**Begin Date:** 9/4/2024 12:01:00 AM -- **Due Date:** 9/13/2024 11:59:00 PM **End Date:** 9/13/2024 11:59:00 PM

**Problem 1:**

When you drive to work on a winding road, the odometer of your car changes from 27123 km to 27134 km.

*Part (a)* **The distance your car traveled is ________.**

The purpose of the odometer is to measure the distance the car travels, which is what this part of the problem is asking. The difference between the two readings is $27134 \text{ km} - 27143 \text{ km} = 11 \text{ km}$.

The distance the car travels is, therefore, ...

$\boxed{\text{exactly 11 km}}$

*Part (b)* **The magnitude of your car's displacement is ________.**

This is a continuation from our solution for part (a).

We learn from our textbook that the magnitude of the displacement equals the straight-line distance between the final position and the initial one.

If the road were straight, the magnitude of the displacement would equal the traveled length of the road, which we found in part (a) to be 11 km. But since the road is winding, the final position is closer than 11 km to the initial position.

The magnitude of the car's displacement is, therefore, ...

$\boxed{\text{less than 11 km}}$

**Problem 2:**

When you drive to work on a winding road, the odometer of your car changes from 27120 km to 27140 km in 30 minutes.

*Part (a)* **Your car's average speed was ________.**

We learn from our textbook that the average speed equals the ratio of the total distance traveled to the total travel time.

In the present case, we know from the odometer readings that the total distance traveled is $20 = 27140 - 27120$ km and we are told that the total travel time is $30 \text{ min} = 0.5 \text{ h}$. Thus, the average speed $v$ is

$$v = \frac{20 \text{ km}}{0.5 \text{ h}} = 40 \text{ km/h} .$$

The correct answer is, therefore, ...

$\boxed{\text{exactly 40 km/h}}$

*Part (b)* **The magnitude of your car's average velocity was ________.**

This is a continuation of our solution for part (a).

We learn from our textbook that the magnitude of the average velocity equals the ratio of the straight-line distance between the final position and the initial one to the total travel time.

Since the road is winding, the straight-distance between the final position and the initial one is less than the length of the traveled road, which we found in part (a) to be 20 km. When we take the ratio of a number less than 20 to the 0.5-h travel time, we find a magnitude of average velocity that is less than 40 km/h.

The correct answer is, therefore, ...

$\boxed{\text{less than 40 km/h}}$

**Problem 3:**

Answer the following question concerning velocity and speed.

*Part (a)* **If a particle has constant velocity, is its speed necessarily constant?**

The velocity of a particle is a vector which has both a scalar magnitude and a direction. If the the velocity is constant, then both the magnitude and the direction are constant. The magnitude is called the speed, and hence the speed is constant.

$\boxed{\text{yes}}$

**Problem 4:**

Answer the following question concerning velocity and speed.

*Part (a)* **If a particle has constant speed, is its velocity necessarily constant?**

The velocity of a particle is a vector which has both a scalar magnitude and a direction. The magnitude is called the speed; it is possible for the speed to be constant while the direction is not, such as when a vehicle is turning a corner. Hence, it is possible for the speed to be constant while the velocity is not.

$\boxed{\text{no}}$

**Problem 5:**

In a biathlon race you first ride a bicycle at an average speed of *18* miles per hour for *15* miles, then you must run for another *3.5* miles.

*Part (a)* **With what average speed, in miles per hour, must you run if your average speed for the entire race is to be *10.1* mi/h?**

The average speed for the entire race is equal to the total distance traveled divided b the total elapsed time.

$$v_{\text{ave}} = \frac{d_1+d_2}{t_1+t_2}$$

where the numerator is the sum of the distances in the two parts of the race and the sum of the elapsed time for the two parts is in the denominator. We are asked to find the quantity $v_2 = d_2/t_2$, the average speed for the running portion of the race.

The time $t_1$ can be found from the information given:

$$t_1 = \frac{d_1}{v_1}$$

Then,

$$t_2 = \frac{d_2}{v_2}$$

Making these substitutions into the first equation, we can carry out the necessary algebraic manipulations to get an expression for the average running speed.

$$v_{\text{ave}} = \frac{d_1+d_2}{t_1+t_2} = \frac{d_1+d_2}{\left(\frac{d_1}{v_1}\right)+\left(\frac{d_2}{v_2}\right)}$$

$$\frac{v_{\text{ave}}}{d_1+d_2} = \frac{1}{\left(\frac{d_1}{v_1}\right)+\left(\frac{d_2}{v_2}\right)}$$

Invert both sides.

$$\frac{d_1+d_2}{v_{\text{ave}}} = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

Then,

$$\frac{d_1+d_2}{v_{\text{ave}}} - \frac{d_1}{v_1} = \frac{d_2}{v_2}$$

Now, we find the average running speed,

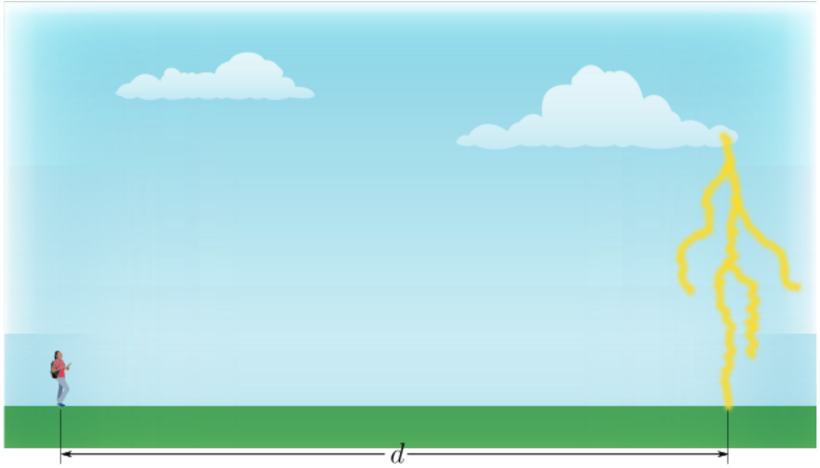
$$v_2 = \frac{d_2}{\frac{d_1+d_2}{v_{\text{ave}}} - \frac{d_1}{v_1}}$$

$$= \frac{3.5 \text{ mi}}{\frac{(3.5 \text{ mi})+(3.5 \text{ mi})}{10.1 \text{ mph}} - \frac{15 \text{ mi}}{18 \text{ mph}}}$$

$$\boxed{v_2 = 3.506 \text{ mph}}$$

**Problem 6:**

A student witnesses a flash of lightning, and, 1.5 s later, the student hears the associated clap of thunder.




