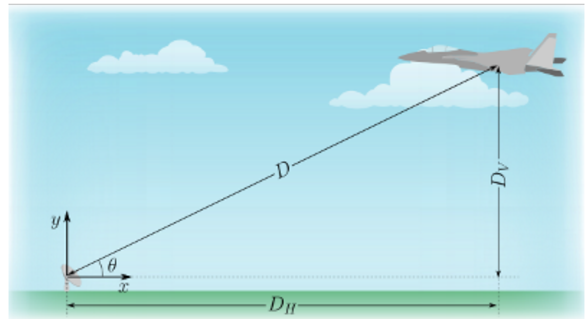
Problem 1:

A plane flies towards a ground-based radar dish. Radar locates the plane at a distance $D = 10.3$ km from the dish, at an angle $\theta = 30.2°$ above horizontal.



Part (a) **What is the plane's horizontal distance, in meters, from the radar dish?**

Note the right triangle created by the horizontal and vertical components of the distance to the plane. The horizontal distance is the length of the side adjacent to the angle θ, and hence the cosine function is indicated.

$$\cos\theta = \frac{D_H}{D}$$

which may be rearranged as

$$\begin{aligned} D_H &= D\cos\theta \\ &= (10.3 \times 10^3 \text{ m})\cos(30.2°) \end{aligned}$$

Note that we have already substituted $1 \text{ km} = 1000 \text{ m}$ because the answer was requested in meters.

$\boxed{D_H = 8902 \text{ m}}$

Part (b) **What is the plane's vertical distance, in meters, above the radar dish?**

Note the right triangle created by the horizontal and vertical components of the distance to the plane. The vertical distance is the length of the side opposite to the angle θ, and hence the sine function is indicated.

$$\sin\theta = \frac{D_V}{D}$$

which may be rearranged as

$$\begin{aligned} D_V &= D\sin\theta \\ &= (10.3 \times 10^3 \text{ m})\sin(30.2°) \end{aligned}$$

Note that we have already substituted $1 \text{ km} = 1000 \text{ m}$ because the answer was requested in meters.

$\boxed{D_V = 5181 \text{ m}}$

Results may be validated by making certain that the components satisfy the Pythagorean theorem. Working in kilometers,

$$\begin{aligned} D &= \sqrt{D_H^2 + D_V^2} \\ &= \sqrt{(8.902 \text{ km})^2 + (5.181 \text{ km})^2} \\ &= 10.30 \text{ km} \end{aligned}$$

This cross-check would catch many errors, but it won't catch the common error where the sine and cosine functions are errantly swapped, hence interchanging the values of the horizontal and vertical components.

Part (c) **Write an expression for the position vector, $\vec{D}$, in rectangular Cartesian unit-vector form using the coordinate axes provided in the drawing in the problem statement.**

Cartesian unit-vector notation multiplies each component with the unit vector in the corresponding direction.

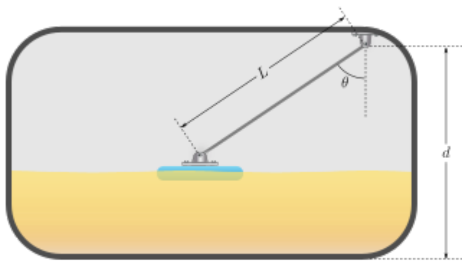
$$\vec{D} = D_H\,\hat{i} + D_V\,\hat{j}$$

Noting how the components were calculated in the previous steps,

$$\boxed{\vec{D} = D\cos\theta\,\hat{i} + D\sin\theta\,\hat{j}}$$

Problem 2:

The fuel tank on a car has a depth $d = 0.302$ m. The fuel level in the tank is detected by a sensor at the end of an arm with length $L = 0.603$ m. The arm is free to rotate about a pivot at an upper corner of the fuel tank. The angle between the arm and the vertical is θ, as indicated in the drawing.



Part (a) **Derive an expression for the sensor height, h, above the horizontal tank bottom as a function of L, d and θ.**

In this problem, we want a function that we can use to say how high above the bottom of the tank the sensor is floating. We can use our knowledge of trigonometry to set up an equation for how far down the tank in height the sensor is as follows.

$$h_d = L\cos\theta$$

What we want, however, is how high from the bottom the sensor is, not how far from the top it is. To account for this, we can subtract the distance from the top that we just found from the total height of the tank. This will give us the height from the bottom of the tank to the sensor.

$$\boxed{h = d - L\cos\theta}$$

Part (b) **Use logic to deduce the value, in degrees, of the angle θ when the fuel tank is full. No computation is required.**

We know that the arm is connected to the top corner of the fuel tank. If the tank was full, then the sensor would be at the same height as the arm. This means that the arm would trace a straight line perpendicular to the wall of the tank out to the sensor. Since the arm would be perpendicular to the wall, the angle between the wall and the sensor would be

$$\boxed{\theta_{\text{full}} = 90^\circ}$$

Part (c) **Calculate the angle, in degrees, associated with a half-full fuel tank.**

Using our results from part (a), we can set up an equation for the angle and solve it for this scenario. The important thing to realize is that we know that if the tank is half full, then the height of the sensor must be $\frac{d}{2}$.

$$h = d - L\cos\theta_{\text{half}}$$

$$\frac{d}{2} = d - L\cos\theta_{\text{half}}$$

$$\frac{d}{2} - d = -L\cos\theta_{\text{half}}$$

$$-\frac{d}{2} = -L\cos\theta_{\text{half}}$$

$$\frac{d}{2L} = \cos\theta_{\text{half}}$$

$$\arccos\left(\frac{d}{2L}\right) = \theta_{\text{half}}$$

$$\theta_{\text{half}} = \arccos\left(\frac{0.302\ \text{m}}{2 \cdot 0.603\ \text{m}}\right)$$

$$\boxed{\theta_{\text{half}} = 75.54^\circ}$$

Part (d) **Note that the arm is longer than the tank is deep, $L > d$. What angle, in degrees, is associated with an empty fuel tank?**

At first, one might be tempted to think that since the tank is empty, the arm would swing all the way back to the wall making an angle of zero degrees. However, if you look at the length of the arm compared to the height of the tank, we see that the arm is longer than the height of the tank such that it's impossible for it to be parallel with the wall. Instead, we must use our results from part (a) to set up an equation for the angle in this case. The important thing to realize is that since the tank is empty, the height of the sensor above the bottom of the tank is zero.

