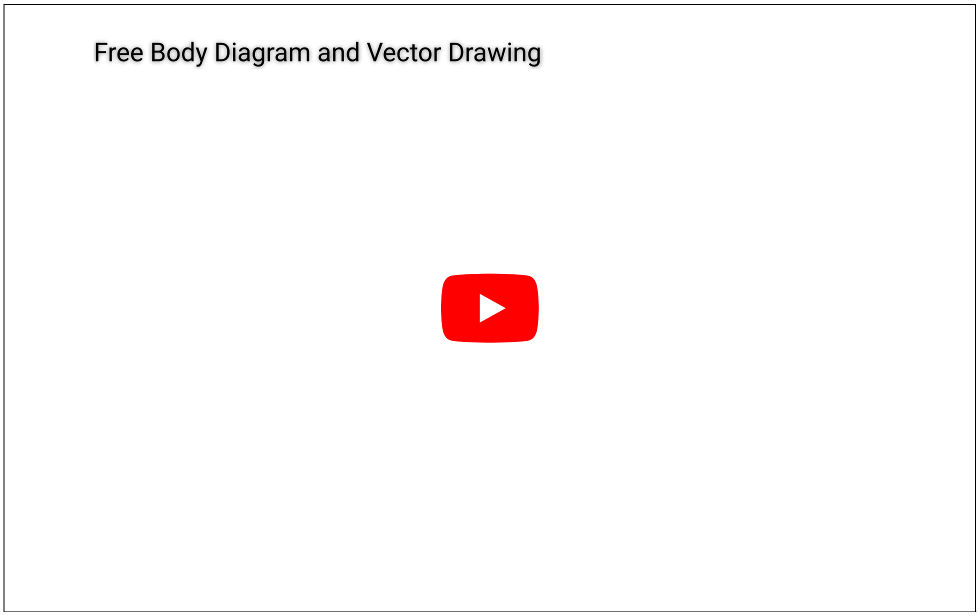
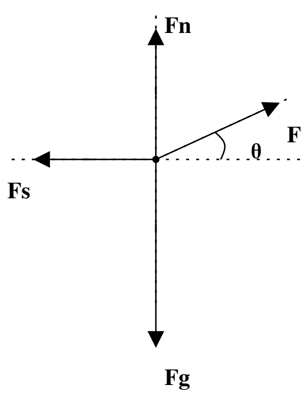
Problem 1:

Full solution not currently available at this time.

This problem is designed to introduce you to Expert TA's Free Body Diagram question type. Please watch the following brief video and then complete Part (a) of this problem. The video will show you how to enter the correct answer for Part (a).

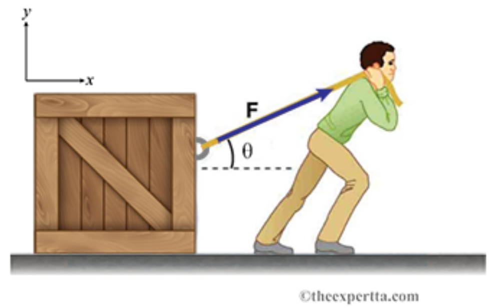


Part (a) **Please use the interactive area below to recreate the Free Body Diagram from the video. The video shows the case where a man is pulling the crate and where friction is involved. You can pause the video around 1 min and 35 seconds in to see the correct FBD.**



Problem 2:

A crate sits on a rough surface. Using a rope, a man applies a force to the crate, as shown in the figure. The force is not enough to move the crate, however, and it remains stationary. Use *f* to represent the force of friction.



Part (a) **Please use the interactive area below to draw the Free Body Diagram for the crate.**

All arrows representing forces should have their tails at the center of the crate. The crate is stationary and is in static equilibrium, so the net force acting on it is zero.

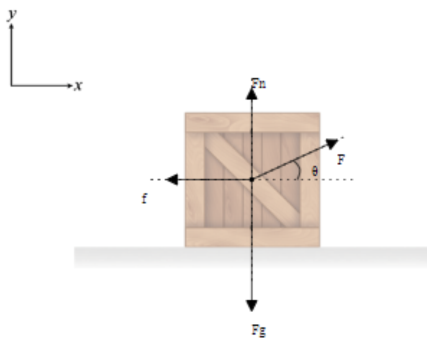
The applied force arrow F points from the crate's center to the right and upward, forming the acute angle θ above the horizontal.

The normal force arrow F_n points from the crate's center directly upward.

The friction force arrow f points from the crate's center directly to the left, opposite the direction of impending motion. The direction of impending motion is the direction of motion in the absence of friction, which is to the right.

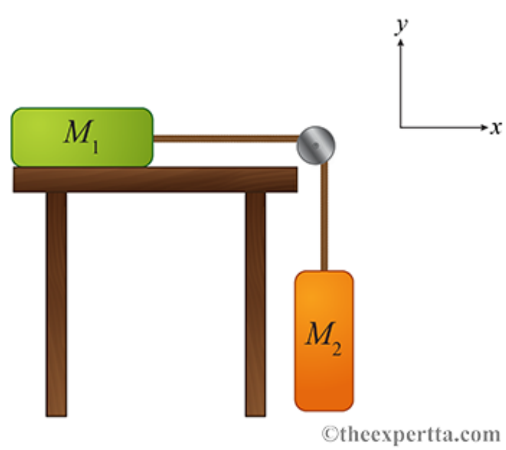
The gravitational force arrow F_g points from the crate's center directly downward.

The net force on the block is zero. Thus, the magnitudes of the f force equals the magnitude of the horizontal component of the F force. That is represented by the equality of the length of the f arrow to the length of the horizontal projection of the F arrow. Similarly, the length of the F_g arrow equals the sum of the lengths of the F_n arrow and the the vertical projection of the F arrow.



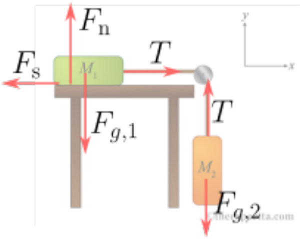
Problem 3:

Two blocks are tied together with a massless string that does not stretch and connected via a frictionless and massless pulley. Mass one, M_1, rests on a table top and is stationary. Denote the force for static friction as *Fs* and the tension in the string by *T*.



Part (a) **Please use the interactive area below to draw the Free Body Diagram for the mass M_1.**

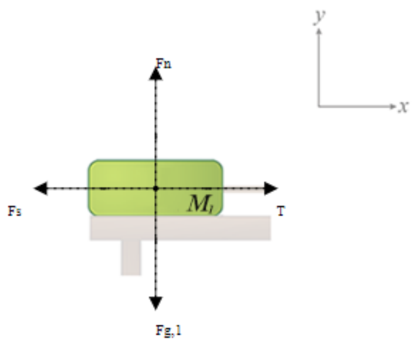
The original drawing has been marked up with forces.



For the block with mass M_1 on the table:

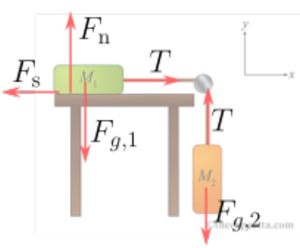
- Its weight, $F_{g,1}$, acts vertically downward.
- The table exerts a normal force, F_n, vertically upward with an equal magnitude to the weight.
- The tension, T, pulls the block towards the right.
- Because the block is not sliding, we know that the force of static friction, F_s, has the same magnitude as the tension, but it is directed towards the left.

Placing these forces on an FBD:



Part (b) **Please use the interactive area below to draw the Free Body Diagram for the mass M_2.**

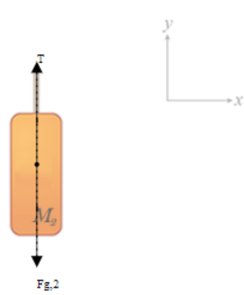
The original drawing has been marked up with forces.