*Part (a)* **Sound travels at** 343 m/s **in the air. What distance from the student, in meters, is the lightning strike?**

speed = 343 m/s

time = 1.5 s

distance = ?

The equation that relates speed, distance, and time is as follows.

$$speed = \frac{distance}{time}$$

$$s = \frac{d}{t}$$

The unknown value is distance, so you must solve for distance. Multiply both sides of the equation by t.

$$s = \frac{d}{t}$$

$$s \cdot t = d$$

$$d = s \cdot t$$

At this point, plug in the known numbers. Make sure to include the units with the numbers.

$$d = s \cdot t$$

$$d = (343\ \mathrm{m/s})(1.5\ \mathrm{s})$$

The units of seconds cancel.

$$\boxed{d = 514.5\ \mathrm{m}}$$

*Part (b)* **Light travels at $3.0 \times 10^8$ m/s in the air. How long, in seconds, did it take the light to reach the student's eyes after the lightning strike?**

If light travels at $3.0 \cdot 10^8 \frac{m}{s}$, how long did it take the light to travel that distance?

speed = $3.0 \cdot 10^8 \frac{m}{s}$

time = ?

distance = $514.5m$

This question is similar, to part a, but now the speed is the speed of light and the time is unknown. The distance was calculated in part a.

$$s = \frac{d}{t}$$

The unkown value is time, so solve for time.

Time is in the numerator, so multiply by t to bring it to the numerator.

$$s = \frac{d}{t}$$

$$t \cdot s = d$$

Divide by speed.

$$t = \frac{d}{s}$$

Plug in numbers and units. Remember that $d$ is the distance calculated in part a and $s$ is the speed of light.

$$t = \frac{d}{s}$$

$$t = \frac{(514.5\ \text{m})}{\left(3.0 \times 10^8\ \text{m/s}\right)}$$

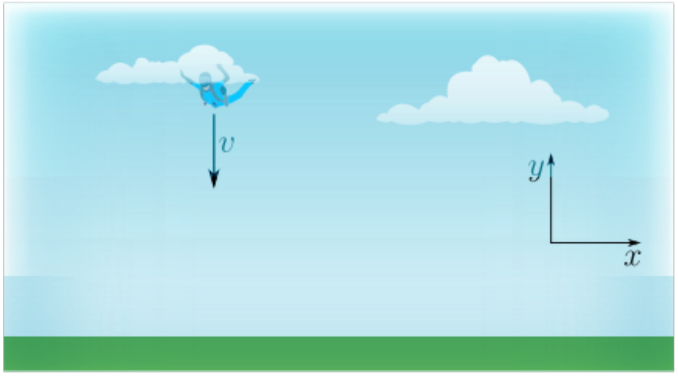
The units of meters cancels. The unit of seconds can be brought to the numerator.

$$t = \frac{(514.5)\ \text{s}}{\left(3.0 \times 10^8\right)}$$

$$\boxed{t = 1.715 \times 10^{-6}\ \text{s}}$$

**Problem 7:**

A skydiver falls $d_1 = 302$ m in $t_1 = 7.5$ s before opening her parachute. After the chute opens, she falls an additional $d_2 = 701$ m in $t_2 = 90.5$ s. A coordinate system is indicated in the figure.




*Part (a)* **Enter an expression for the skydiver's average speed, $v_1$, during the period before opening the chute in terms of the given quantities.**

$d_1 = 302m$

$t_1 = 7.5s$

$d_2 = 701m$

$t_2 = 90.5s$

Write an expression for the skydiver's average speed during the period before the parachute opens.

$$|v_y| = \frac{|\Delta y|}{|\Delta t|}$$

We are looking at the speed in region 1 of the motion.

$$v_1 = \frac{|\Delta y|}{|\Delta t|}$$

The magnitude of the change in the y position was $d_1$. The total time was $t_1$.

$$\boxed{v_1 = \frac{d_1}{t_1}}$$

*Part (b)* **Enter an expression for the skydiver's average speed, $v_2$, during the period after opening the chute in terms of the given quantities.**

$d_1 = 302m$

$t_1 = 7.5s$

$d_2 = 701m$

$t_2 = 90.5s$

Write an expression for the skydiver's average speed during the period after the parachute opens.

$$|v_y| = \frac{|\Delta y|}{|\Delta t|}$$

We are looking at the speed in region 2 of the motion.

$$v_2 = \frac{|\Delta y|}{|\Delta t|}$$

The magnitude of the change in the y position was $d_2$. The total time was $t_2$.

$$\boxed{v_2 = \frac{d_2}{t_2}}$$

***Part (c)*** **Enter an expression for the skydiver's average speed during the entire fall, $v_{avg}$, in terms of the given quantities.**

$d_1 = 302m$

$t_1 = 7.5s$

$d_2 = 701m$

$t_2 = 90.5s$

Write an expression for the skydiver's average speed during the entire fall.

$$|v_y| = \frac{|\Delta y|}{|\Delta t|}$$

We are looking at the average speed for the entire fall.

$$v_a = \frac{|\Delta y|}{|\Delta t|}$$

The magnitude of the total change in the y position includes both distances.

$$|\Delta y| = d_1 + d_2$$

The total time that it takes to fall is the sum of both individual times.

$$|\Delta t| = t_1 + t_2$$

Plug both of these into the equation for average velocity.

$$v_a = \frac{|\Delta y|}{|\Delta t|}$$

$$\boxed{v_a = \frac{(d_1 + d_2)}{(t_1 + t_2)}}$$

***Part (d)*** **Calculate the skydiver's average speed, $v_{avg}$, in meters per second, during the entire fall.**

$d_1 = 302m$

$t_1 = 7.5s$

$d_2 = 701m$

$t_2 = 90.5s$

Calculate the average speed over the entire fall using the result from part c.

$$v_a = \frac{(d_1 + d_2)}{(t_1 + t_2)}$$

At this point, plug in numbers and units.

$$v_a = \frac{(d_1 + d_2)}{(t_1 + t_2)}$$

$$v_a = \frac{(302m + 701m)}{(7.5s + 90.5s)}$$

Add the terms in the numerator and denomenator independently.

$$v_a = \frac{1003m}{98.00s}$$

Perform the quotient. Notice what happens with the units.

$$\boxed{v_a = 10.23\frac{m}{s}}$$

**Problem 8:**

A particle's position along the $x$-axis is described by the function

$$x(t) = A\,t + B\,t^2,$$

where $t$ is in seconds, $x$ is in meters, and the constants $A$ and $B$ are given below.

