = \left(\frac{3}{4}\frac{m}{(\rho\pi)}\right)^{\frac{1}{3}}$$

Plugging in numbers and converting units as needed,

$$r = R = \left(\frac{\left(\frac{3}{4} \cdot 10 \cdot 1.99 \cdot 10^{30}\ \text{kg}\right)}{\left(10^{18}\ \text{kg/m}^3 \cdot \pi\right)} \right)^{\frac{1}{3}} \cdot 10^{-3}\ \text{km/m}$$

$$\boxed{R = 16.81\ \text{km}}$$

Problem 12:

Consider these points:

$$A = (-9, 2)$$
$$B = (2, -9)$$
$$C = (-9, -9)$$

which are placed in three different quadrants of a Cartesian coordinate system. Convert each set of Cartesian coordinates to polar coordinates of the form (r, θ) where the angle is measured counterclockwise, in degrees, from the positive x axis.

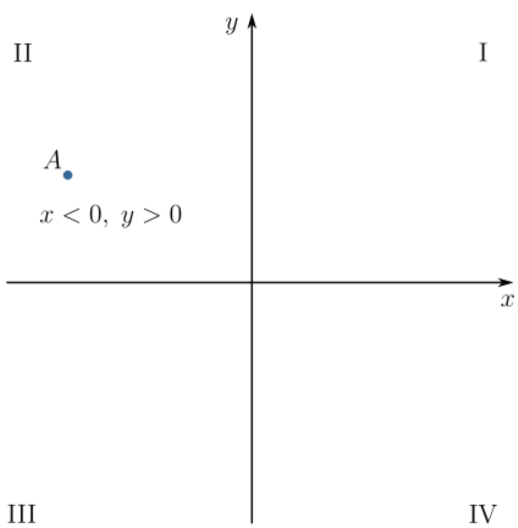
$$0^\circ \leq \theta < 360^\circ$$

Part (a) **What quadrant does point A occupy?**

For the point

$$A = (-9, 2)$$

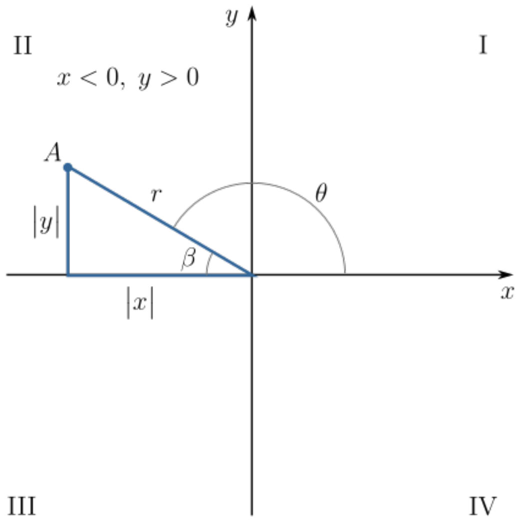
the x coordinate is negative, placing the point in the left half-plane. The y coordinate is positive, placing the point in the upper half-plane. Together this places the point in quadrant II as shown below. The graph below is qualitatively correct, but based upon your randomized coordinates, the precise placement within the quadrant may be a little different.



Quadrant II

Part (b) **What is the radial coordinate, r_A, for point A?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point A.



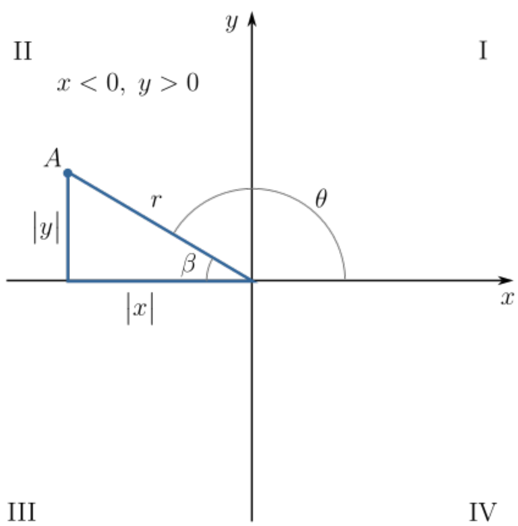
The radial coordinate is the hypotenuse of that triangle. Since the lengths are squared in the formula, we can drop the absolute-value bars. We find,

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-9)^2 + (2)^2}$$

$r = 9.220$

Part (c) **What is the angular coordinate, θ_A , for point A, expressed in degrees measured counterclockwise from positive x?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point A .