For the block with mass M_2 suspended at the lower end of the string:

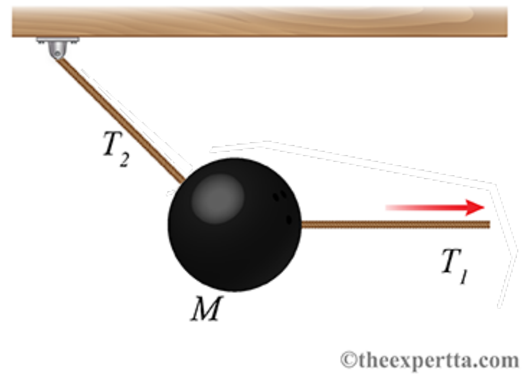
- Its weight, $F_{g,2}$, acts vertically downward.
- The tension, T, pulls the block upwards with a magnitude equal to its weight.

Placing these forces on an FBD:



Problem 4:

Consider a bowling ball of mass M attached to two ropes. One rope is being pulled horizontally. The second rope is tied to the ceiling (see figure) and is at a 45° with respect to the ceiling.The bowling ball is stationary.



Part (a) **Please use the interactive area below to draw the Free Body Diagram for the bowling ball.**

All arrows representing forces should have their tails at the center of the ball. The ball is stationary and is in equilibrium, so the net force acting on it is zero.

The gravitational force arrow F_g points from the ball's center directly downward.

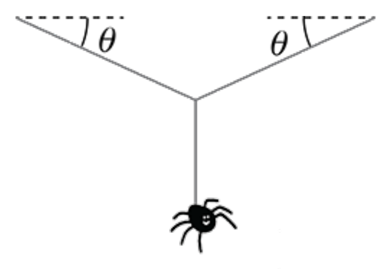
The tension force arrow T_1 points from the ball's center directly to the right.

The tension force arrow T_2 points from the ball's center upward and to the left, forming a 45° angle left of the vertical.

The net force on the ball is zero. Thus, the magnitude of the vertical component of the T_2 force equals the magnitude of the F_g force. That is represented by the equality of the length of the vertical projection of the T_2 arrow to the length of the F_g arrow. Similarly, the horizontal projection of the length of the T_2 arrow equals the length of the T_1 arrow.

Problem 5:

A spider with a mass of $m = 2.01 \times 10^{-5}$ kg hangs motionless from three stands of web. One strand is vertical, and the other two each make an angle of $\theta = 10.1°$ with the horizontal ceiling, as shown in the diagram. Let T_v be the tension in the vertical strand of the web, and let T_o be the tension in either of the other strands. In numeric responses, use $9.80\ m/s^2$ for the acceleration due to gravity.



Part (a) **Write an expression for the tension in the vertical strand, T_v.**

The tension in a light string is, by definition, the magnitude of the force it exerts whatever it is connected to. The net force on the spider is zero, so the web must pull upward on the spider with a force equal in magnitude to the weight of the spider, so

$$\boxed{T_v = mg}$$

Part (b) **What is the numeric value, in newtons, of the tension in the vertical vertical strand, T_v?**

Substituting numbers,

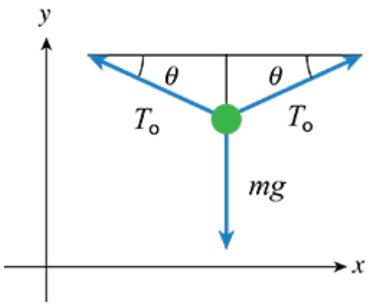
$$T_v = \left(2.01 \times 10^{-5}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)$$

Calculating,

$$\boxed{T_v = 1.970 \times 10^{-4\ \text{N}}}$$

Part (c) **Write an expression for the tension in a strand that attaches to the ceiling, T_o.**

The diagram is a free-body diagram of the connection point of the three strands, where y is vertically upward.



From $\Sigma F_y = 0$, we have $2T_o \sin\theta - mg = 0$, so

$$\boxed{T_o = \frac{mg}{2\sin\theta}}$$

Part (d) **What is the numeric value, in newtons, of the tension T_o?**

Substituting numbers,

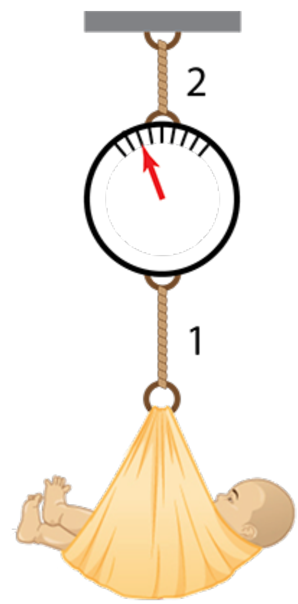
$$T_o = \frac{(2.01 \times 10^{-5}\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{2\sin(10.1°)}$$

Calculating,

$$\boxed{T_o = 5.616 \times 10^{-4}\ \text{N}}$$

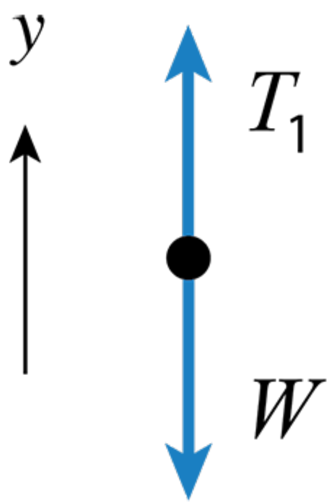
Problem 6:

A baby is being weighed in the the manner depicted in the figure. The upper end of rope 1 is connected to the bottom of the scale, and the baby and basket are attached to the lower end of the same rope. The scale, which has a mass ***0.5*** kg, hangs from rope 2, whose upper end is connected to the ceiling. The scale reads ***50.1*** N.



Part (a) **What is the tension, in newtons, in rope 1?**

The diagram is a free-body diagram of the baby, where the $+y$ direction is vertically upward.

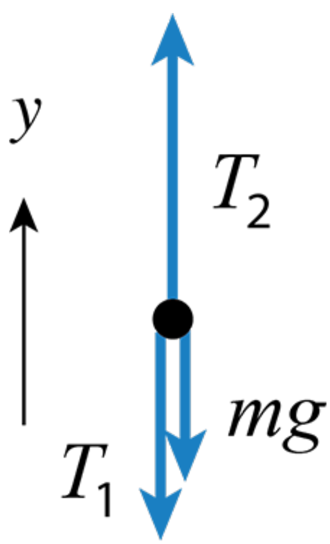


The tension, T, in a light string or rope is, by definition, the magnitude of the force it exerts whatever it is connected to. The net force on the baby is zero, so from $\Sigma F_y = 0$, we have $T_1 - W = 0$, so $T_1 = W$. Thus,

$$\boxed{T_1 = 50.10\,\mathrm{N}}$$

Part (b) **What is the tension, in newtons, in rope 2?**

The diagram is a free-body diagram of the scale, where the $+y$ direction is vertically upward.