**Randomized Variables**

$A = -4.9$ m/s

$B = 2.1$ m/s$^2$

*Part (a)* **Enter an expression, in terms of $A$, $B$, and $t$, for the velocity of the particle as a function of time.**

The velocity function is the time derivative of the position function. We take the time derivative of the given function and obtain the expression.

$$v(t) = \frac{d}{dt}\left(At + Bt^2\right)$$

$$\boxed{v(t) = A + 2Bt}$$

*Part (b)* **At what time, in seconds, is the particle's velocity zero?**

In part (a) we found the velocity function

$$v(t) = A + 2Bt$$

Equate this function to zero and solve for the elapsed time.

$$0 = A + 2Bt$$

$$t = -\frac{A}{2B}$$

$$= -\frac{-4.9\text{ m/s}}{2\left(2.1\text{ m/s}^2\right)}$$

$\boxed{t = 1.167 \text{ s}}$

**Problem 9:**

A particle's position along the *x*-axis is described by
$x(t) = A\,t + B\,t^2$,
where $t$ is in seconds, $x$ is in meters, and the constants $A$ and $B$ are given below.

**Randomized Variables**

$A = -4.9$ m/s
$B = 2.1$ m/s$^2$

*Part (a)* **What is the velocity, in meters per second, of the particle at the time $t_1$ = 3.0 s?**

The velocity function is the time derivative of the position function. We take the time derivative of the given function and obtain the expression

$$v(t) = A + 2Bt$$

Now substitute the given time, $t_1 = 3.0$ s.

$$v(t_1) = (-4.9 \text{ m/s}) + 2\left(2.1 \text{ m/s}^2\right)(3.0 \text{ s})$$

$$\boxed{v(t_1) = 7.700 \text{ m/s}}$$

*Part (b)* **What is the velocity, in meters per second, of the particle when it is at the origin ($x = 0$) at time $t_0 > 0$?**

To find at what positive time $t_0$ the particle is at the origin, we equate the given position function to zero and solve for the nonzero value of the time.

$$x(t) = At + Bt^2 = 0$$

Dividing all of the terms by $t$, we have

$$A + Bt = 0$$

$$\begin{aligned} t &= -\tfrac{A}{B} \\ &= \tfrac{-4.9 \text{ m/s}}{2.1 \text{ m/s}^2} \\ &= 1.111 \text{ s} \end{aligned}$$

This is indeed a positive value, as specified in the question, so we can proceed.

In part (a) we found the velocity function

$$v(t) = A + 2Bt$$

Substitute the time $t_0$ found above in for $t$.

$$\begin{aligned} v(t_0) &= A + 2Bt_0 \\ &= -4.9 \text{ m/s} + 2\left(2.1 \text{ m/s}^2\right)(1.111 \text{ s}) \end{aligned}$$

$$\boxed{v(t_0) = -0.2333 \text{ m/s}}$$

**Problem 10:**

A particle moves in one dimension according to $x(t) = A\,t + B\cos(t)$, where $x$ is in meters and $t$ is in seconds. You can assume that both $A$ and $B$ are greater then zero.

*Part (a)* **Carefully consider the function x(t). Which of the graphs below best represents it?**

We are given the position as a function of time as

$$x(t) = At + B\cos(t)$$

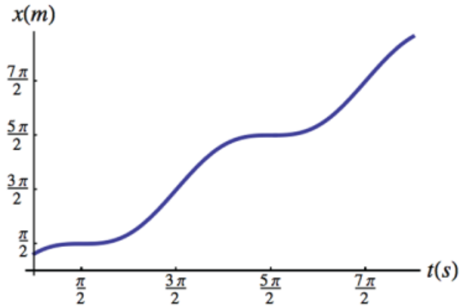
where $A$ and $B$ are positive constants.

The first term, $At$, describes a straight line starting at $x = 0$ m and rising with increasing values of $t$.

The second term represents an oscillation about $x = 0$ m.

When these behaviors are combined, we get an oscillation about the straight line represented by the first term. The graph rises from left to right as time increases. Two graphs show that behavior, but only one is non-zero at time $t = 0$ s, which it must be because the cosine function has its maximum value at time equals 0 seconds.

Therefore, the answer is




***Part (b)*** **Write an equation for the instantaneous velocity, *v(t)*.**

We are given the position as a function of time as

$$x\,(t) = At + B\cos\,(t)$$

where A and B are positive, non-zero constants.

The velocity function is obtained from the position function by taking its time derivative.

$$v\,(t) = \frac{\mathrm{d}}{\mathrm{d}t}[At + B\cos\,(t)]$$

$$\boxed{v\,(t) = A - B\sin\,(t)}$$

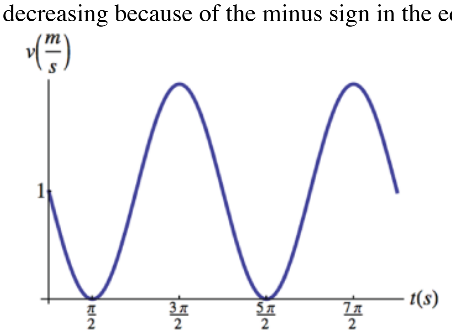
***Part (c)*** **When, for value of time $t \geq 0$ is the velocity equal to zero m/s? You may assume that $A = B$.**