First find the vertex angle of that triangle at the origin.

$$\tan \beta = \frac{|y|}{|x|}$$
$$= \frac{|2|}{|-9|}$$
$$= 0.2222$$

and

$$\beta = \tan^{-1}\left(0.2222\right)$$
$$= 12.53^\circ$$

Note that the two angles shown are supplementary, meaning they sum to a line.

$$\theta_A + \beta = 180^\circ$$

so

$$\theta_A = 180^\circ - \beta$$
$$= 180^\circ - 12.53^\circ$$

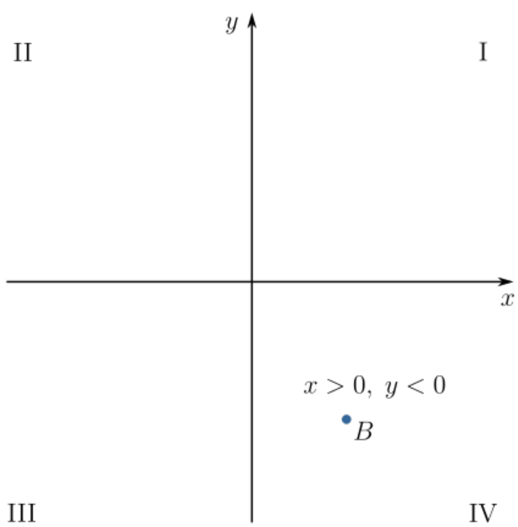
$\theta_A = 167.5^\circ$

Part (d) **What quadrant does point B occupy?**

For the point

$$B = (2, -9)$$

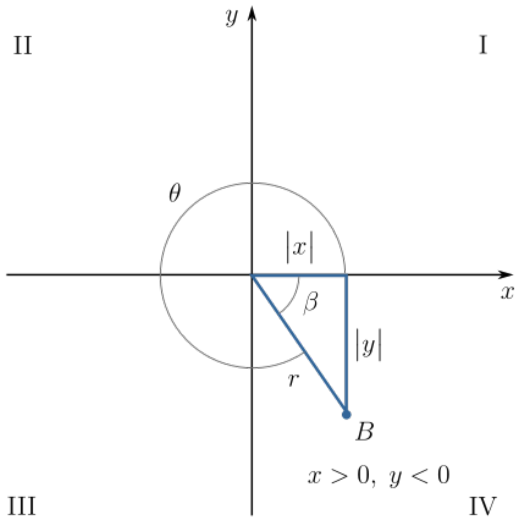
the x coordinate is positive, placing the point in the right half-plane. The y coordinate is negative, placing the point in the lower half-plane. Together this places the point in quadrant IV as shown below. The graph below is qualitatively correct, but based upon your randomized coordinates, the precise placement within the quadrant may be a little different.



Quadrant IV

Part (e) **What is the radial coordinate, r_B, for point B?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point B.



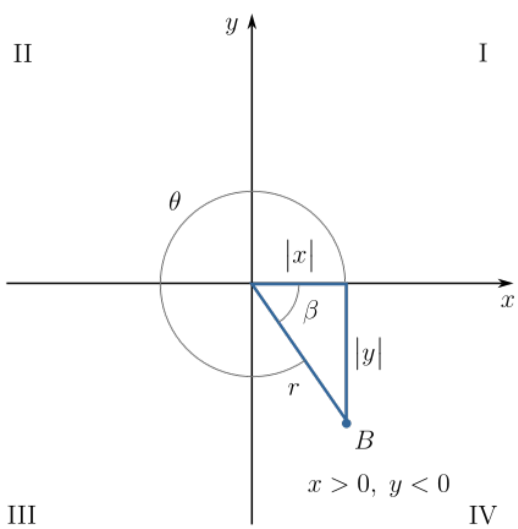
The radial coordinate is the hypotenuse of that triangle. Since the lengths are squared in the formula, we can drop the absolute-value bars. We find,

$$r_B = \sqrt{x^2 + y^2}$$
$$= \sqrt{(2)^2 + (-9)^2}$$

$r_B = 9.220$

Part (f) **What is the angular coordinate, θ_B, for point B, expressed in degrees measured counterclockwise from positive x?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point B.