$$h = d - L\cos\theta_{\text{empty}}$$

$$0 = d - L\cos\theta_{\text{empty}}$$

$$-d = -L\cos\theta_{\text{empty}}$$

$$\frac{d}{L} = \cos\theta_{\text{empty}}$$

$$\arccos\left(\frac{d}{L}\right) = \theta_{\text{empty}}$$

$$\theta_{\text{empty}} = \arccos\left(\frac{0.302\ \text{m}}{0.603\ \text{m}}\right)$$

$$\boxed{\theta_{\text{empty}} = 59.95^\circ}$$

Problem 3:

The time-dependent position of a particle is given by

$$\vec{r} = \left(5.0\ \text{m/s}^2\right)t^2\ \hat{\imath} + \left(4.0\ \text{m/s}^2\right)t^2\ \hat{\jmath}$$

Part (a) **What is the magnitude, in meters, of the particle's distance from the origin at $t = 0.0$ s?**

We are given the object's position vector as a function of time:

$$\vec{r} = r_x t^2\ \hat{\imath} + r_y t^2\ \hat{\jmath}$$

The object's distance d from the origin is the magnitude of its position vector,

$$d = \sqrt{(r_x t^2)^2 + (r_y t^2)^2}$$

We substitute $t = 0.0$ s and obtain the requested distance.

$$\boxed{d(0.0\text{ s}) = 0.0\text{ m}}$$

Part (b) **What is the magnitude, in meters, of the particle's distance from the origin at $t = 2.0$ s?**

We are given the object's position vector as a function of time.

$$\vec{r} = r_x t^2\,\hat{i} + r_y t^2\,\hat{j}$$

The object's distance from the origin equals the magnitude of its position vector,

$$d = \sqrt{(r_x t^2)^2 + (r_y t^2)^2}$$

We substitute the given values, including $t = 2.0$ s.

$$d(2.0\text{ s}) = \sqrt{\left[\left(5.0\text{ m/s}^2\right)\left(2.0\text{ s}\right)^2\right]^2 + \left[\left(4.0\text{ m/s}^2\right)\left(2.0\text{ s}\right)^2\right]^2}$$

$$\boxed{d\left(2.0\text{ s}\right) = 25.61\text{ m}}$$

Part (c) **Provide an expression in Cartesian unit-vector notation for the velocity vector of the particle. Do not include the units, but assume that units of length are meters, and units of time are meters.**

The velocity is the derivative with respect to time of the position. Use the property that the derivative of a sum is the sum of the derivatives.

$$\frac{\mathrm{d}}{\mathrm{d}t}(f+g) = \frac{\mathrm{d}f}{\mathrm{d}t} + \frac{\mathrm{d}g}{\mathrm{d}t}$$

Also use the property that the derivative of the product of a constant and a function is the product of the constant with the derivative of the function.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(Cf\right) = C\frac{\mathrm{d}f}{\mathrm{d}t}$$

Applied to the position vector,

$$\begin{aligned}
\vec{v} &= \frac{\mathrm{d}}{\mathrm{d}t}\vec{r} \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(5.0\text{ m/s}^2\right)t^2\,\hat{i} + \left(4.0\text{ m/s}^2\right)t^2\,\hat{j}\right] \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(5.0\text{ m/s}^2\right)t^2\,\hat{i}\right] + \frac{\mathrm{d}}{\mathrm{d}t}\left[\left(4.0\text{ m/s}^2\right)t^2\,\hat{j}\right] \\
&= \left(5.0\text{ m/s}^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}t^2\right)\,\hat{i} + \left(4.0\text{ m/s}^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}t^2\right)\,\hat{j}
\end{aligned}$$

Next use the rule for differentiating a monomial power.

$$\frac{\mathrm{d}}{\mathrm{d}t}t^n = nt^{n-1}$$

With an application to the case $n = 2$,

$$\begin{aligned}
\vec{v} &= \left(5.0\text{ m/s}^2\right)\left(2t\right)\,\hat{i} + \left(4.0\text{ m/s}^2\right)\left(2t\right)\,\hat{j} \\
&= \left(10.0\text{ m/s}^2\right)t\,\hat{i} + \left(8.0\text{ m/s}^2\right)t\,\hat{j}
\end{aligned}$$

For the sake of entry of the answer,we suppress the units.

$$\boxed{\vec{v} = 10.0t\,\hat{i} + 8.0t\,\hat{j}}$$

Part (d) **What is the speed, in meters per second, of the particle when $t = 0.0$ s?**

The velocity at the given time is

$$\begin{aligned}\vec{v}(0.0\text{ s}) &= \left(10.0\text{ m/s}^2\right)\left(0.0\text{ s}\right)\hat{\imath} + \left(8.0\text{ m/s}^2\right)\left(0.0\text{ s}\right)\hat{\jmath}\\ &= \left(0.0\text{ m/s}\right)\hat{\imath} + \left(0.0\text{ m/s}\right)\hat{\jmath}\end{aligned}$$

The speed is the magnitude of the velocity vector.

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2}\\ &= \sqrt{\left(0.0\text{ m/s}\right)^2 + \left(0.0\text{ m/s}\right)^2}\end{aligned}$$

$$\boxed{v\left(0.0\text{ s}\right) = 0.0\text{ m/s}}$$

Part (e) **What is the speed, in meters per second, of the particle when $t = 2.0$ s?**

The velocity at the given time is

$$\begin{aligned}\vec{v}(2.0\text{ s}) &= \left(10.0\text{ m/s}^2\right)\left(2.0\text{ s}\right)\hat{\imath} + \left(8.0\text{ m/s}^2\right)\left(2.0\text{ s}\right)\hat{\jmath}\\ &= \left(20.\text{ m/s}\right)\hat{\imath} + \left(16\text{ m/s}\right)\hat{\jmath}\end{aligned}$$

The speed is the magnitude of the velocity vector.

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2}\\ &= \sqrt{\left(20.\text{ m/s}\right)^2 + \left(16\text{ m/s}\right)^2}\end{aligned}$$

$$\boxed{v\left(0.0\text{ s}\right) = 25.61\text{ m/s}}$$

Problem 4:

A particle has a constant acceleration given by

$$\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath}$$

and initially, at $t = 0$, the particle is at rest at the origin.