The net force on the scale is zero, so from $\Sigma F_y = 0$, we have $T_2 - T_1 - mg = 0$, so

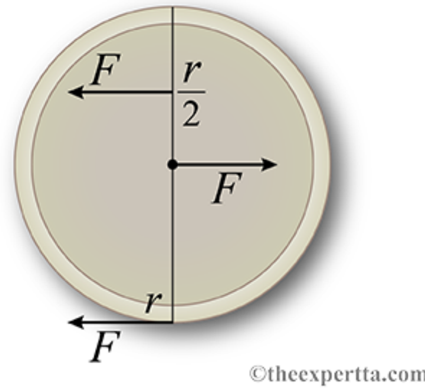
$$T_2 = mg + T_1 = (0.5\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right) + (50.1\,\mathrm{N})$$

Calculating,

$$\boxed{T_2 = 55.00\,\mathrm{N}}$$

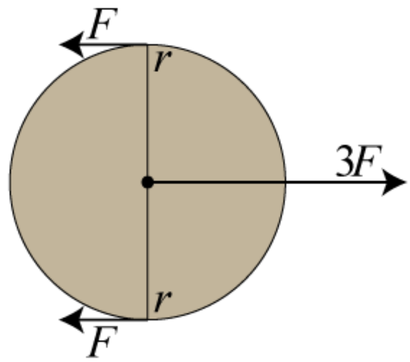
Problem 7:

A circular air hockey puck of radius r slides across a frictionless air hockey table and is subjected to several forces as shown below. The magnitude and direction of each force is given. Forces are applied at either the center of mass of the puck, the outer edge (a distance *r* from the center), or a distance halfway (*r*/2) between the center and the outer edge.



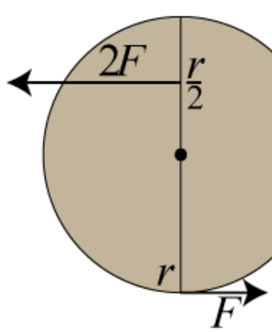
Part (a) **Select the examples below that have a net torque of zero about the axis perpendicular to the page and extending from the center of the puck.**

To answer this question, we need to calculate the net torque for each image, recalling that the torque is equal to the force multiplied by the radius. For our purposes, let's let counterclockwise torques be positive and clockwise torques be negative.



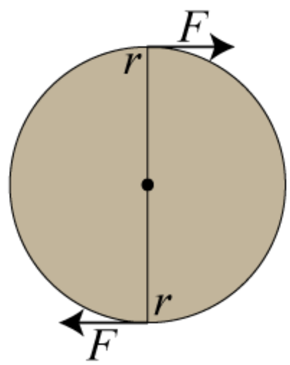
$$\tau_1 = Fr + 3F \cdot 0 - Fr$$

$$\tau_1 = 0$$



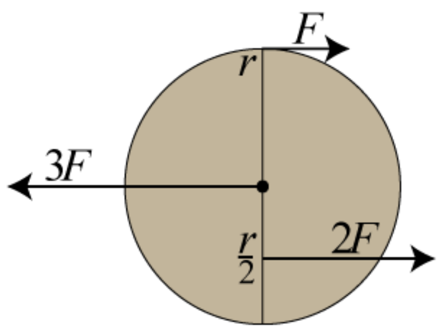
$$\tau_2 = 2F \cdot \frac{r}{2} + Fr$$

$$\tau_2 = 2Fr$$



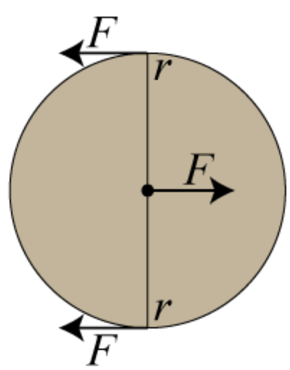
$$\tau_3 = -Fr - Fr$$

$$\tau_3 = -2Fr$$



$$\tau_4 = -Fr + 3F \cdot 0 + 2F \cdot \frac{r}{2}$$

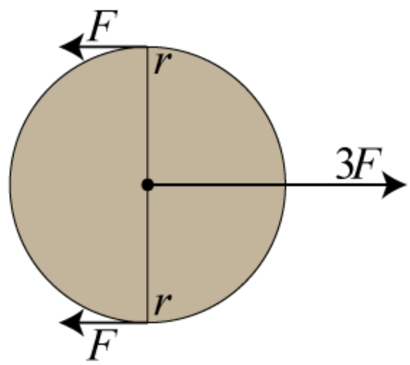
$$\tau_4 = 0$$

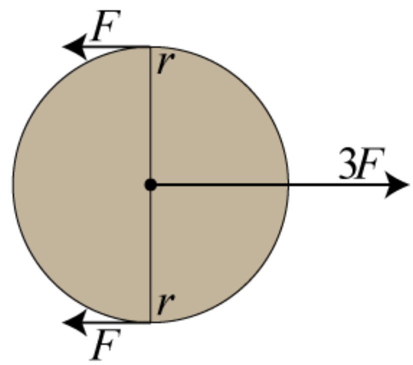


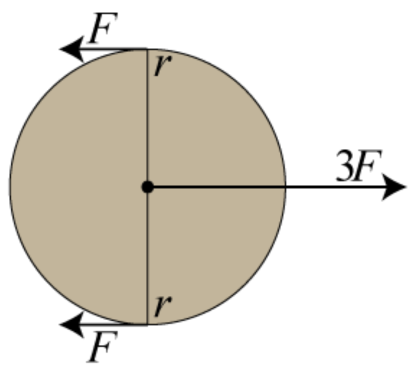
$$\tau_5 = Fr + F \cdot 0 - Fr$$

$$\tau_5 = 0$$

Based on these results, the correct images are:







Problem 8:

Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the surface of the door. One child pushes with a force of *15* N at a distance of *0.575* m from the hinges, and the second child pushes at a distance of *0.425* m.

Part (a) **What is the magnitude of the force, in newtons, the second child must exert to keep the door from moving? Assume friction is negligible.**

In order for the door to remain stationary, both the net torque and the net force on it must be zero. Since the door is attached to the wall by its hinges, the hinges will exert sufficient force to ensure the door does not experience motion that isn't rotational. Further, since the hinges are located at the axis of rotation, any force they exert on the door will not produce a torque. As a consequence, the only required condition for the door to remain stationary is that there is no net torque. Let's write an equation for the net torque and assign the torque generated by the second child the positive direction.

$$\tau_2 - \tau_1 = 0$$

Since both forces are being exerted perpendicular to the lever arm (which is the door in this case), the torque will be equal to the force multiplied by the distance from the hinges.

$$F_2 d_2 - F_1 d_1 = 0$$

$$F_2 d_2 = F_1 d_1$$

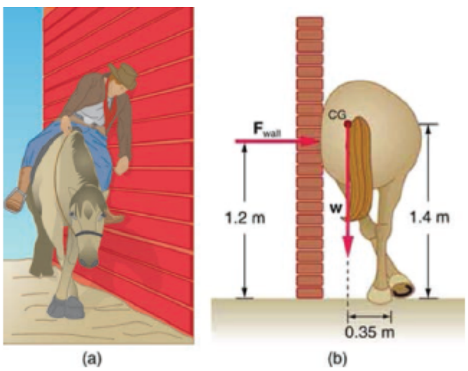
$$F_2 = \frac{F_1 d_1}{d_2}$$

$$F_2 = \frac{15 \text{ N } \cdot 0.575 \text{ m}}{0.425 \text{ m}}$$

$$\boxed{F_2 = 20.29 \text{ N}}$$

Problem 9:

Suppose a horse leans against a wall as shown in the figure. The total mass of the horse and rider is ***475*** kg.