First find the vertex angle of that triangle at the origin.

$$\tan\beta = \frac{|y|}{|x|}$$
$$= \frac{|-9|}{|2|}$$
$$= 4.500$$

and

$$\beta = \tan^{-1}\left(4.500\right)$$
$$= 77.47^\circ$$

Note that the two angles shown sum to a full circle.

$$\theta_B + \beta = 360^\circ$$

so

$$\theta_B = 360^\circ - \beta$$
$$= 360^\circ - 77.47^\circ$$

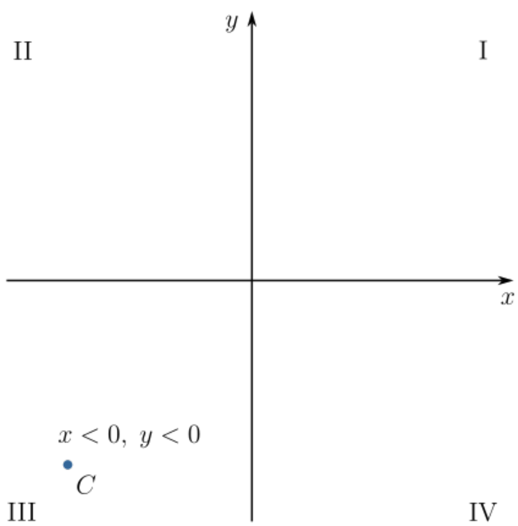
$\boxed{\theta_B = 282.5^\circ}$

Part (g) **What Quadrant does point C occupy?**

For the point

$$C = (-9, -9)$$

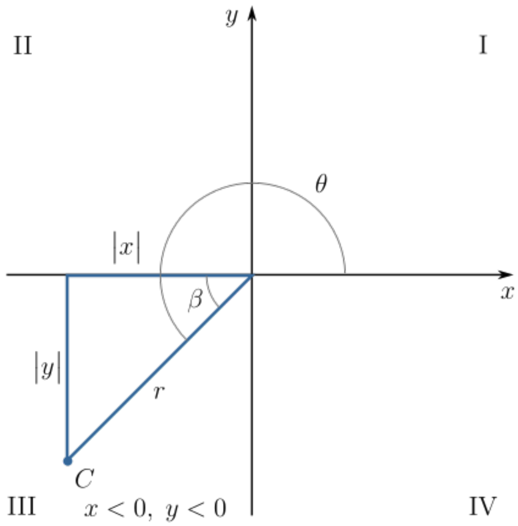
the x coordinate is negative, placing the point in the left half-plane. The y coordinate is negative, placing the point in the lower half-plane. Together this places the point in quadrant III as shown below. The graph below is qualitatively correct, but based upon your randomized coordinates, the precise placement within the quadrant may be a little different.



$\boxed{\text{Quadrant III}}$

Part (h) **What is the radial coordinate, r_C, for point C?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point C.



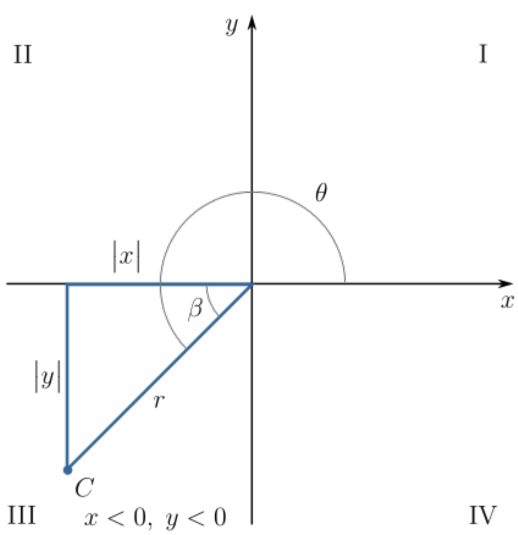
The radial coordinate is the hypotenuse of that triangle. Since the lengths are squared in the formula, we can drop the absolute-value bars. We find,

$$\begin{aligned} r_C &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-9)^2 + (-9)^2} \end{aligned}$$

$$\boxed{r_C = 12.73}$$

Part (i) **What is the angular coordinate, θ_C, for point C, expressed in degrees measured counterclockwise from positive x?**

The graph below is qualitatively correct, but it may not reflect the precise randomized coordinates in your version of the problem. Note that the side lengths of the triangle shown are given by the magnitudes of the x and y coordinates of point C.