Part (a) **What is the particle's position in Cartesian unit-vector notation as a function of time?**

The position vector of a particle in the $x - y$ plane is given by

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

When the acceleration is constant,

$$\begin{aligned}x(t) &= x_0 + v_{0,x}t + \frac{1}{2}a_x t^2\\ y(t) &= y_0 + v_{0,y}t + \frac{1}{2}a_y t^2\end{aligned}$$

With the particle initially at rest at the origin,

$$\begin{aligned}x_0 &= 0\\ y_0 &= 0\\ v_{0,x} &= 0\\ v_{0,y} &= 0\end{aligned}$$

Putting it all together,

$$\vec{r}(t) = \left(\frac{1}{2}a_x t^2\right)\hat{i} + \left(\frac{1}{2}a_y t^2\right)\hat{j}$$

Part (b) **What is the particle's velocity in Cartesian unit-vector notation as a function of time?**

The velocity vector of a particle in the $x - y$ plane is given by

$$\vec{v}(t) = v_x(t)\,\hat{i} + v_y(t)\,\hat{j}$$

When the acceleration is constant,

$$v_x(t) = v_{0,x} + a_x t$$
$$v_y(t) = v_{0,y} + a_y t$$

Since the particle is initially at rest,

$$v_{0,x} = 0$$
$$v_{0,y} = 0$$

Putting it all together,

$$\vec{v}(t) = (a_x t)\,\hat{i} + (a_y t)\,\hat{j}$$

Part (c) **What is the particle's path, expressing the y coordinate as a function of x? Your expression will be independent of time.**

From Part (a),

$$x = \frac{1}{2}a_x t^2$$
$$y = \frac{1}{2}a_y t^2$$

Taking the ratio of the second equation to the first,

$$\frac{y}{x} = \frac{a_y}{a_x}$$

Note that the time has canceled. Upon solving for the y coordinate,

$$y = \frac{a_y}{a_x}x$$

Problem 5:

Taking north to be the positive y direction and east to be the positive x direction, a particle's position is given by

$$r(t) = (20.2\ \mathrm{m/s})t\,\hat{i} + (4.01\ \mathrm{m/s^2})t^2\,\hat{j}$$

Part (a) **In what direction is the particle traveling at $t = 0.0$ s?**

The instantaneous direction of the motion is given by the velocity vector at the specified time. The velocity is the derivative of the position.

$$\begin{aligned}
v(t) &= \frac{\mathrm{d}}{\mathrm{d}t}r(t) \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left[(20.2\ \mathrm{m/s})t\,\hat{i} + (4.01\ \mathrm{m/s^2})t^2\,\hat{j}\right] \\
&= (20.2\ \mathrm{m/s})\left(\frac{\mathrm{d}}{\mathrm{d}t}t\right)\hat{i} + (4.01\ \mathrm{m/s^2})\left(\frac{\mathrm{d}}{\mathrm{d}t}t^2\right)\hat{j} \\
&= (20.2\ \mathrm{m/s})(1)\,\hat{i} + (4.01\ \mathrm{m/s^2})(2t)\,\hat{j} \\
&= (20.2\ \mathrm{m/s})\,\hat{i} + 2(4.01\ \mathrm{m/s^2})t\,\hat{j}
\end{aligned}$$

Evaluated at the the specified time,

$$\begin{aligned} v(0.0\text{ s}) &= (20.2\text{ m/s})\,\hat{\imath} + 2(4.01\text{ m/s}^2)(0.0\text{ s})\,\hat{\jmath} \\ &= (20.2\text{ m/s})\,\hat{\imath} \end{aligned}$$

At the specified time, the particle is moving in the $\hat{\imath}$ direction, or east.

$\boxed{\text{east}}$

Part (b) **At what time, in seconds, is the particle traveling *exactly* northeast?**

The instantaneous direction of the motion is given by the velocity vector at the specified time. The velocity which is the derivative of the position, was determined as part of the solution to Part (a).

$$v(t) = (20.2\text{ m/s})\,\hat{\imath} + 2(4.01\text{ m/s}^2)t\,\hat{\jmath}$$

The particle is moving northwest when the coefficients of $\hat{\imath}$ and $\hat{\jmath}$ are equal.

$$20.2\text{ m/s} = 2(4.01\text{ m/s}^2)t$$

Solving for the time when this is the case,

$$t = \frac{20.2\text{ m/s}}{2(4.01\text{ m/s}^2)}$$

$\boxed{t = 2.519\text{ s}}$

Problem 6:

An airplane starts at rest and accelerates at *5.6* m/s^2 at an angle of *25*° south of west.

Part (a) **After *7* s, how far, in meters, in the westerly direction has the airplane traveled?**

We are told that the airplane begins at rest and travels at an angle south of west. We must determine the horizontal component (due west) of its displacement.

Let x represent motion towards the west, which we'll consider to be the positive x-direction. The horizontal component can be found using the following kinematic equation:

$$x = v_{x,0}t + \tfrac{1}{2}a_x t^2$$

where $v_{x,0}$ = 0 m/s is the initial velocity component.

The x-component of the acceleration is $a_x = a\cos\theta$, so we can find the position using the given information.

$$\begin{aligned} x &= \tfrac{1}{2}a_x t^2 \\ &= \tfrac{1}{2}a\cos(\theta)\,t^2 \\ &= \tfrac{1}{2}\left(5.6\text{ m/s}^2\right)\cos(25^\circ)\,(7\text{ s})^2 \end{aligned}$$

$\boxed{x = 124.3\text{ m}}$

Part (b) **After *7* s, how far, in meters, in the southerly direction has the airplane traveled?**

Let y represent motion towards the south, which we'll consider to be the positive y-direction. The appropriate kinematic equation for this situation is

$$y = v_{y,0}t + \tfrac{1}{2}a_y t^2$$

where $v_{y,0} = 0$ m/s. Since we know the acceleration, the angle, and the elapsed time, we can find the displacement toward the south.

$$\begin{aligned} y &= v_{y,0}t + \tfrac{1}{2}a_y t^2 \\ &= v_{y,0}t + \tfrac{1}{2}a\cos(\theta)\,t^2 \\ &= \tfrac{1}{2}\left(5.6\text{ m/s}^2\right)\sin(25^\circ)\,(7\text{ s})^2 \end{aligned}$$

$\boxed{y = 57.98\text{ m}}$

Problem 7:

A boat leaves the dock at $t = 0.00$ s, and, starting from rest, maintains a constant acceleration of $(0.202\ \mathrm{m/s^2})\,\hat{\imath}$ relative to the water. Due to currents, however, the water itself is moving with a velocity of $(0.203\ \mathrm{m/s})\,\hat{\imath} + (1.01\ \mathrm{m/s})\,\hat{\jmath}$.