The velocity function is zero for the first time when $t = \frac{\pi}{2}$ and is zero again every time $t$ increases by $2\pi$. The requested times are therefore

$$\boxed{t = \left(\frac{1}{2} + 2n\right)\pi,\ n = 0, 1, 2, \ldots}$$

***Part (d)*** **Which of the options below shows a graph of the velocity of the particle as a function of time?**

Earlier, we found the expression for the instantaneous velocity, v(t) = A - B \text{ sin}(t) At time equals zero seconds, the velocity is $A$ m/s. As time increases, the function oscillates about the line $v(t) = A$. Only one graph represents this behavior with the velocity initially decreasing because of the minus sign in the equation. It is this graph, which assumes that $A = B = 1$,




**Problem 11:**

There are two factors that contribute to the total stopping distance for a traveling vehicle, the perception-reaction distance and the braking distance. When an event occurs that requires an emergency stop, the vehicle continues to travel at its initial velocity for the perception-reaction distance while the driver reacts to the event. Once the brakes are applied, the vehicle undergoes constant acceleration as it travels the braking distance. Historically, engineers used a perception-reaction time of 0.75 second, but now they assume a perception-reaction time of 1.0 second for the average driver.

*Part (a)* **A vehicle has an initial speed $v_0$. The driver has a perception-reaction time of $t_{\text{react}}$. When the driver begins braking, the magnitude of the vehicle's acceleration is $a$. Enter an expression for the total stopping distance of the vehicle.**

There are two parts to the situation, so there are two contributing terms for the total distance.

For the first term, the vehicle is traveling with constant speed for a specific period of reaction time.

$$x_{\text{react}} = v_0 t_{\text{react}}$$

For the second term, we do not know the time for braking, but we do know the initial and final speeds. We apply the equations of kinematics to find the braking distance.

$$v^2 - v_0^2 = 2ax$$

The final speed is $v = 0$ m/s. The initial speed in this part is the same as for the first term, $v_0$. Then,

$$x_{\text{brake}} = -\frac{v_0^2}{2a}$$

Note that the acceleration is negative, resulting in a positive value for this distance.

Combining the terms, using the magnitude of the acceleration, we find

$$\boxed{x_{\text{total}} = v_0 t_{\text{react}} + \frac{v_0^2}{2a}}$$

*Part (b)* **A vehicle has an initial speed $v_0$. The driver has a perception-reaction time of $t_{\text{react}}$. As the vehicle is driven down the road, a tree falls onto the road a distance $x_{\text{f}}$ in front of the vehicle. When the driver begins braking, the magnitude of the vehicle's acceleration is $a$. While the driver has enough time to be able to react and begin braking, the car will not stop before it hits the tree. Write an expression for the speed of the vehicle as it hits the tree.**

This time the velocity in the instant before the car hits the tree is not zero.

The distance to the tree has two contributing terms:

For the first term, the vehicle is traveling with constant speed for the reaction time.

$$x_{\text{react}} = v_0 t_{\text{react}}$$

We assume that this distance is less than the total distance to the tree.

The second term for $x_{\text{brake}}$, involves a constant acceleration, so we can apply the equations of kinematics.

$$v_f^2 - v_0^2 = -2ax_{\text{break}}$$

where

$$x_{\text{brake}} = x_{\text{f}} - v_0 t_{\text{react}}$$

and $x_{\text{f}}$ is the total distance when the car is at time $t = 0$ seconds.

$$v_f^2 = v_0^2 - 2a\left(x_f - v_0 t_{\text{react}}\right)$$

$$\boxed{v_f = \sqrt{v_0^2 - 2a\left(x_f - v_0 t_{\text{react}}\right)}}$$

*Part (c)* **A vehicle has an initial speed $v_0 = 30.2$ m/s when a tree falls on the roadway $x_f = 80.1$ m in front of the vehicle. The driver has a perception-reaction time of $t_{\text{react}} = 0.75$ s, and, when the driver begins braking, the magnitude of the vehicle's acceleration is $a = 6.03$ m/s$^2$. Calculate the speed of the vehicle, in meters per second, when it hits the tree.**

Substitute numbers into the expression previously obtained.

$$\begin{aligned} v_f &= \sqrt{v_0^2 - 2a(x_f - v_0\, t_{\text{react}})} \\ &= \sqrt{(30.2 \text{ m/s})^2 - 2\left(6.03 \text{ m/s}^2\right)\left[80.1 \text{ m} - (30.2 \text{ m/s})(0.75 \text{ s})\right]} \end{aligned}$$

$$\boxed{v_f = 14.81 \text{ m/s}}$$

Recall that there were some assumptions made when obtaining the expression that was just evaluated. Since we did not obtain a negative value inside the square root, and because the final speed is less than the initial speed, we may be confident that the assumptions were valid.

*Part (d)* **A vehicle has an initial speed $v_0 = 30.2$ m/s when a tree falls on the roadway $x_f = 80.1$ m in front of the vehicle. The driver has a perception-reaction time of $t_{react} = 1.00$ s, and, when the driver begins braking, the magnitude of the vehicle's acceleration is $a = 6.03$ m/s$^2$. Calculate the speed of the vehicle, in meters per second, when it hits the tree.**

Substitute numbers into the expression previously obtained.

$$\begin{aligned} v_f &= \sqrt{v_0^2 - 2a(x_f - v_0\, t_{react})} \\ &= \sqrt{(30.2 \text{ m/s})^2 - 2\left(6.03 \text{ m/s}^2\right)\left[80.1 \text{ m} - (30.2 \text{ m/s})(1.00 \text{ s})\right]} \end{aligned}$$

$$\boxed{v_f = 17.61 \text{ m/s}}$$

Recall that there were some assumptions made when obtaining the expression that was just evaluated. Since we did not obtain a negative value inside the square root, and because the final speed is less than the initial speed, we may be confident that the assumptions were valid.

**Problem 12:**

A particle is initially at rest at the origin at $t = 0.00$ s. From $t = 0.00$ s to $t = 5.00$ s its velocity changes according to $v(t) = (3.20 \text{ m/s}^2)t$. From $t = 5.00$ s to $t = 11.00$ s its velocity changes according to $v(t) = (16.00 \text{ m/s}) - (1.50 \text{ m/s}^2)(t - 5.00 \text{ s})$. From $t = 11.00$ s to $t = 20.00$ s its velocity does not change.