First find the vertex angle of that triangle at the origin.

$$\begin{aligned} \tan\beta &= \frac{|y|}{|x|} \\ &= \frac{|-9|}{|-9|} \\ &= 1.000 \end{aligned}$$

and

$$\beta = \tan^{-1}(1.000)$$
$$= 45.00^\circ$$

Note that the difference of the two angles shown is a line.

$$\theta_C - \beta = 180^\circ$$

so

$$\theta_C = 180^\circ + \beta$$
$$= 180^\circ + 45.00^\circ$$

$$\boxed{\theta_C = 225.0^\circ}$$

Problem 13:

A fly enters through an open window and zooms around the room. In a Cartesian coordinate system with three axes along three edges of the room, the fly changes its position from point (4.00 m, 1.50 m, 2.50 m) to (***0.51*** m, ***2.5*** m, ***0.51*** m).

Part (a) **What is the magnitude of the fly's displacement?**

The vector $\Delta\vec{r}$ that starts at the first point and ends at the second is

$$\Delta\vec{r} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$$

$$= (0.51 \text{ m} - 4.00 \text{ m})\,\hat{i} + (2.5 \text{ m} - 1.50 \text{ m})\,\hat{j} + (0.51 \text{ m} - 2.50 \text{ m})\,\hat{k}$$

$$= (-3.490 \text{ m})\,\hat{i} + (1.000 \text{ m})\,\hat{j} + (-1.990 \text{ m})\,\hat{k}$$

Applying the Pythagorean theorem to the components gives the magnitude:

$$\Delta r = \sqrt{(-3.490 \text{ m})^2 + (1.000 \text{ m})^2 + (-1.990 \text{ m})^2}$$

Calculating,

$$\boxed{\Delta r = 4.140 \text{ m}}$$

Problem 14:

The distance from San Francisco to Sacramento is shown in the figure.

Part (a) **Find the component in the northern direction of the displacement from San Francisco to Sacramento in kilometers.**

The northern component of the displacement is related to the displacement and the angle with respect to east by the expression

$$\sin(\theta) = \frac{S_N}{S}$$

where S_N is the northern component in km, S is the displacement in km, and θ is the angle north of east. Therefore,

$S_N = Ssin(\theta)$

Plugging in numbers and converting units as needed,

$S_N = 123 \cdot sin(45°)$

$$\boxed{S_N = 86.97 \text{ km}}$$

Part (b) **Find the component in the eastern direction of the displacement from San Francisco to Sacramento in kilometers.**

The eastern component of the displacement is related to the displacement and the angle with respect to east by the expression

$$cos(\theta) = \frac{S_E}{S}$$

where S_E is the eastern component in km, S is the displacement in km, and θ is the angle north of east. Therefore,

$S_E = Scos(\theta)$

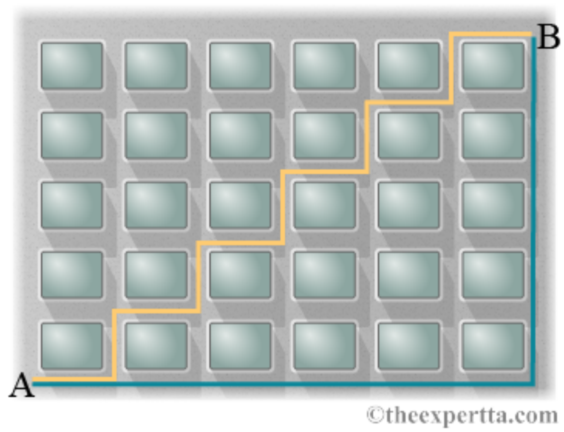
Plugging in numbers and converting units as needed,

$S_E = 123 \cdot cos(45°)$

$$\boxed{S_E = 86.97 \text{ km}}$$

Problem 15:

You are in Manhattan and need to walk from point A to point B. The person you are with suggests that you will get there quicker if you walk directly towards your destination. Given that buildings are in the way, the most "direct" route results in zig-zagging as shown in the figure, since you are only able to travel exactly east or exactly north. Consider the figure and compare the yellow zig-zag path with the blue-green single-turn path.