Part (a) **How fast, in meters per second, is the boat moving at $t = 4.01$ s?**

Assume the dock is the origin. The total velocity is the velocity due to the acceleration plus the velocity of the water itself. From $v_x = v_{0x} + a_x t$ with $v_{0x} = 0$, we have $v_x = a_x t$. Thus, the velocity at $t = 4.01$ s is

$$\begin{aligned}\vec{v} &= \left[\left(0.202\ \mathrm{m/s^2}\right)(4.01\ \mathrm{s}) + (0.203\ \mathrm{m/s})\right]\hat{\imath} + (1.01\ \mathrm{m/s})\,\hat{\jmath} \\ &= (1.013\ \mathrm{m/s})\,\hat{\imath} + (1.010\ \mathrm{m/s})\,\hat{\jmath}\end{aligned}$$

Speed is the magnitude of the velocity, so

$$v = \sqrt{(1.013\ \mathrm{m/s})^2 + (1.010\ \mathrm{m/s})^2}$$

Caclulating,

$$\boxed{v = 1.430\ \mathrm{m/s}}$$

Part (b) **How far, in meters, is the boat from the dock at $t = 4.01$ s?**

The total displacement is the displacement due to the movement of the water, plus the displacement due to the movement *through* the water. From $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ with zero initial position and zero acceleration, $x = v_{0x}t$ and $y = v_{0y}t$ describe the displacment of the water, while $x = \frac{1}{2}a_x t^2$ is the displacment through the water. Thus, the total displacment at $t = 4.01$ s is

$$\begin{aligned}\Delta r &= \left[\tfrac{1}{2}\left(0.202\ \mathrm{m/s^2}\right)(4.01\ \mathrm{s})^2 + (0.203\ \mathrm{m/s})(4.01\ \mathrm{s})\right]\hat{\imath} + \left[(1.01\ \mathrm{m/s})(4.01\ \mathrm{s})\right]\hat{\jmath} \\ &= (2.438\ \mathrm{m})\,\hat{\imath} + (4.050\ \mathrm{m})\,\hat{\jmath}\end{aligned}$$

The distance from the dock is the magnitude of the displacement,

$$\text{distance} = \sqrt{(2.438\ \mathrm{m})^2 + (4.050\ \mathrm{m})^2}$$

Caclulating,

$$\boxed{\text{distance} = 4.727\ \mathrm{m/s}}$$

Problem 8:

A spaceship is traveling at a velocity of

$$\vec{v}_0 = (20.1\ \mathrm{m/s})\,\hat{\imath}$$

when its rockets fire, giving it an acceleration of

$$\vec{a} = (2.02\ \mathrm{m/s^2})\,\hat{\imath} + (4.03\ \mathrm{m/s^2})\,\hat{\jmath}$$

Part (a) **How fast, in meters per second, is the rocket moving 3.04 s after the rockets fire?**

Since the acceleration is constant, we have

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t \\ &= (20.1\ \mathrm{m/s})\,\hat{\imath} + \left[\left(2.02\ \mathrm{m/s^2}\right)\hat{\imath} + \left(2.02\ \mathrm{m/s^2}\right)\hat{\jmath}\right](3.04\ \mathrm{s})\end{aligned}$$

$$= (26.24\ \mathrm{m/s})\,\hat{i} + (12.25\ \mathrm{m/s})\,\hat{j}$$

Speed is the mangitude of the velocity:

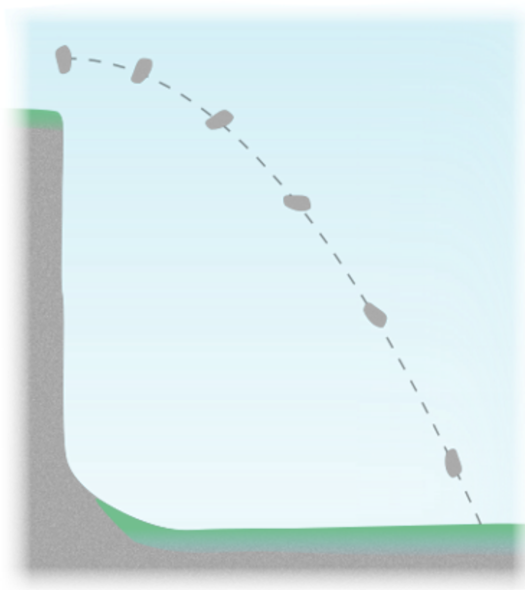
$$v = \sqrt{(26.24\ \mathrm{m/s})^2 + (12.25\ \mathrm{m/s})^2}$$

Calculating,

$$\boxed{v = 28.96\ \mathrm{m/s}}$$

Problem 9:

A stone is thrown off of a cliff with an initial velocity that is horizontal. There is no air resistance, and it follows the path shown which is typical for projectile motion.



Part (a) **What direction is the acceleration of the stone?**

The acceleration due to gravity is always downward. If air resistance were present, it would tend to slow the speed of the stone, implying an acceleration in the direction opposite to the velocity, but air resistance is assumed to be irrelevant. (If the speed of the stone is not too great, then the assumption is reasonable.) The only contribution is the downward acceleration due to gravity.

The correct choice is

Problem 10:

A projectile is launched from ground level at an angle of 30 degrees above the horizontal. Neglect air resistance and consider the motion from just after the moment it is launched to just before the moment it lands on the ground.

Part (a) **When is the projectile's velocity equal to zero?**

In projectile motion with a non-vertical launch angle, as in the present case, the velocity has a constant nonzero horizontal component. Thus, no matter what the velocity's vertical component is, the velocity is never zero.

The correct choice is, therefore, ...

The projectile's velocity is never zero.

Part (b) **When does the projectile have the smallest speed?**

In projectile motion with a non-vertical launch angle, as in the present case, the velocity has a constant nonzero horizontal component. Thus the projectile has the smallest speed, when the vertical component of its velocity is zero, which it is at the trajectory's highest point. Since the launch angle is above the horizontal, the trajectory does indeed have a highest point.

The correct choice is, therefore, ...

At the highest point.

Part (c) **When does the projectile's speed equal its launch speed?**

In the present case, with the projectile launched from and landing on the ground, the evolution of its speed is this: During the ascent from the launch to the trajectory's highest point, the speed decreases to a minimum. Then, during the descent to the ground, the speed increases back to its value at launch.

The correct choice is, therefore, ...

Just before landing on the ground.

Part (d) **After launch, when does the projectile's velocity equal its launch velocity?**

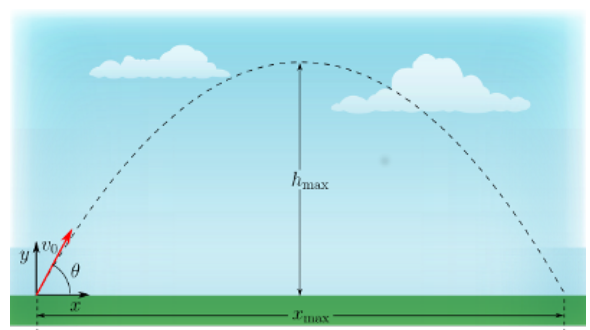
In the present case, with the projectile launched from and landing on the ground, the evolution of its velocity is this: During the ascent from the launch to the trajectory's highest point, the velocity is directed above the horizontal and its speed decreases to a minimum. Then, during the descent to the ground, the velocity is directed below the horizontal, and its speed increases back to its value at launch. Nowhere does the velocity have both the same direction and the same speed.