*Part (a)* **What is the position of the particle at $t = 2.01$?**

Velocity is the time derivative of position, so $v = dx/dt$, and $dx = v\,dt$. Thus, $x(t) = \int v(t)\,dt + C$, so

$$x(t) = \int \left(3.20 \text{ m/s}^2\right) t\,dt + C = \tfrac{1}{2}\left(3.20 \text{ m/s}^2\right) t^2 + C$$

Since $x(0) = 0$, we have $C = 0$, so

$$x(2.01 \text{ s}) = \tfrac{1}{2}\left(3.20 \text{ m/s}^2\right)(2.01 \text{ s})^2$$

Calculating,

$$\boxed{x(2.01 \text{ s}) = 6.464 \text{ m}}$$

*Part (b)* **What is the position of the particle at $t = 6.01$?**

Integrating again,

$$\begin{aligned} x(t) &= \int \left[(16.0 \text{ m/s}) - \left(1.50 \text{ m/s}^2\right)(t - 5.00 \text{ s})\right] dt + C \\ &= (23.5 \text{ m/s})\,t - \tfrac{1}{2}\left(1.50 \text{ m/s}^2\right) t^2 + C \end{aligned}$$

Using the results from part (a), at $t = 5.00$ s. the position is equal to $\dfrac{1}{2}\left(3.20 \text{ m/s}^2\right)(5.00 \text{ s})^2 = 40.0$ m, so

$$(23.5 \text{ m/s})(5.00 \text{ s}) - \tfrac{1}{2}\left(1.50 \text{ m/s}^2\right)(5.00 \text{ s})^2 + C = 40.0 \text{ m}$$

From this we have $C = -58.75$ m, so

$$x(6.01 \text{ s}) = (23.5 \text{ m/s})(6.01 \text{ s}) - \tfrac{1}{2}\left(1.50 \text{ m/s}^2\right)(6.01 \text{ s})^2 - 58.75 \text{ m}$$

Calculating,

$$\boxed{x(6.01 \text{ s}) = 55.39 \text{ m}}$$

**Part (c) What is the position of the particle at $t$ = 12?**

Using the results from part (b), the position at $t = 11.00$ s is

$$x(11.00\text{ s}) = (23.5\text{ m/s})(11.00\text{ s}) - \tfrac{1}{2}\left(1.50\text{ m/s}^2\right)(11.00\text{ s})^2 - 58.75\text{ m} = 109\text{ m}$$

The velocity at $t = 11.00$ s is

$$v(11.00\text{ s}) = (16.00\text{ m/s}) - \left(1.50\text{ m/s}^2\right)(11.00\text{ s} - 5.00\text{ s}) = 7.00\text{ m/s}$$

From $t = 11.00$ s to $t = 12$ s, then, the particle's displacement is

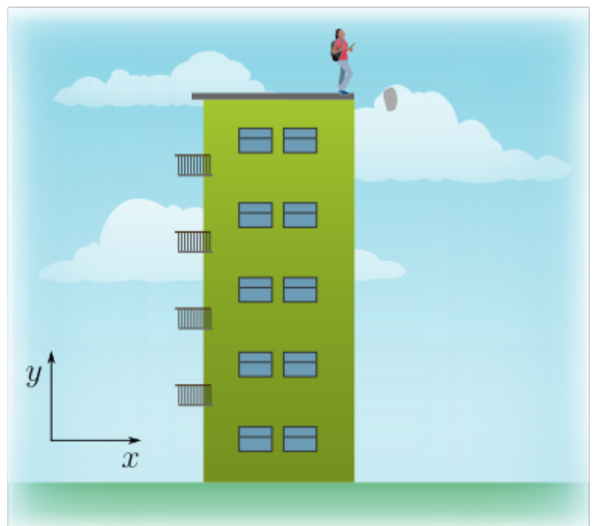
$$(12\text{ s} - 11.00\text{ s})(7.00\text{ m/s}) = 7.000\text{ m}$$

Adding this to the position at $t = 11.00$ s yields

$$\boxed{x(12\text{ s}) = 116.0\text{ m}}$$

**Problem 13:**

A stone is dropped from rest from the top of a building. It takes $\Delta t = 2.04$ s for it to reach the ground. Use the coordinate axes provide in the image.




**Part (a) What is the initial velocity, in meters per second, of the stone?**

$\Delta$t = 2.04 s

$v_i = 0.0$ m/s

What is the initial velocity, $v_i$ of the stone?

The problem statement says that the stone begins at rest at the top of the building. The means that initially it is not moving and it's initial velocity is zero.

$$\boxed{v_i = 0.0\text{ m/s}}$$

**Part (b) What is the magnitude, in meters per squared second, of the acceleration?**

$\Delta$t = 2.04 s

$$v_i = 0\frac{m}{s}$$

What is the magnitude of the acceleration?

The stone is in free-fall, so the acceleration is the acceleration due to gravity for earth.

$$|a| = |g|$$

$$\boxed{|a| = 9.81\frac{m}{s^2}}$$

*Part (c)* **Enter an expression for the final velocity, $v_f$, of an object that has an initial velocity, $v_i$, and accelerates with acceleration $a$ for a time period $\Delta t$.**

$\Delta t = 2.04$ s

$$v_i = 0\frac{m}{s}$$

What is the equation for the final velocity of an object as a function of acceleration, time, and initial velocity?

This is the equation that determines the relationship among these values.

The equation should still be in it's general form without any substitutions made.

$$\boxed{v_f = v_i + a\Delta t}$$

*Part (d)* **Calculate the magnitude, in meters per second, of the final velocity of the stone.**

$\Delta t = 2.04$ s

$$v_i = 0\frac{m}{s}$$

Calculate the magnitude of $v_f$ .