Part (a) **Which is the true statement?**

In general, if the blocks have a width a_i m and a length b_i m, the path with one turn will be a distance of

$$d_1 = \left(\sum a_i + \sum b_i\right) \text{ m}$$

But going around block i means a length $a_i + b_i$, so

$$d_2 = \sum (a_i + b_i) = \sum a_i + \sum b_i$$

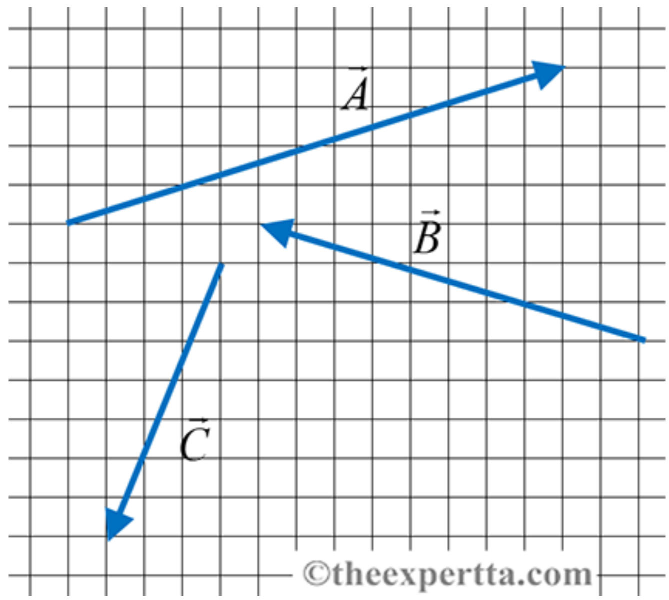
Thus,

$$d_1 = d_2$$

and the smae distance is traveled.

Problem 16:

When two vectors are added together graphically, one can draw the vectors so that the tail of one vector touches the tip of the other. Consider the three vectors shown in the figure labeled $\vec{A}, \vec{B},$ and $\vec{C}$.



Part (a) **If the three vectors in the figure are added together $\vec{A} + \vec{B} + \vec{C}$ using the graphical method, in approximately what direction will the resultant be?**

In this problem, we are investigating **vector addition**.

For this case, adding the vectors graphically gives zero displacement, meaning that the final point of the path coincides with the initial point (in the figure below, the vectors form a closed triangle).

Therefore, this resultant vector has no length and therefore no direction.

Images

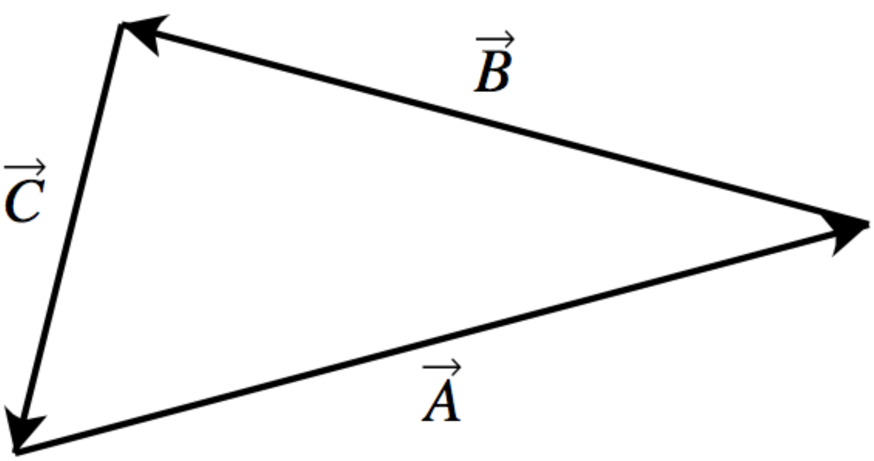
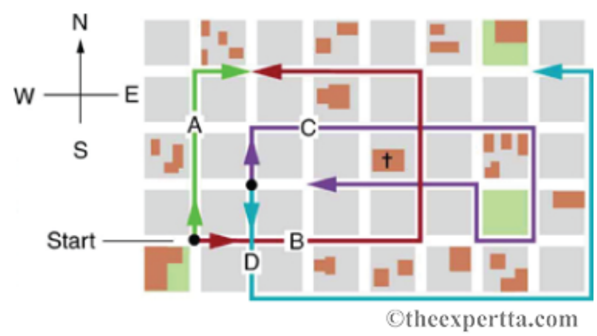


Figure 1. The three vectors, added graphically, give a resultant vector of zero.