The correct choice is, therefore, ...

The projectile's velocity is never equal to its launch velocity after launch.

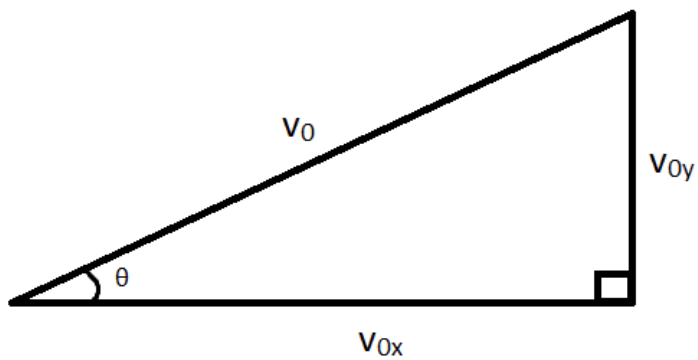
Problem 11:

During a baseball game, a baseball is struck, at ground level, by a batter. The ball leaves the baseball bat with an initial speed $v_0 = 25.7$ m/s at an angle $\theta = 15.5^\circ$ above the horizontal. Let the origin of the Cartesian coordinate system be the ball's position the instant it leaves the bat. Ignore air resistance throughout this problem.



Part (a) **Express the magnitude of the ball's initial horizontal velocity component, $v_{0,x}$, in terms of v_0 and θ.**

Let's start by drawing a triangle from the horizontal and vertical components of the initial velocity:



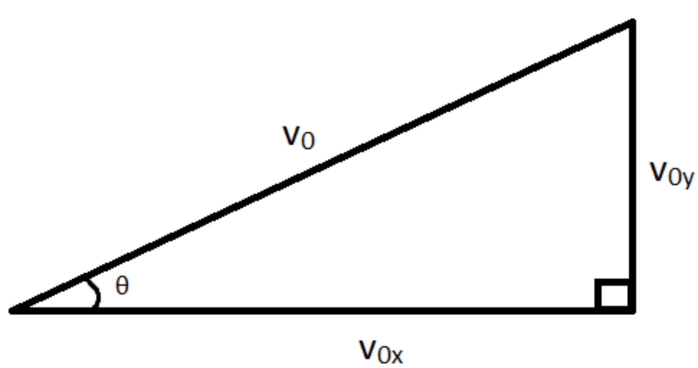
Recall that the cosine function of an angle in a right triangle gives the adjacent side divided by the hypotenuse. We can use this to write an equation to relate the angle, initial velocity, and the initial horizontal velocity.

$$\frac{v_{0x}}{v_0} = cos\,(\theta)$$

$$\boxed{v_{0x} = v_0 cos\,(\theta)}$$

Part (b) **Express the magnitude of the ball's initial vertical velocity component, $v_{0,y}$, in terms of v_0 and θ.**

Let's start by drawing a triangle from the horizontal and vertical components of the initial velocity:



Recall that the sine function of an angle in a right triangle gives the opposite side divided by the hypotenuse. We can use this to write an equation to relate the angle, initial velocity, and the initial vertical velocity.

$$\frac{v_{0y}}{v_0} = sin\,(\theta)$$

$$\boxed{v_{0y} = v_0 sin\,(\theta)}$$

Part (c) **Find the ball's maximum vertical height, h_{max}, in meters, above the ground.**

To find the maximum vertical height of the ball, we only need to look at the vertical motion of the ball. We will need to use a kinematic equation for this, so let's set up our system of coordinates to have the origin at the ground and up as the positive direction. Now, let's go over our known values. We found an expression for the initial vertical velocity of the ball in part (a), we know that the ball will experience acceleration downward due to gravity, and we know that the ball will have a final vertical acceleration of zero at the moment when it reaches its maximum height. We can therefore use the following kinematic equation to find a value for how high the ball traveled:

$$v^2 = v_0^2 + 2ad$$

Now, let's plug values for our system into this equation and solve for the maximum height.

$$(0 \text{ m/s})^2 = (25.7 \text{ m/s} \cdot sin\,(15.5^\circ))^2 + 2 \cdot -9.8 \text{ m/s}^2 \cdot h_{max}$$

$$0 = (25.7 \text{ m/s} \cdot sin\,(15.5^\circ))^2 - 19.6 \text{ m/s}^2 \cdot h_{max}$$

$$19.6 \text{ m/s}^2 \cdot h_{max} = (25.7 \text{ m/s} \cdot sin\,(15.5^\circ))^2$$

$$h_{max} = \frac{(25.7 \text{ m/s} \cdot sin\,(15.5^\circ))^2}{19.6 \text{ m/s}^2}$$

$$\boxed{h_{max} = 2.402 \text{ m}}$$

Part (d) **Enter an expression in terms of v_0, θ, and g for the time it takes the ball to travel to its maximum vertical height.**

As in part (c), we will only need to concern ourselves with the vertical motion of the ball to solve this problem. We can use the same coordinate system we used in part (c) and we also have the same known variables with the addition of the ball's maximum height. Based on our known values, we can use the following kinematic equation:

$$v = v_0 + at$$

Now, let's plug in our variables, recalling that the velocity of the ball in the vertical direction will be zero when it reaches its maximum height.

$$0 \text{ m/s} = v_0 sin\,(\theta) - gt$$

$$-v_0 sin\,(\theta) = -gt$$

$$\boxed{t = \frac{v_0 sin\,(\theta)}{g}}$$

Part (e) **Calculate the horizontal distance, $x_{\max}$, in meters, that the ball has traveled when it returns to ground level.**

To solve this problem, we will obviously need to use another kinematic equation. We can let our coordinate system be the same as it was in parts (c) and (d). We know the initial horizontal velocity of the ball from part (a). We also know that the ball experiences no horizontal acceleration during its flight. Finally, in part (d) we found an expression for the amount of time it takes for the ball to reach its maximum height. Due to the symmetry of this problem, the ball will take just as long to land as it did to go up, meaning that the total time the ball is in flight is equal to twice the expression we found in part (d). With this information, we can set up the following kinematic equation for the ball's horizontal motion:

$$d = v_0 t + \frac{1}{2} a t^2$$

Let's begin plugging in variables (remembering that the time will be twice the expression that we found in part (d)) and solve for the total horizontal distance.