Part (a) **Calculate the force, in newtons, exerted on the wall, assuming that force is horizontal and using the data in the schematic representation of the situation.**

To begin, let's note that the force exerted on the wall will be equal in magnitude to the force the wall exerts on the horse based on Newton's Third Law. We therefore need to solve for the force that the horse exerts on the wall, which we can find by noting that the net torque the horse experiences must be zero for it to be stationary. Let's write an equation for the torque about the point of contact between the horse's hooves and the ground, letting counterclockwise be the positive direction for rotation.

$$\tau_{\text{weight}} - \tau_{\text{wall}} = 0$$

$$\tau_{\text{weight}} = \tau_{\text{wall}}$$

To find the torques, we can multiply the force exerted by the wall by the vertical distance from the hooves and the force exerted by the horse's weight by the horizontal distance from the center of mass, as this will multiply the forces by the perpendicular component of the lever arm for both cases.

$$wd_{\text{center of mass}} = F_{\text{wall}}\, h_{\text{wall}}$$

$$mgd_{\text{center of mass}} = F_{\text{wall}}\, h_{\text{wall}}$$

$$mgd_{\text{center of mass}} = F_{\text{wall}}\, h_{\text{wall}}$$

$$\frac{mgd_{\text{center of mass}}}{h_{\text{wall}}} = F_{\text{wall}}$$

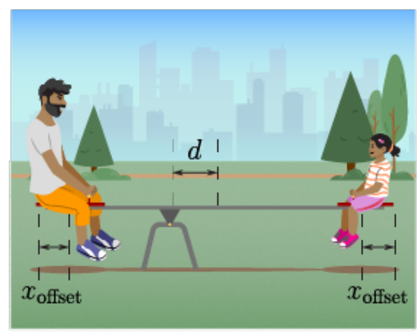
Now, let's plug in values and solve for the magnitude of the force exerted both by and on the wall.

$$F_{\text{wall}} = \frac{475 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 0.35 \text{ m}}{1.2 \text{ m}}$$

$$\boxed{F_{\text{wall}} = 1358 \text{ N}}$$

Problem 10:

You have been hired to design a family-friendly see-saw. Your design will feature a uniform board of mass $M = 5.01$ kg and length $L = 7.02$ m that can be moved so that the pivot is a distance d from the center of the board. This will allow riders to achieve static equilibrium even if they are of different mass, as most people are. You have decided that each rider will be positioned so that his/her center of mass will be a distance $x_{\text{offset}} = 5.03$ cm from the end of the board when seated, as shown.



You have selected a child of mass $m = 20.4$ kg and an adult of mass $n = 4$ times the mass of the child to test out your prototype.

Part (a) **Determine the distance d, in meters.**

For this problem, you want to find the pivot location such that the board (b) is in equilibrium. Since the adult (ad) is more massive than the child (ch), the pivot location has to be closer to the adult for the resulting torques around the pivot point to cancel, as shown in the picture. The general equation for torque is

$$\tau = rFsin\,(\theta) \text{ N m}$$

where r is the distance the force is being applied from the pivot point in meters, F is the force applied in Newtons, and θ is the angle the force and distance make with respect to one another in rad. The forces involved are the force exerted by gravity on the adult, child, and board. For equilibrium, the total torque about the pivot point must equal zero. Therefore,

$$\sum \tau = \tau_{ad} + \tau_{ch} + \tau_b = 0$$

For simplicity, let's set the zero point of the coordinate system at the pivot point. The adult will have a torque in the counterclockwise (positive) direction and the board and child in the clockwise (negative) direction. Expanding the torque equation

$$\sum \tau = r_{ad}F_{ad}sin\,(\theta_{ad}) - r_{ch}F_{ch}sin\,(\theta_{ch}) + r_bF_bsin\,(\theta_b) = 0$$

Since the board is uniform, the center of mass of the board is at its midpoint and its distance from the pivot is d. The adult is a distance $\frac{L}{2} - x_{offset} - d$ m from the pivot and the child is $\frac{L}{2} - x_{offset} + d$ m from the pivot. Since the force is from gravity, the angle that all three forces have with respect to the board is 90 degrees (the sine term is 1). Substituting into the equation and using g as the acceleration of gravity,

$$0 = \left(\frac{L}{2} - x_{offset} - d\right) nmg - \left(\frac{L}{2} - x_{offset} + d\right) mg - dMg$$

Dividing both sides by g and grouping the d terms together

$$0 = \left(\frac{L}{2} - x_{offset}\right) nm - \left(\frac{L}{2} - x_{offset}\right) m - d\left(M + m + nm\right)$$

Solving for d and simplifying

$$d\left(M + m + nm\right) = \left(\frac{L}{2} - x_{offset}\right) nm - \left(\frac{L}{2} - x_{offset}\right) m$$

$$d = \frac{\left(\left(\frac{L}{2} - x_{offset}\right) nm - \left(\frac{L}{2} - x_{offset}\right) m\right)}{\left(M + m + nm\right)}$$

Plugging in numbers

$$d = \frac{\left(\left(\frac{7.02 \text{ m}}{2} - \frac{(5.03 \text{ cm})}{100 \text{ cm/m}}\right) 4 \cdot 20.4 \text{ kg} - \left(\frac{7.02 \text{ m}}{2} - \frac{(5.03 \text{ cm})}{100 \text{ cm/m}}\right) 20.4 \text{ kg}\right)}{\left(5.01 \text{ kg} + 20.4 \text{ kg} + 4 \text{ kg} \cdot 20.4 \text{ kg}\right)}$$

$$\boxed{d = 1.979 \text{ m}}$$

Part (b) **Determine the magnitude, in newtons, of the force exerted on the pivot point by the see-saw.**

For this problem, the pivot point supports the total weight of the board, child, and adult since the see-saw is in equilibrium (not moving). Therefore the sum of all the forces equal zero. Choosing up to be positive,

$$\sum F = F_{pivot} - F_{ad} - F_{ch} - F_b = 0$$

solving for the force of the pivot,

$$F_{pivot} = F_{ad} + F_{ch} + F_b = nmg + mg + Mg$$

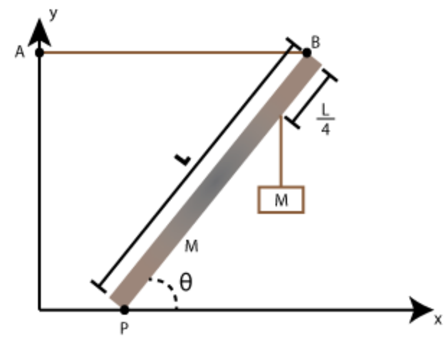
where g = 9.81 m/s^2. Factoring out g and plugging in numbers

$$F_{pivot} = \left(4 \cdot 20.4 \text{ kg} + 20.4 \text{ kg} + 5.01 \text{ kg}\right) \cdot 9.81 \text{m/s}^2$$

$$\boxed{F_b = 1050. \text{ N}}$$

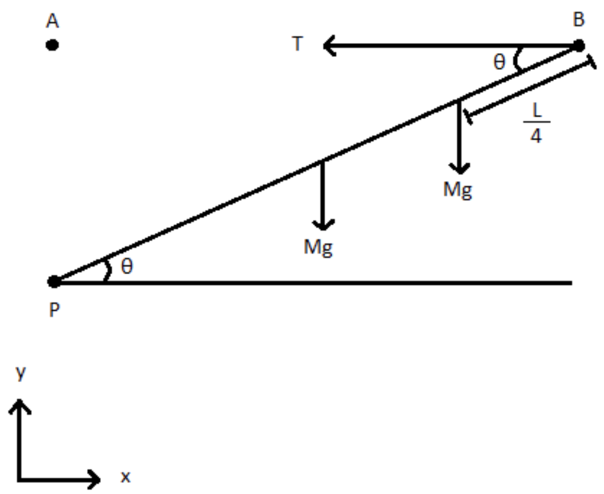
Problem 11:

A uniform beam of length L and mass M has its lower end pivoted at P on the floor, making an angle θ with the floor. A horizontal cable is attached at its upper end B to a point A on a wall. A box of mass M is suspended from a rope that is attached to the beam one-fourth L from its upper end.



Part (a) **Write an expression for the *y*-component P_y of the force exerted by the pivot on the beam.**

Let's begin by drawing a free-body diagram of the beam.