Begin with the equation found in part e.

$$v_f = v_i + a\Delta t$$

Note that $v_i = 0$

$$v_f = 0 + a\Delta t$$

$$v_f = a\Delta t$$

Plug in numbers for $a$ and $t$.

$$v_f = \left(-9.81\frac{m}{s^2}\right) \cdot (2.04s)$$

$$v_f = -20.01\frac{m}{s}$$

Notice that you are asked for the magnitude of the final velocity, so take the absolute value.

$$v_f = -20.01\frac{m}{s}$$

$$|v_f| = |-20.01|\frac{m}{s}$$

$$\boxed{|v_f| = 20.01\frac{m}{s}}$$

***Part (e)*** **Enter an expression for the displacement, $\Delta y$, of an object that has an initial velocity, $v_i$, and accelerates with acceleration $a$ for a time period $\Delta t$.**

$\Delta t = 2.04$ s

$$v_i = 0\frac{m}{s}$$

Express the displacement $\Delta y$ of an object as a function of acceleration, time, and initial velocity.

Begin with the equation for the final y position.

$$y_f = y_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

You are asked to express the displacement $\Delta y$ instead of the final position $y_f$ . Your goal is to use the following substitution.

$$\Delta y = y_f - y_i$$

To use this substitution, begin with the equation and subtract $y_i$ from each side.

$$y_f = y_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$y_f - y_i = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

Make the substitution above.

$$\boxed{\Delta y = v_i\Delta t + \frac{1}{2}a(\Delta t)^2}$$

***Part (f)*** **Calculate the magnitude, in meters, of the displacement of the stone.**

$\Delta t = 2.04$ s

$$v_i = 0\frac{m}{s}$$

Calculate the magnitude of $\Delta y$.

Begin with the equation you derived in part e.

$$\Delta y = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

Notice that $v_i = 0$

$$\Delta y = 0 \cdot \Delta t + \frac{1}{2}a(\Delta t)^2$$

$$\Delta y = \frac{1}{2}a(\Delta t)^2$$

Plug in numbers.

$$\Delta y = \frac{1}{2}\left(9.81\frac{m}{s^2}\right) \cdot (2.04s)^2$$

Notice that $s^2$ will cancel leaving only meters.

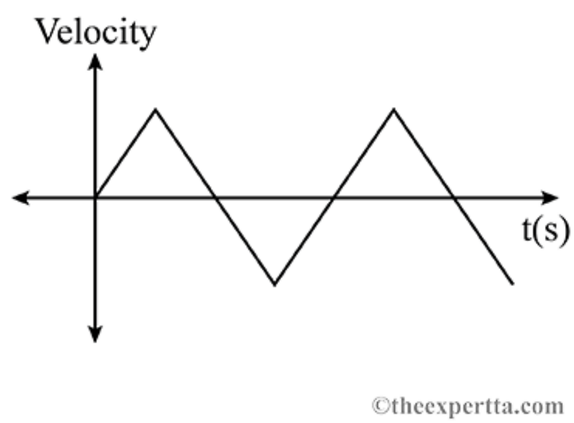
$$\Delta y = \frac{1}{2}(9.81) \cdot (2.04)^2 m$$

$$\Delta y = (4.905) \cdot (4.162)\, m$$

$$\boxed{\Delta y = 20.41m}$$

**Problem 14:**

The velocity versus time graph for the motion of an object is shown in the figure.

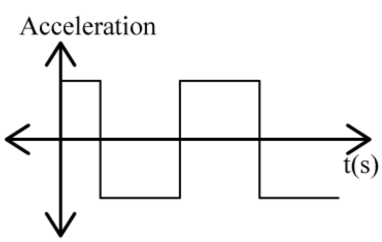



*Part (a)* **Choose the correct acceleration versus time graph for the object.**

The acceleration is related to the velocity and time by the expression

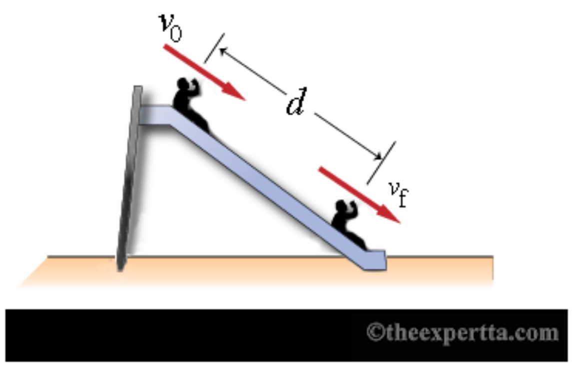
$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

so while the velocity is increasing at a constant rate in the beginning, the acceleration is a constant positive value. Since the slope of the velocity changes instantaneously, the acceleration does as well (actually undefined there). The velocity graph switches between an increase in velocity to a decrease, so a positive to negative acceleration. Only one graph exhibits this behavior:



**Problem 15:**

A 20-kg boy slides down a playground slide with a constant acceleration $a = 1.1$ m/s$^2$ parallel to the surface of the slide. The boy starts sliding with an initial speed of $v_0$ . Refer to the figure.




*Part (a)* **Enter an expression for the child's speed squared, $v_f^2$, at the bottom of the slide in terms of $a$, $v_0$, and the length of the slide, $d$.**

We use the constant-acceleration kinematic equation

$$2ad = v_f^2 - v_0^2$$

for an object accelerating with acceleration $a$ from initial speed $v_0$ to final speed $v_f$ over distance $d$.

We solve for $v_f^2$ and obtain the requested expression ...

$$\boxed{v_f^2 = v_0^2 + 2ad}$$

*Part (b)* **If the slide is 4.0 m long and the boy's final speed is $v_f$ = 5.0 m/s, what is his initial speed, in meters per second?**

We use the constant-acceleration kinematic equation

$$2ad = v_f^2 - v_0^2$$

for an object accelerating with acceleration $a$ from initial speed $v_0$ to final speed $v_f$ over a distance $d$.

We solve for the initial speed,

$$v_0 = \sqrt{v_f^2 - 2ad}$$

and substitute the given values,

$$v_0 = \sqrt{(5.0 \text{ m/s})^2 - 2\left(1.1 \text{ m/s}^2\right)(4.0 \text{ m})} .$$

$$\boxed{v_0 = 4.025 \text{ m/s}}$$

*Part (c)* **When starting from rest, how long, in seconds, does it take the boy to reach the bottom of the slide?**

We use the constant-acceleration kinematic equation

$$d = 1/2at^2$$

for an object accelerating from rest with acceleration $a$ over distance distance $d$ during time interval $t$.