$$x_{max} = v_0 cos\,(\theta) \cdot \left(2 \cdot \frac{v_0 sin\,(\theta)}{g}\right) + \frac{1}{2} \cdot 0\ \mathrm{m/s}^2 \cdot \left(2 \cdot \frac{v_0 sin\,(\theta)}{g}\right)^2$$

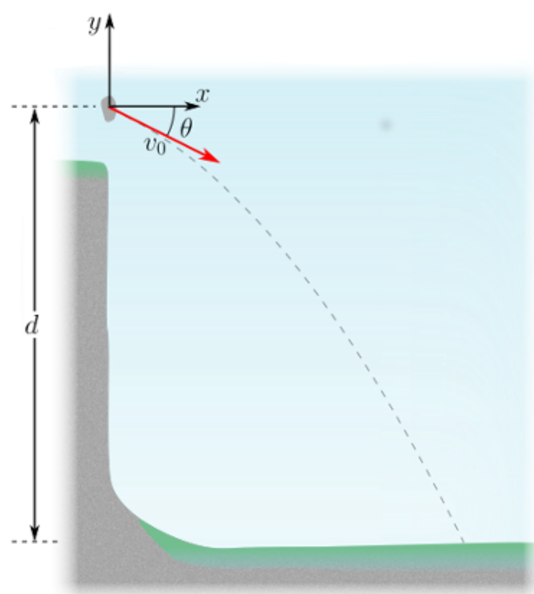
$$x_{max} = 2 \cdot \frac{v_0^2 cos\,(\theta)\, sin\,(\theta)}{g}$$

$$x_{max} = 2 \cdot \frac{(25.7\ \mathrm{m/s})^2 \cdot cos\,(15.5^\circ) \cdot sin\,(15.5^\circ)}{9.81\ \mathrm{m/s}^2}$$

$$\boxed{x_{max} = 34.66\ \mathrm{m}}$$

Problem 12:

A student standing on a cliff throws a stone from a vertical height of $d = 8.0\ \mathrm{m}$ above the level ground with velocity $v_0 = 15\ \mathrm{m/s}$ at an angle $\theta = 15^\circ$ *below* the horizontal, as shown. It moves without air resistance. Use a Cartesian coordinate system with the origin at the initial position of the stone.



Part (a) **With what speed, in meters per second, does the stone strike the ground?**

The horizontal component of the stone's velocity throughout its flight does not change, since there is no acceleration. The expression describing this is

$$v_x = v_0 cos\,(\theta)$$

The vertical component of the stone's velocity does vary with time because of the downward acceleration due to gravity that acts on it throughout its flight. Applying an appropriate kinematic equation gives the relationship between the vertical component of the velocity and the height from which the stone originates.

$$v_y^2 = v_0^2 sin^2(\theta) + 2gd$$

Note that the acceleration vector and the vertical displacement vector both point downward, so the minus signs associated with g and d cancel each other.

The magnitude of the stone's final velocity is found using the Pythagorean theorem.

$$v_f = \sqrt{v_x^2 = v_y^2} = \sqrt{v_0^2 cos^2(\theta) + v_0^2 sin^2(\theta) + 2gd}$$

Note that $cos^2(\theta) + sin^2(\theta) = 1$, which simplifies the above equation.

$$\begin{aligned} v_f &= \sqrt{v_0^2 + 2gd} \\ &= \sqrt{(15\ \mathrm{m/s})^2 + 2\left(9.81\ \mathrm{m/s^2}\right)(8.0\ \mathrm{m})} \end{aligned}$$

$$\boxed{v_f = 19.54\ \mathrm{m/s}}$$

Part (b) **If the stone had been thrown from the clifftop with the same initial speed and the same angle, but *above* the horizontal, would its impact velocity be different?**

From part a, we see that angle that the stone is thrown doesn't enter into the final equation for the final velocity.

$$v_f = \sqrt{v_0^2 + 2gd}$$

Therefore, the answer will be the same no matter how the stone is shown with that initial velocity. Later, you'll learn about the conservation of mechanical energy and this will become further clarified.

$$\boxed{\text{no}}$$

Problem 13:

A quarterback can run with a speed of v_q = 15 miles per hour. He throws the football with a speed v_{ball} = 35 miles per hour at an unknown angle θ that is between 40 and 70 degrees, measured relative to horizontal plane. Neglect air resistance.

Part (a) **Is it possible for him to catch his own pass?**

As the ball flies through the air, it has both horizontal and vertical velocity components. The quarter back can only run horizontally. If the velocity of the quarterback is greater than or equal to the horizontal component of the ball's velocity, then it is possible for him to catch his own thrown ball.

$\boxed{\text{Yes}}$

Part (b) **In this part we'll assume the quarterback stops and throws the ball just as a receiver runs past just beside him running at 15 miles per hour. If the quarterback releases his pass at that instant, what would be the angle in degrees be that he would have to throw the ball for the receiver to catch it?**

With the information given, we can determine the minimum angle at which the football should be through by the quarterback.

$$v_{\mathrm{q}} \geq v_{\mathrm{ball}} \cos\ (\theta)$$

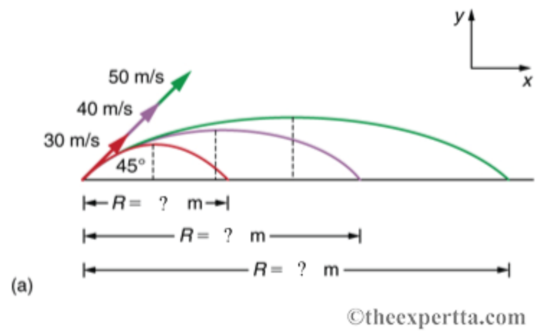
Then,

$$\theta = \cos^{-1}\left(\frac{v_{\mathrm{q}}}{v_{\mathrm{ball}}}\right) = \cos^{-1}\left(\frac{15}{35}\right)$$

$$\boxed{\theta = 64.6^{\circ}}$$

Problem 14:

Find the ranges for the projectiles shown in the figure at an elevation angle of 45° and the given initial speeds.