Referring to our free-body diagram, we can see that in order for the forces on the beam to be balanced, the pivot must be exerting enough force to balance the force exerted by gravity on both the beam and the box. We can therefore write the following equaiton for the force exerted by the pivot:

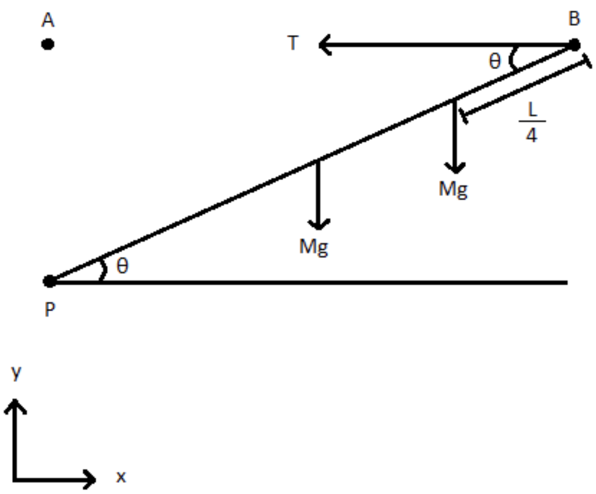
$$P_y - Mg - Mg = 0$$

$$P_y - 2Mg = 0$$

$$\boxed{P_y = 2Mg}$$

Part (b) **Write an expression for the tension *T* in the horizontal cable AB**

Let's begin by examining a free-body diagram.



Using translational kinematics, we lack sufficient information to calculate the tension in the horizontal cable. As a result, we must instead turn to rotational kinematics. We know that the beam should be stationary, meaning that the net torque must be zero. Allowing counterclockwise to be the positive direction for torque, we can write the following equation based on our free-body diagram:

$$\tau_{\text{cable}} - \tau_{\text{weight}} - \tau_{\text{box}} = 0$$

Now, let's use the equation relating the torque, force exerted, length of the lever arm, and the angle between the force and lever arm. Note that, since the sine function is equal to the cosine of the complementary angle, we can take the cosine of θ to find the sine of the angles between the gravitational forces and the lever arm.

$$TLsin\left(\theta\right) - Mg\frac{L}{2} \cdot cos\left(\theta\right) - Mg\left(L - \frac{L}{4}\right) \cdot cos\left(\theta\right) = 0$$

$$TLsin\left(\theta\right) - \left(Mg\frac{2L}{4} \cdot cos\left(\theta\right) + Mg\frac{3L}{4} \cdot cos\left(\theta\right)\right) = 0$$

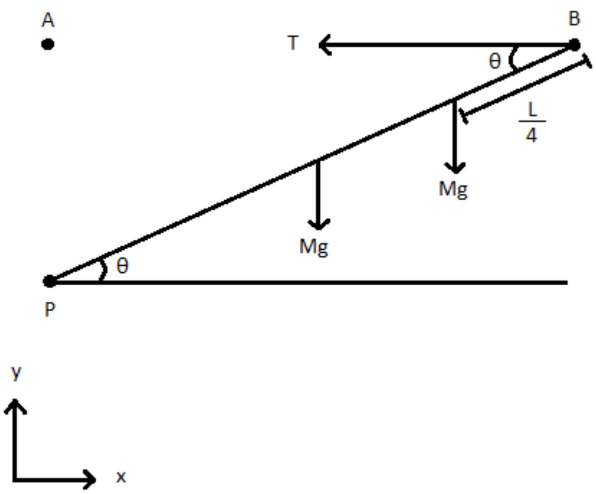
$$TLsin\left(\theta\right) - Mg\frac{5L}{4} \cdot cos\left(\theta\right) = 0$$

$$TLsin\left(\theta\right) = Mg\frac{5L}{4} \cdot cos\left(\theta\right)$$

$$\boxed{T = \frac{5}{4}Mg \cdot cotan\left(\theta\right)}$$

Part (c) **Write an expression for the x-component P_x of the force exerted by the pivot on the beam, in terms of T.**

Once again, let's start with a free-body diagram.



Looking at the free-body diagram, we see that there are only two forces acting horizontally. For the forces on the beam to be balanced, the force exerted by the tension in the cable and the force exerted by the pivot must be equal in magnitude and opposite in direction. The correct anwer is therefore:

$$\boxed{P_x = T}$$

Part (d) **What is the tension in the horizontal cable, in newtons, if the mass of the beam is *25* kg, the length of the beam is *6* m, and the angle is *21*°?**

To find the answer, we can plug the given values into the equation that we found in part (b) and solve.

$$T = \frac{5}{4} Mg \cdot cotan\,(\theta)$$

$$T = \frac{5}{4} \cdot 25\text{ kg } \cdot 9.81\text{ m/s}^2 \cdot cotan\,(21°)$$

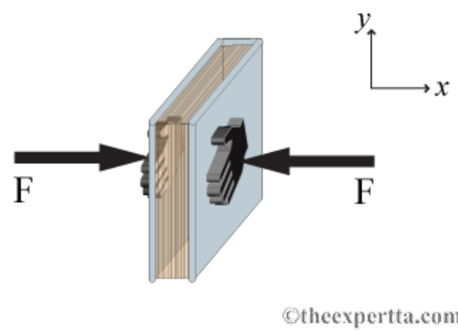
$$\boxed{T = 798.6\text{ N}}$$

Problem 12:

A woman holds a book by placing it between her hands such that she presses at right angles to the front and back covers. The book has a mass of m = *0.6* kg and the coefficient of static friction between her hand and the book is μ_s = *0.51*.

Randomized Variables

m = *0.6* kg
μ_s = *0.51*



Part (a) **What is the weight of the book, F_{gb} in Newtons?**

The weight of the book will be equal to the mass of the book multiplied by the acceleration due to gravity.