We solve for $t$,

$$t = \sqrt{\frac{2d}{a}}$$

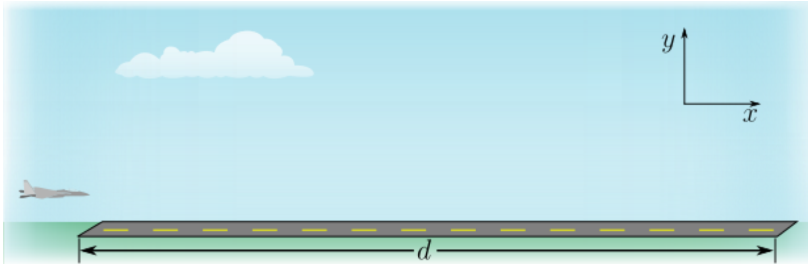
and substitute the given values,

$$t = \sqrt{\frac{2(4.0 \text{ m})}{1.1 \text{ m/s}^2}}$$

$$\boxed{t = 2.697 \text{ s}}$$

**Problem 16:**

At the equator, the radius of the Earth is approximately 6370 km. A jet flies at a very low altitude at a constant speed of $v = 201$ m/s. Upon landing, the jet can produce an average deceleration of magnitude $a = 15$ m/s$^2$ .




*Part (a)* **How long, in seconds, will it take the jet to circle the Earth at the equator?**

Speed is defined as the distance traveled divided by the time.

$$v = \frac{\Delta x}{\Delta t}$$

The distance traveled is equal to the circumference of the earth in meters. Inserting this into the above equation and solving for the time gives,

$$\Delta t = \frac{2\pi R}{v}$$

$$= \frac{2\pi(6370\text{ km})\left(\frac{1000\text{ m}}{1\text{ km}}\right)}{(201\text{ m/s})}$$

$$\boxed{\Delta t = 1.991 \times 10^5\text{ s}}$$

*Part (b)* **What is the minimum landing distance, in meters, this jet needs to come to rest? Assume that, when the jet *initially* touches the ground, it is moving at the same speed as it was when it was flying.**

Considering the equations of kinematics and the given information, we can find the distance using

$$v_f^2 = v^2 - 2ad$$

The jet comes to rest, so the final speed is equal to zero m/s. Solving for the distance, we find,

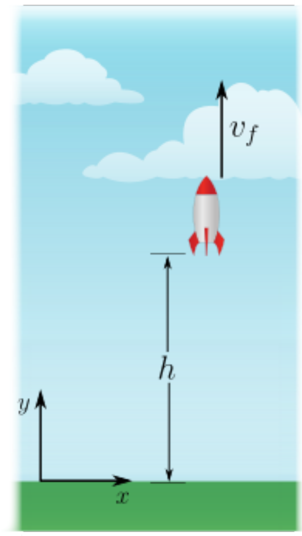
$$d = \frac{v^2}{2a}$$

$$= \frac{(201\text{ m/s})^2}{\left(2\left(15\text{ m/s}^2\right)\right)}$$

$$\boxed{d = 1347\text{ m}}$$

**Problem 17:**

A student launches a small rocket which starts from rest at ground level. At a height $h = 1.04$ km, the rocket reaches a speed of $v_f = 201$ m/s. At that height, the rocket runs out of fuel, so there is no longer any thrust propelling it. Take the positive direction to be upward. The drawing is *not* to scale.




*Part (a)* **Assuming constant acceleration, what is the rocket's acceleration, in meters per squared second, during the period from its launch until it runs out of fuel?**

The kinematic equation for velocity with constant acceleration over a distance with an initial velocity, in general, is

$$v_f^2 = v_i^2 + 2ad$$

where a is the acceleration in m/s$^2$ , d is the distance in m, and $v_{f,i}$ are the final and initial velocities in m/s. Since the rocket started from rest,

$$v_i = 0$$

Therefore,

$$v_f^2 = 2a_1 h$$

Thus,

$$a_1 = \frac{v_f^2}{2h}$$

Plugging in numbers and converting units as needed,

$$a = a_1 = \frac{(201\text{ m/s})^2}{(2)\left(1.04 \times 10^3\text{ m}\right)}$$

$$\boxed{a_1 = 19.42\text{ m/s}^2}$$

*Part (b)* **After the rocket's engine turns off at a height of $h = 1.04$ km, it continues to move upward due to the velocity that it reached. What is the rocket's acceleration, in meters per squared second, during the period from engine shutoff until it returns to the ground? Ignore air resistance.**

During freefall, since gravity is the only force acting on the rocket, its acceleration is

$$a_2 = -g = -9.81\text{ m/s}^2$$

$$\boxed{a_2 = -9.810\text{ m/s}^2}$$

*Part (c)* **Calculate the maximum height, in meters above ground level, that the rocket reaches.**

Since the rocket already traveled a distance h, we need to determine the extra height after the rocket's engine shuts off. The kinematic equation for velocity with constant acceleration over a distance with an initial velocity, in general, is

$$v_f^2 = \left(v_i^2 + 2ad\right)\ \mathrm{m^2/s^2}$$

where a is the acceleration in $\mathrm{m/s^2}$, d is the distance in m, and $v_{f,i}$ are the final and initial velocities in m/s. Since the rocket starts with the velocity stated in the problem and at the max height it will stop,

$$v_f = 0\ \mathrm{m/s}$$

$$v_i = 201\ \mathrm{m/s}$$

Therefore,

$$-v_i^2 = 2ad$$

Thus,

$$d = -\frac{v_i^2}{(2a_2)}$$

The total distance is therefore,

$$h_t = h + d = h + \left(-\frac{v_i^2}{(2a_2)}\right)$$

Plugging in numbers and converting units as needed,

$$h_t = 1.04 \cdot 1000\ \mathrm{m} + \left(-\frac{(201\ \mathrm{m/s})^2}{(2 \cdot (-9.81)\ \ \mathrm{m})}\right)$$

$$\boxed{h_t = 3099\ \mathrm{m}}$$

**Problem 18:**

A child throws a penny off of the Eiffel tower ($h = 324$ m) with a downward velocity of magnitude $V_0 = 2$ m/s.