Part (a) **Find the range, in meters, for the projectile with a speed of 30 m/s.**

The horizontal distance the projectile travels during its flight is given by the range formula.

$$R = v_0^2 \frac{sin(2\theta)}{g}$$

$$= (30\ \mathrm{m/s})^2 \frac{sin(2{\cdot}45^{\circ})}{\left(9.8\ \mathrm{m/s}^2\right)}$$

$$\boxed{R = 91.8\ \mathrm{m}}$$

Part (b) **Find the range, in meters, for the projectile with a speed of 40 m/s.**

The horizontal distance the projectile travels during its flight is given by the range formula.

$$R = v_0^2 \frac{sin(2\theta)}{g}$$

$$= (40\ \mathrm{m/s})^2 \frac{sin(2{\cdot}45^{\circ})}{\left(9.8\ \mathrm{m/s}^2\right)}$$

$$\boxed{R = 163\ \mathrm{m}}$$

Part (c) **Find the range, in meters, for the projectile with a speed of 50 m/s.**

$$R = v_0^2 \frac{sin(2\theta)}{g}$$

$$= (50\ \mathrm{m/s})^2 \frac{sin(2{\cdot}45^{\circ})}{\left(9.8\ \mathrm{m/s}^2\right)}$$

$$\boxed{R = 255\ \mathrm{m}}$$

Problem 15:

Great Problems in Physics Series

Dr. Jennifer Carter is an assistant professor at Susquehanna University where she teaches both astronomy and physics courses. Her main research interests are in astrophysics and data analysis. Currently her focus is on exoplanet research, in which she is using Bayesian data analysis techniques to test current models of reflected light and thermal emissions of exoplanets against novel models she is developing.

I chose this problem because many students struggle with the quadratic and symmetric nature of projectile motion. In particular, they may struggle in choosing which of the two possible solutions applies to a given situation. This problem will require students to use the context of the problem statement to determine which of two possible solutions to choose, but what makes this problem unique compared to many other problems is that there will be two positive solutions to the amount of time that has passed; therefore, students cannot rely on eliminating the negative time as being physically unreasonable and must instead carefully consider the context of the problem

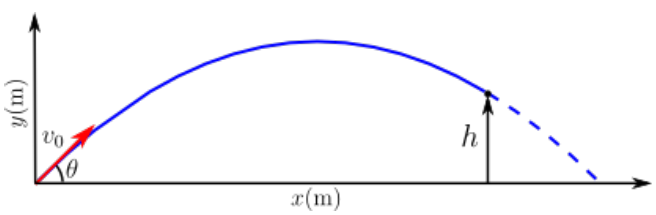
Read more about Dr. Carter and her research here.

View Dr. Carter's LinkedIn profile here.

Some children are practicing baseball and accidentally throw a ball onto the roof of one of their houses. Suppose that the baseball was thrown at an angle of ***51***° above the horizontal at a speed of ***19*** m/s. The ball rises to its maximum height before falling onto the roof of the house ***3.5*** m above the level at which it was thrown.

Part (a) **Determine the total amount of time, in seconds, the baseball spends in the air.**

First, let's construct a picture of the situation.



Second, let's list what variables are known and which must be determined.

Horizontal Direction:

$$x = ?$$
$$v_{0,x} = v_0 \cos(\theta)$$
$$= (19\ \mathrm{m/s}) \cos(51^\circ)$$
$$v_x = v_{0,x}$$
$$a_x = 0\ \mathrm{m/s^2}$$
$$t = ?$$

Vertical Direction:

$$y = h$$
$$= 3.5\ \mathrm{m}$$
$$v_{0,y} = v_0 \sin(\theta)$$
$$= (19\ \mathrm{m/s}) \sin(51^\circ)$$
$$v_y = ?$$
$$a_y = -g$$

$$= -9.81 \text{ m/s}^2$$

$$t = ?$$

Looking at the variables corresponding to the vertical direction, we see that there is enough information to determine time. There are two methods that may be employed. First, we can use the equation of vertical position versus time.

$$y = \tfrac{1}{2}a_y t^2 + v_{0,y}t$$

Inserting our knowns we find,

$$h = -\tfrac{1}{2}gt^2 + v_0 \sin(\theta)t$$

Subtracting the height, h, from both sides,

$$0 = -\tfrac{1}{2}gt^2 + v_0 \sin(\theta)t - h$$

This is a quadratic equation of the form

$$0 = at^2 + bt + c$$

with

$$a = -\tfrac{1}{2}g \quad , \quad b = v_0 \sin(\theta) \quad \text{and} \quad c = -h$$

which has two solutions given by

$$\begin{aligned} t_\pm &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-v_0 \sin(\theta) \pm \sqrt{\left(v_0 \sin(\theta)\right)^2 - 4\left(-\frac{1}{2}g\right)(-h)}}{2\left(-\frac{1}{2}g\right)} \\ &= \frac{v_0 \sin(\theta) \mp \sqrt{\left(v_0 \sin(\theta)\right)^2 - 2gh}}{g} \\ &= \frac{(19 \text{ m/s}) \sin(51^\circ) \mp \sqrt{\left((19 \text{ m/s}) \sin(51^\circ)\right)^2 - 2(9.81 \text{ m/s}^2)(3.5 \text{ m})}}{\frac{9.81 \text{ m}}{\text{s}^2}} \\ &= 0.2594 \text{ s} \quad \text{or} \quad 2.751 \text{ s} \end{aligned}$$

Both of these solutions are physically possible, so we must turn to the context of the problem to determine which applies to our situation. The problem states that the ball has risen to its maximum height and is now falling. You should also recall that projectile motion is symmetric in nature. The smaller of the two times corresponds to the point at which the ball rose to height h, and the larger of the two corresponds to the time it takes for the ball to rise to its maximum height and then fall to a height h; therefore, we should choose the larger of the two times.

$$\boxed{t = 2.751 \text{ s}}$$

The second approach to determining the time to rise and fall to height h is to first determine the final velocity in the vertical direction and then the amount of time to reach said velocity. Using

$$v_y^2 = v_{0,y}^2 + 2a_y y$$

we take the square root and substitute numbers.

$$\begin{aligned} v_y &= \pm\sqrt{v_{0,y}^2 + 2a_y y} \\ &= \pm\sqrt{\left(v_0 \sin(\theta)\right)^2 + 2\,(-g)\,h} \\ &= \pm\sqrt{\left((19 \text{ m/s}) \sin(51^\circ)\right)^2 - 2\,(9.81 \text{ m/s}^2)\,(3.5 \text{ m})} \\ &= \pm 12.22 \text{ m/s} \end{aligned}$$

The problem states that the baseball is traveling downward, so we should use the negative solution. Now using the equation for velocity versus time

$$v_y = v_{0,y} + a_y t$$

and substituting the numbers that we have

$$-12.22 \text{ m/s} = (19 \text{ m/s}) \sin(51^\circ) - \left(9.81 \text{ m/s}^2\right) t$$

and solving for the time,

$$t = \frac{-12.22 \text{ m/s} - (19 \text{ m/s}) \sin(51^\circ)}{-9.81 \text{ m/s}}$$

$$\boxed{t = 2.751 \text{ s}}$$

which, within round-off error, agrees with the previous result.