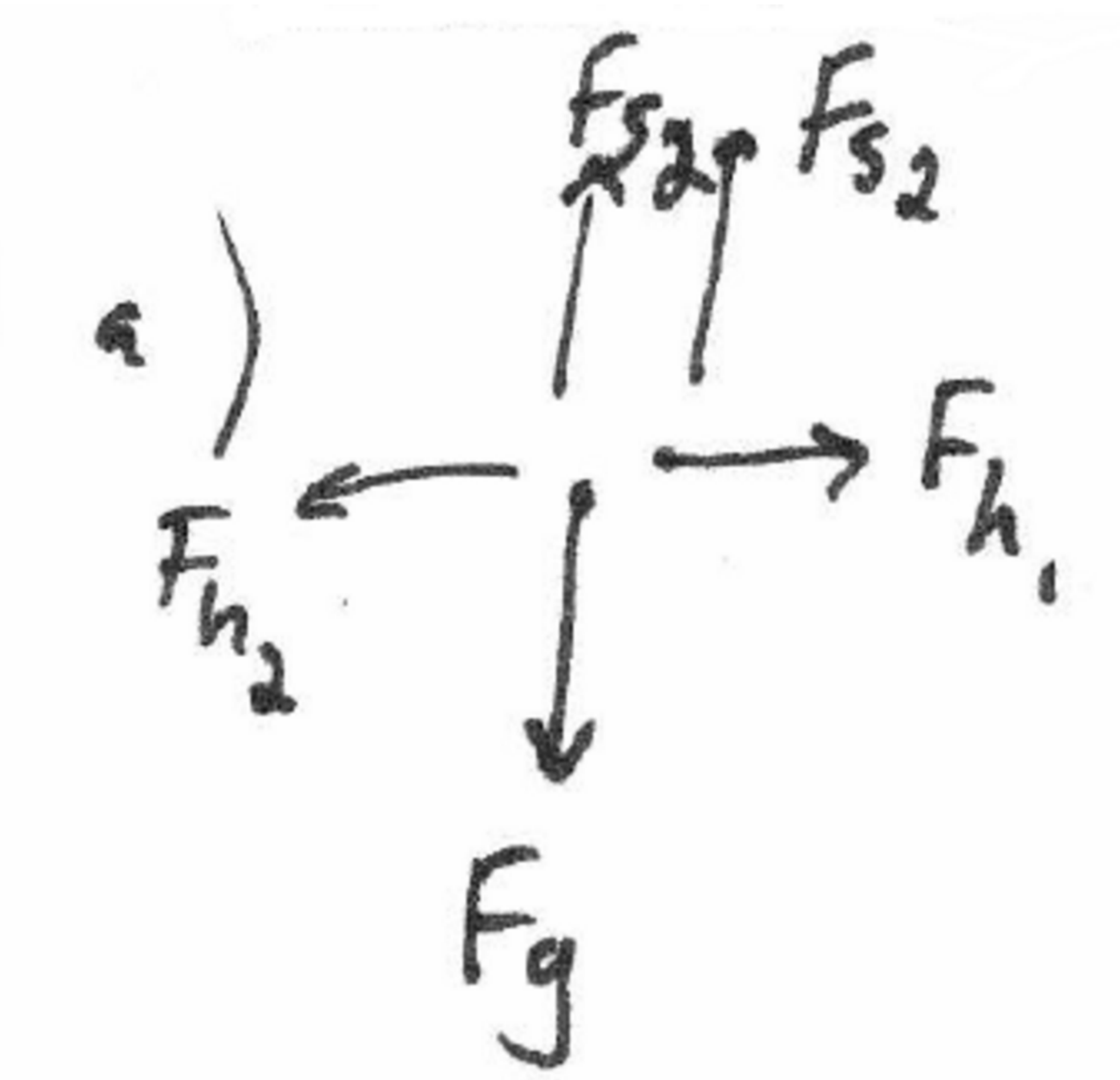
$$F_{gb} = mg$$

$$F_{gb} = 0.6\ \text{kg}\ \cdot 9.8\ \text{m/s}^2$$

$$\boxed{F_{gb} = 5.880\ \text{N}}$$

Part (b) **What is the minimum force she must apply with each of her hands, F_{min} in Newtons, to keep the book from falling?**

Let's start with a free-body diagram of the book.



In order for the book to remain in position, the maximum force of static friction must have at least the same magnitude as the weight of the book. The total force of static friction will be given by the coefficient of static friction multiplied by the total normal force. Assuming that the woman is exerting the minimum force possible on the book, the normal forces will each be equal to the minimum possible force. We can use this information to set up the following equation:

$$F_{min}\mu_s + F_{min}\mu_s = F_{gb}$$

$$2F_{min}\mu_s = mg$$

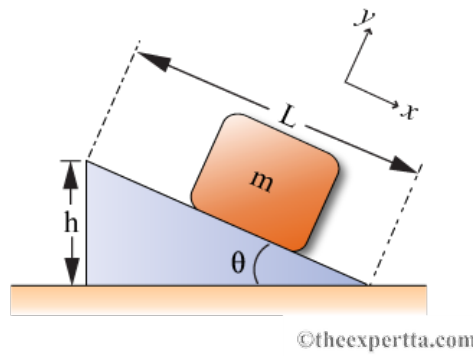
$$F_{min} = \frac{mg}{2\mu_s}$$

$$F_{min} = \frac{0.6\ \text{kg}\ \cdot 9.8\ \text{m/s}^{\ 2}}{2 \cdot 0.51}$$

$$\boxed{F_{min} = 5.765\ \text{N}}$$

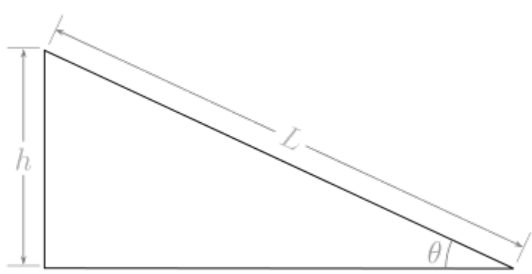
Problem 13:

A block of mass $m = 10.2$ kg rests on an inclined plane with a coefficient of static friction of $\mu_s = 0.07$ between the block and the plane. The inclined plane has length $L = 6.03$ m and height $h = 3.04$ m as indicated in the drawing.



Part (a) **What is the angle, in degrees, between the surface of the plane and the horizontal?**

The geometry of the inclined plane is recreated in the image below.



The side opposite angle θ has length h, and the hypotenuse has length L. These sides are related to the angle using the sine function.

$$\sin\theta = \frac{h}{L}$$

Taking the inverse sine of both sides,

$$\sin^{-1}\left(\sin\theta\right) = \sin^{-1}\left(\frac{h}{L}\right)$$

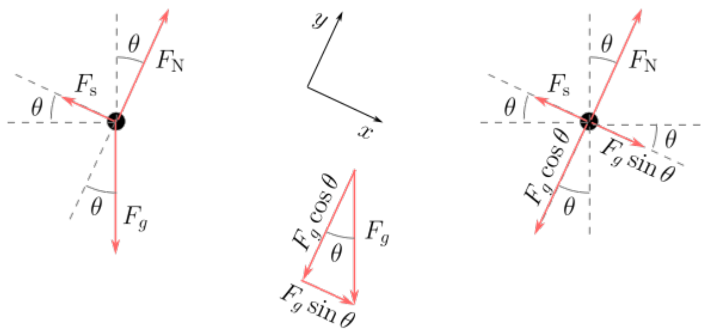
and

$$\theta = \sin^{-1}\left(\frac{h}{L}\right)$$
$$= \sin^{-1}\left(\frac{3.04 \text{ kg}}{6.03 \text{ m}}\right)$$
$$= \sin^{-1}(0.5041)$$

$\theta = 30.27°$

Part (b) **What is the magnitude, in newtons, of the normal force that acts on the block?**

Let's begin with a free-body diagram, as shown below on the left.



In the previous step we found that the angle has the value $\theta = 30.27°$. Focusing on the FBD on the left, notice that the weight vector, $\vec{F}_g$, is directed vertically downward, making an angle θ with the negative y direction. The normal force, $\vec{F}_\text{N}$, is perpendicular to the incline and aligned with positive y. The force of static friction is directed up the plane, resisting acceleration of the block, and is aligned with the negative x direction.

The diagram in the center shows the decomposition of the weight vector into its components. The y component, perpendicular to the incline, is adjacent to the angle θ, so the cosine function is indicated. The x component, parallel to the incline, is opposite the angle θ, so the sine function is indicated.

The diagram on the right recreates the FBD, but the weight vector is replaced by its components. We can easily form the sum of the components in the y direction. Noting that the normal force is directed toward positive y while the y component of the weight is directed towards negative y,

$$F_{\text{net},y} = \Sigma_i F_{y,i}$$
$$= F_\text{N} - mg\cos\theta$$

Since there is no acceleration perpendicular to the incline, the component of the net force in that direction must be zero.

$$F_{\text{net},y} = 0$$
$$F_\text{N} - mg\cos\theta = 0$$

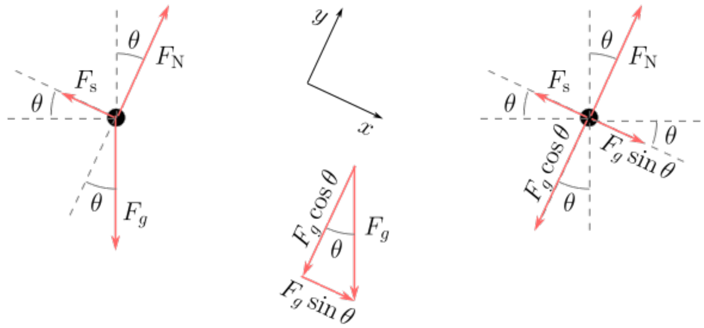
Solving for the magnitude of the normal force,

$$F_\text{N} = mg\cos\theta$$
$$= (10.2 \text{ kg})(30.27°9.81 \text{ m/s}^2)\cos(30.27°)$$

$F_\text{N} = 86.42 \text{ N}$

Part (c) **What is the x component, in newtons, of the force of gravity acting on the block?**

Let's recall free-body diagram, as presented in the previous step.