*Part (a)* **Select from below the correct expression for the time *t*, it takes for the penny to reach the ground. Neglect air resistance.**

This problem involves the application of the **equations of motion in one dimension** in the case of **free fall**.

In this part, we are asked to find how long it takes for a penny to reach the ground when thrown off of the Eiffel tower. As in any case of free fall, the mass of the falling object (here, the penny) doesn't matter!

The solution thus involves three steps:

- determining which equation of motion to use
- making substitutions, using the **appropriate signs** for our known variables, into the selected equation of motion
- solving for time

Let's get started!

**Choosing an Equation of Motion**

When choosing which equation of motion is the most appropriate (or easiest) to use for a problem, it's important identify which variable we want and which variables we know. In this case, we want *time*, and we know 3 variables: the initial velocity, the distance traveled, and the acceleration.

The equation that gives us time with the known variables is

$$y - y_0 = v_0 t + \frac{1}{2} a_y t^2$$

**Making Substitutions**

Taking $+y$ as upward, we can make the following substitutions,

$$y - y_0 = -h$$

$$v_0 = -V_0$$

$$a_y = -g$$

where the negative signs indicate, respectively, that the final position is lower than the initial position, the object was initially thrown *downward*, and the acceleration due to gravity points *downward*. With these substitutions, our equation for motion becomes

$$-h = -V_0 t - \frac{1}{2} g t^2$$

or, more simply,

$$h = V_0 t + \frac{1}{2} g t^2$$

**Solving for $t$**

Note this is a quadratic equation! Let's rearrange it so that it appears in its more familiar form,

$$\frac{1}{2} g t^2 + V_0 t - h = 0$$

It may be painful, but we'll have to apply the quadratic formula. If we write the solution for $t$ in *its* familiar form, we have

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

$$a = \frac{1}{2} g$$

$$b = V_0$$

$$c = -h$$

Therefore, we have two potential solutions

$$t = \frac{-V_0 \pm \sqrt{V_0^2 - 4\left(\frac{1}{2} g\right)(-h)}}{2\left(\frac{1}{2} g\right)} = \frac{-V_0 \pm \sqrt{V_0^2 + 2gh}}{g}$$

We know that there should be only *one* physical solution, the time it takes for the penny to hit the ground. So, how do we go about deciding which time is correct, given the following options?

$$t_1 = \frac{-V_0 + \sqrt{V_0^2 + 2gh}}{g}$$

$$t_2 = \frac{-V_0 - \sqrt{V_0^2 + 2gh}}{g}$$

In each case, the sign of our result is determined by the numerator (the denominator is positive, $g = 9.81\ \text{m/s}^2$), and the first term in the numerator is negative. With this in mind, it is easy to see that the first option will give a value for time that is positive, and the second will give us a value that is negative.

Because the penny should hit the ground *after* it is thrown, time must be positive! Though the latter option would make a great story, the first option is the only one that makes physical sense.

Our final answer is therefore

$$t = \frac{-V_0 + \sqrt{V_0^2 + 2gh}}{g}$$

$$\boxed{t = \frac{-V_0 + \sqrt{V_0^2 + 2gh}}{g}}$$

*Part (b)* **Calculate the time $t$, in seconds, that it takes for the penny to fall.**

In this case, we simply evaluate the time it takes the penny to hit the ground using the equation from *Part (a)*,

$$t = \frac{-V_0 + \sqrt{V_0^2 + 2gh}}{g} = \frac{-\left(2\ \text{m/s}\right) + \sqrt{\left(2\ \text{m/s}\right)^2 + 2\left(9.81\ \text{m/s}^2\right)\left(324\ \text{m}\right)}}{9.81\ \text{m/s}^2} = 7.926\ \text{s}$$

$$\boxed{t = 7.926\ \text{s}}$$

*Part (c)* **Find the speed at which the penny strikes the ground $V_f$, in meters per second, using the initial velocity, $V_0$, and the drop distance, $h$.**

In this part, we are asked to find the speed of the penny as it strikes the ground.

Because they are so useful in solving problems that involve motion, let's repeat the steps we took in *Part (a)*, here solving for the final velocity,

- determining which equation of motion to use
- making substitutions, using the **appropriate signs** for our known variables, into the selected equation of motion
- solving for the final velocity

**Choosing an Equation of Motion**

To avoid compounding rounding errors that often occur when using values found in intermediate steps, let's not use the time we found in *Part (b)*. Instead, let's used the original known variables: the initial velocity, the distance traveled, and the acceleration.

The equation that gives us final velocity most easily is

$$v_f^2 = v_0^2 + 2a_y\left(y - y_0\right)$$

**Making Substitutions**

Making the same substitutions, as in *Part (a)*,

$$y - y_0 = -h$$

$$v_0 = -V_0$$

$$v_f = V_{\mathrm{f}}$$

$$a_y = -g$$

These substitutions give

Unmatched Grouping: , at 10 ) expected right curly brace

or, more simply,

$$V_{\mathrm{f}}^2 = V_0^2 + 2gh$$

**Solving for $V_{\mathrm{f}}$**

Taking the square root of both sides of our equation, we have

$$V_{\mathrm{f}} = \pm\sqrt{V_0^2 + 2gh}$$

where we are again confronted with two possible signs in our solution. We know that the penny is traveling *downward*, and if we had been asked to find the final velocity of the penny (a vector), the negative sign would apply. In this case, however, we're asked to find the speed (a positive scalar). Hence, our answer is

$$V_{\mathrm{f}} = \sqrt{V_0^2 + 2gh}$$

Evaluating the result,

$$V_{\mathrm{f}} = \sqrt{(2\ \mathrm{m/s})^2 + 2\left(9.81\ \mathrm{m/s^2}\right)(324\ \mathrm{m})} = 79.76\ \mathrm{m/s}$$

Oh, dear. This penny would carry a punch!

$$\boxed{V_{\mathrm{f}} = \sqrt{(2\ \mathrm{m/s})^2 + 2\left(9.81\ \mathrm{m/s^2}\right)(324\ \mathrm{m})} = 79.76\ \mathrm{m/s}}$$