Part (b) **Calculate the horizontal distance, in meters, between the baseball's launch point and its landing point.**

The horizontal position is determined, using the time from part (a), via

$$x = v_{0,x}t + \frac{1}{2}a_x t^2$$
$$= v_0 \cos(\theta)t$$
$$= (19\ \mathrm{m/s})\cos(51^\circ)\,(2.751\ \mathrm{s})$$

$$\boxed{x = 32.89\ \mathrm{m}}$$

Problem 16:

You are the passenger in a car with your friend, who is driving. At one point in the journey, the your friend is following a curve on the highway. Concerned for your safety, you ask your friend how fast they are going.

Part (a) **Your friend responds, "I am going around this curve at a constant velocity of 55 mph." Is their statement physically correct?**

Your friend is using physics jargon incorrectly. Although the speed might be a constant 55 mph, the direction the car is moving is changing as it follows the curve.

Since velocity is defined not only by speed, but also by the direction of motion, a change in only one (in this case direction) is sufficient to change the velocity. Hence, velocity is changing although speed is staying the same.

$$\boxed{\text{No, the word 'velocity' is being used incorrectly.}}$$

Problem 17:

A particle is traveling along a circular path.

Part (a) **Which of the following is associated with a change in the speed of the particle?**

We learn from our textbook that it is only tangential acceleration that is associated with a change in the particle's speed.

The correct answer is, therefore, ...

$$\boxed{\text{Tangential acceleration}}$$

Part (b) **Which of the following is associated with a change in the velocity of the particle?**

Velocity involves both speed and direction. Centripetal acceleration changes direction and tangential acceleration changes speed. Thus, both accelerations are associated with a change in the particle's velocity.

The correct answer is, therefore, ...

$$\boxed{\text{Both centripetal and tangential acceleration will change the velocity.}}$$

Problem 18:

A particle travels in a circular path of radius *10.1* m at a constant speed of *20.1* m/s.

Part (a) **What is the magnitude of the acceleration of the particle in m/s^2?**

In circular motion at constant speed (uniform circular motion) the acceleration is directed toward the center of the circle. It is called centripetal acceleration, with a magnitude $a_c = v^2/r$, where v is the speed and r is the radius of the circular path. Thus,

$$a_c = \frac{(20.1\ \mathrm{m/s})^2}{10.1\ \mathrm{m}}$$

Calculating,

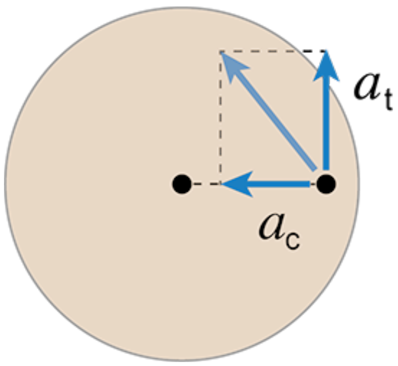
$$\boxed{a_c = 40.00\ \mathrm{m/s^2}}$$

Problem 19:

A propeller blade, measured from the rotational axis to the tip, has a length of 1.51 m. Starting from rest, the tip of the blade has a tangential acceleration of 2.02 $\mathrm{m/s^2}$.

Part (a) **What is the magnitude of the total acceleration of the tip of the blade, in meters per squared second, 0.53 s after the blade begin to rotate?**

In circular motion with changing speed there are two components to the acceleration. The centripetal acceleration is due to the velocity changing direction and is directed toward the center of the circle with a magnitude $a_c = v^2/r$, where v is the speed and r is the radius of the circular path. The tangential acceleration a_t is due to changing speed and is tangent to the circular path. The diagram illustrates the situation:



The centripetal acceleration is perpendicular to the tangential acceleration, so we apply the Pythagorean theorem, as follows, to find the magnitude of the total acceleration:

$$a = \sqrt{a_t^2 + a_c^2}$$

Substituting the expression for centripetal acceleration,

$$a = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$$

The speed of the tip of the blade, however, is the tangential acceleration times the time, so

$$a = \sqrt{\left(2.02\ \mathrm{m/s^2}\right)^2 + \left(\frac{\left[\left(2.02\ \mathrm{m/s^2}\right)(0.53\ \mathrm{s})\right]^2}{1.51\ \mathrm{m}}\right)^2}$$

Calculating,

$$\boxed{a = 2.158\ \mathrm{m/s^2}}$$

Problem 20:

A race car entering the curved part of the track drops its speed from *45* m/s to *35* m/ s in *2.01* s.

Part (a) **If the radius of the curved part of the track is *280* m, calculate the magnitude of the total acceleration, in m/s^2, of the race car just after it has begun to reduce its speed.**

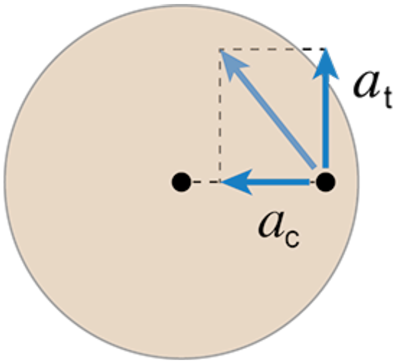
In circular motion with changing speed there are two components to the acceleration. The centripetal acceleration is due to the velocity changing direction and is directed toward the center of the circle with a magnitude $a_c = v^2/r$, where v is the speed and r is the radius of the circular path. The tangential acceleration a_t is due to changing speed and is tangent to the circular path. We start by calculating the magnitude of the centripetal acceleration:

$$a_c = \frac{(45\ \mathrm{m/s})^2}{280\ \mathrm{m}} = 7.232\ \mathrm{m/s^2}$$

Next, the magnitude of the tangential acceleration:

$$a_{\mathrm{t}} = \left| \frac{35\ \mathrm{m/s} - 45\ \mathrm{m/s}}{2.01\ \mathrm{s}} \right| = 4.975\ \mathrm{m/s}^2$$

The diagram illustrates the situation:



The centripetal acceleration is perpendicular to the tangential acceleration, so we apply the Pythagorean theorem, as follows, to find the magnitude of the total acceleration:

$$a = \sqrt{a_{\mathrm{t}}^2 + a_{\mathrm{c}}^2} = \sqrt{\left(4.975\ \mathrm{m/s}^2\right)^2 + \left(7.232\ \mathrm{m/s}^2\right)^2}$$

Calculating,

$$\boxed{a = 8.778\ \mathrm{m/s}^2}$$