Recall we found that the angle has the value $\theta = 30.27^\circ$.

The diagram on the right recreates the FBD, but the weight vector is replaced by its components. Note that the x component is directed towards positive x, so a positive sign is indicated.

$$\begin{aligned} F_{g,x} &= F_g \sin\theta \\ &= mg \sin\theta \\ &= \left(10.2 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right) \sin\left(30.27^\circ\right) \end{aligned}$$

$$\boxed{F_{g,x} = 50.45 \text{ N}}$$

Part (d) **Will the block slide?**

When the force of static friction is at its maximum value, then it is proportional to the normal force, and the coefficient of static friction is the constant of proportionality. We have an expression for the normal force that we already used in the solution to Part (b).

$$\begin{aligned} F_{\text{s}}^{\text{max}} &= \mu_{\text{s}} F_{\text{N}} \\ &= \mu_{\text{s}} mg \cos\theta \end{aligned}$$

For small enough angles, the block will not slide. There is an angle where the force of friction is at its maximum value, and any greater angle would cause the block to slide. At that special angle, the maximum force of static friction has the same magnitude as the component of the weight of the block that is acting down the incline.

$$\begin{aligned} F_{\text{s}}^{\text{max}} &= F_{g,x} \\ \mu_{\text{s}} mg \cos\theta &= mg \sin\theta \end{aligned}$$

so

$$\mu_{\text{s}} = \tan\theta$$

and

$$\begin{aligned} \theta_{\text{max}} &= \tan^{-1}(0.07) \\ &= 4.004^\circ \end{aligned}$$

Compare the angle found in Part (a) to this maximum angle.

$$30.27^\circ > 4.004^\circ$$

Yes, the block will slide.

Problem 14:

A book is placed on a table, and the table is placed on the floor.

Part (a) **On what does the floor exert a normal force?**

A normal force is a contact force that acts in a direction that is perpendicular to the interface between two surfaces. Contrast this with friction, which is also a contact force, but it acts parallel to the surface.

- The floor and the table are in contact.

- The book does not contact the floor.

  The floor exerts a normal force on

the table only

  Note that the normal force of the floor on the table is directed upwards.

Part (b) **On what does the table exert a normal force?**

A normal force is a contact force that acts in a direction that is perpendicular to the interface between two surfaces. Contrast this with friction, which is also a contact force, but it acts parallel to the interface.

- The floor and the table are in contact.
- The book and the table are also in contact.

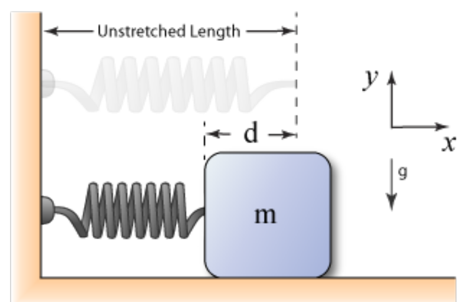
  The floor exerts a normal force on

both the floor and the book

  Note that the normal force of the table on the floor is directed downwards while the normal force of the table on the book is directed upwards.

Problem 15:

A spring with a spring constant of $k = 151$ N/m is initially compressed by a block a distance $d = 0.22$ m from its unstretched length. The block is on a horizontal surface with coefficients of static and kinetic friction μ_s and μ_k, respectively, and it has a mass of $m = 6.3$ kg. Refer to the figure.



Part (a) **The block is released from the initial position and begins to move to the right. Enter an expression for the sum of the forces in the x direction in the configuration shown above, in terms of defined quantities and the acceleration due to gravity, g.**

The forces in the x-direction include the restoring force of the spring (pointing right) and the kinetic friction force (pointing left). Therefore,

$$\sum F_x = (F_s - F_k)\ \text{N}$$

where F_s is the force from the spring in N and F_k is the kinetic friction force. Both of these force can be expressed by the equations

$$F_s = kd$$

$$F_k = \mu_k F_N$$

where k is the spring constant in N/m, d is the distance in m, μ_k is the coefficient of kinetic friction, and F_N is the normal force. In this case, the normal force is

$$F_N = mg$$

since it is on a horizontal surface (m is mass in kg and g is the acceleration of gravity in m/s^2). Substituting everything in,

$$\sum F_x = (k\ \mathrm{N/m}) \cdot (d\ \mathrm{m}) - \mu_k \cdot (m\ \mathrm{kg}) \cdot \left(g\ \mathrm{m/s^2}\right)$$

$$\boxed{\sum F_x = (kd - \mu_k mg)\ \mathrm{N}}$$

Part (b) **Calculate the smallest value for the coefficient of static friction, μ_s, that would keep the block from moving.**

To keep it from moving, the forces in the x-direction would be the restoring force of the spring (pointing right) and the static friction force (pointing left). Therefore,

$$\sum F_x = F_s - F_f = 0\ \mathrm{N}$$

where F_s is the force from the spring in N and F_f is the static friction force. Both of these force can be expressed by the equations

$$F_s = kd$$

$$F_f <= \mu_s F_N$$

where k is the spring constant in N/m, d is the distance in m, μ_s is the coefficient of static friction, and F_N is the normal force. In this case, the normal force is

$$F_N = mg$$

since it is on a horizontal surface (m is mass in kg and g is the acceleration of gravity in m/s^2). For the static friction to just keep the block from moving, the previous inequality turns into an equality. Substituting everything in,

$$\sum F_x = k \cdot d - \mu_s \cdot m \cdot g = 0$$

$$\mu_s = \frac{kd}{mg}$$

Plugging in numbers and converting units as needed,

$$\mu_s = \frac{(151\ \mathrm{N/m} \cdot 0.22\ \mathrm{m})}{\left(6.3\ \mathrm{kg} \cdot 9.81\ \mathrm{m/s^2}\right)}$$

$$\boxed{\mu_s = 0.5375}$$

Part (c) **Assuming the block has just begun to move and the coefficient of kinetic friction is $\mu_s = 0.2$, what is the block's acceleration in meters per squared second?**

From part a,

$$\sum F_x = k \cdot d - \mu_k \cdot m \cdot g = ma$$

$$a = k \cdot \frac{d}{m} - \mu_k \cdot g$$

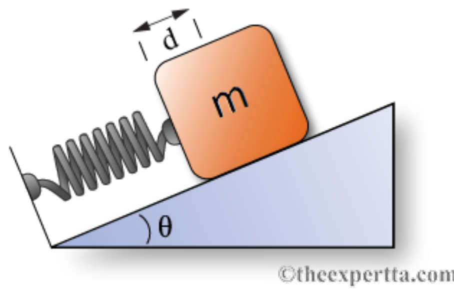
Plugging in numbers and converting units as needed,

$$a = \frac{(151\ \text{N/m} \cdot 0.22\ \text{m})}{(6.3\ \text{kg})} - 0.2 \cdot 9.81\ \text{m/s}^2$$

$$\boxed{a = 3.311\ \text{m/s}^2}$$

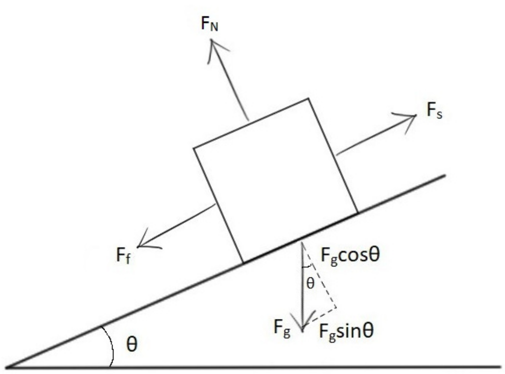
Problem 16:

A block of mass $m = 104$ kg rests against a spring with a spring constant of $k = 505$ N/m on an inclined plane which makes an angle of θ with the horizontal. Assume the spring has been compressed a distance d from its neutral position. Refer to the figure.