Problem 17:

The various lines represent paths taken by different people walking in a city. All blocks are square and 120 m on a side.



Part (a) **Find the total distance traveled for path D in kilometers.**

The total distance walked is the sum of each of the steps taken along path D. This includes 2 blocks (south) plus 6 blocks (east) plus 4 blocks (north) plus 1 block (west). We are not interested in the directions indicated, just the total number of blocks, which will be converted to kilometers.

$$d = (2 + 6 + 4 + 1 \text{ blocks})\left(\frac{120 \text{ m}}{1 \text{ block}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)$$

$$\boxed{d = 1.560 \text{ km}}$$

Part (b) **Find the magnitude of the displacement on path D from start to finish in meters.**

The end point is 5 blocks east and 2 blocks north of the starting point. These distances are two sides of a right triangle and the length of the hypotenuse is the magnitude of the displacement. Using the Pythagorean theorem, we find

$$D = \sqrt{(5 \text{ blocks})^2 + (2 \text{ blocks})^2}\left(\frac{120 \text{ m}}{1 \text{ block}}\right)$$

$$\boxed{D = 646.2 \text{ m}}$$

Part (c) **Find the angle, in degrees, of the direction of the displacement measured north of east.**

Assume due East is the positive x-axis and due North is the positive y-axis, the angle of the displacement is some number of °s north of east. The tangent of the angle is equal to the number of blocks due north divided by the number of blocks due East. Therefore,

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)\\ &= \tan^{-1}\left(\frac{2 \text{ blocks}}{5 \text{ blocks}}\right)\\ &= \tan^{-1}\left(\frac{2}{5}\right)\end{aligned}$$

$$\boxed{\theta = 21.80°}$$

Problem 18:

A delivery man starts at the post office, drives 40.0 km north, then *11* km west, then *61* km northeast, and finally *21* km north to stop for lunch. (The direction “northeast” is 45 degrees east of north.)

Part (a) **What is the magnitude of his net displacement vector, in units of km?**

Taking north to be the positive y direction, and east to be the positive x direction, the four displacements are

$$(0\ \text{km})\hat{i} + (40\ \text{km})\hat{j}$$

$$(-11\ \text{km})\hat{i} + (0\ \text{km})\hat{j}$$

$$(61\ \text{km})\cos(45^\circ)\hat{i} + (61\ \text{km})\cos(45^\circ)\hat{j}$$

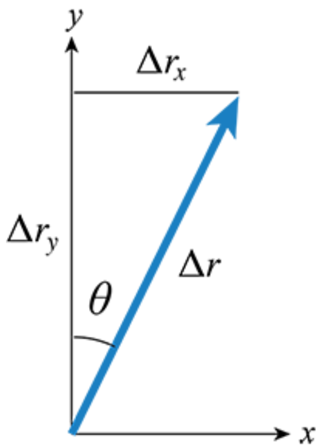
$$(0\ \text{km})\hat{i} + (21\ \text{km})\hat{j}$$

The net displacment is found by adding these vectors, and the result is

$$\Delta\vec{r} = \Delta\vec{r}_x\hat{i} + \Delta\vec{r}_y\hat{j}$$

$$= (32.13\ \text{km})\hat{i} + (104.1\ \text{km})\hat{j}$$

The diagram illustrates the situation



The magnitude of the displacment is

$$\Delta r = \sqrt{(32.13\ \text{km})^2 + (104.1\ \text{km})^2}$$

Calculating,

$$\boxed{\Delta r = 109.0\ \text{km}}$$

Part (b) **What is the direction of his net displacement? Express your answer an angle measured in degrees east of north.**

In the diagram the angle θ measures the direction east of north, where $\tan\theta = \dfrac{\Delta r_x}{\Delta r_y}$, so

$$\theta = \tan^{-1}\left(\frac{32.13\ \text{km}}{104.1\ \text{km}}\right)$$

Calculating,

$$\boxed{\theta = 17.15^\circ}$$