Part (a) **Set your coordinates to have the x axis along the surface of the plane, with up the plane as positive, and the y axis normal to the plane, with out of the plane as positive. Enter an expression for the normal force, F_N, that the plane exerts on the block in terms of quantities defined in the problem statement, and use g for the acceleration due to gravity.**

First, let's look at a free-body diagram of the problem.



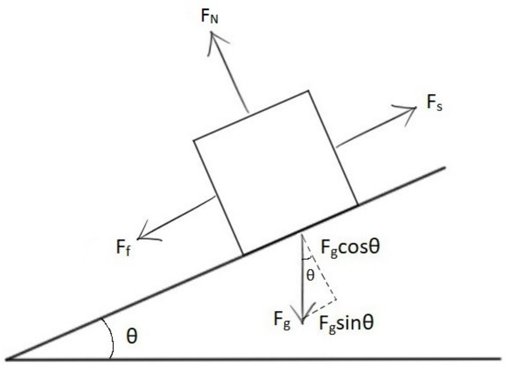
As you can see, the force of gravity can be decomposed using trigonometry into a pair of forces, one in the x-direction and one in the y-direction of the coordinate axis we have chosen. Now, let's consider the physics for a moment. The gravitational force in the y-direction is trying to pull the block through the ramp. In order for the block not to sink into the ramp, the ramp must exert a normal force that counteracts the force of gravity on the block in the y-direction. Therefore, we know that the normal force is equal to the y-component of the gravitational force.

$$F_N = F_g \cos(\theta)$$

$$\boxed{F_N = mg\cos(\theta)}$$

Part (b) **Denoting the coefficient of static friction by μ_s, write an expression for the sum of the forces in the x direction just before the block begins to slide up the inclined plane in terms of quantities defined in the problem statement, and use g for the acceleration due to gravity.**

Let's look at a free-body diagram of the box.



As we see, the force of gravity can be decomposed into two forces that align with the coordinate plane we have defined for this problem. We will need to include to portion of gravity acting in the x-direction as one of the forces acting on the block in the x-direction. We can also see that the forces of friction and gravity are acting on the block in the x-direction. Recalling that up the plane is positive and down the plane is negative according to our coordinate axis, we can sum the forces in the x-direction as

$$\Sigma F_x = F_s - F_f - F_g \sin(\theta)$$

Now, we will want to simplify these forces. We know that gravitational force is mass times the gravitational constant, the force of a spring is the spring constant times distance, and that the maximum force of static friction is the gravitational constant multiplied by the normal force (which we found in the previous part of this problem). Since we are looking at the block just before it moves, static friction will be acting with its maximum possible force. With all this in mind, let's now write the sum of forces in terms of our permitted variables.

$$\boxed{\Sigma F_x = kd - \mu_s mg \cos(\theta) - mg \sin(\theta)}$$

Part (c) **Assuming the plane is frictionless, what will be the angle, in degrees, of the plane if the spring is compressed by gravity a distance** $0.1\ \mathrm{m}$**?**

In part (b), we found an equation for the forces on the block just before it begins to move. By plugging values into that equation and solving for the angle when there is zero net force, we can find the angle just before the block begins to move. Recall, however, that we are assuming that the plane is frictionless for this part of the problem, meaning that the force of friction will be zero in our equation.

$$\Sigma F_x = kd - \mu_s mgcos\left(\theta\right) - mgsin\left(\theta\right)$$

$$0 = kd - 0 - mgsin\left(\theta\right)$$

$$mgsin\left(\theta\right) = kd$$

$$sin\left(\theta\right) = \frac{kd}{mg}$$

$$\theta = arcsin\left(\frac{kd}{mg}\right)$$

$$\theta = arcsin\left(\frac{505\ \mathrm{N/m}\ \cdot 0.1\ \mathrm{m}}{104\ \mathrm{kg}\ \cdot 9.81\mathrm{m/s}^2}\right)$$

$$\boxed{\theta = 2.839^\circ}$$

Part (d) **Assuming $\theta = 45^\circ$ and the surface is frictionless, how far, in meters, will the spring be compressed?**

For this problem, we will want to use the same strategy we did in part (c), except this time instead of plugging in a value for distance and assuming zero net force, we will be plugging in an angle and assuming zero net force. As in part (c), we will once again begin with the equation for forces in the x-direction that we found in part (b) and are also assuming once again that the plane is frictionless.

$$\Sigma F_x = kd - \mu_s mg\cos(\theta) - mg\sin(\theta)$$

$$0 = kd - 0 - mg\sin(\theta)$$

$$-kd = -mg\sin(\theta)$$

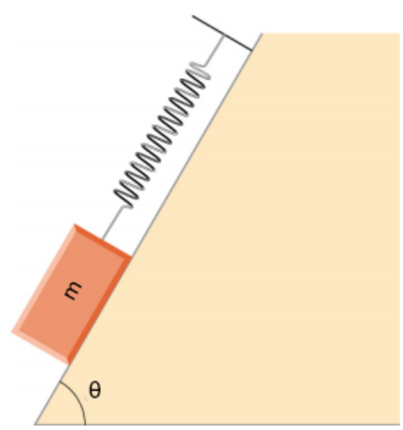
$$d = \frac{mg\sin(\theta)}{k}$$

$$d = \frac{104\text{ kg} \cdot 9.81\text{ m/s}^2 \cdot \sin(45^\circ)}{505\text{ N/m}}$$

$$\boxed{d = 1.429\text{ m}}$$

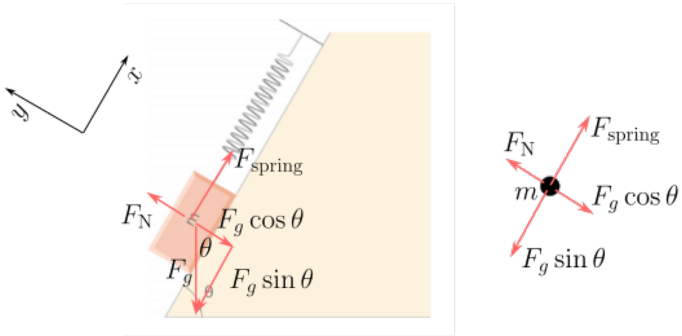
Problem 17:

Shown to the right is a block of mass $m = 20.1$ kg resting on a frictionless ramp inclined at $\theta = 50.1^\circ$ to the horizontal. The block is held by a spring that is stretched by a distance $d = 4.04$ cm after the block is attached to it.



Part (a) **Write an equation for the force constant of the spring in terms of the variables from the problem statement, and use g for the acceleration due to gravity.**

The weight of the block acts vertically downward. The spring force acts up the incline and parallel to its surface. A normal force acts on the block in the direction perpendicular to the incline as shown in the image on the left below.



Notice the coordinate system that has been added toward the upper left. The weight vector, F_g, is decomposed into its components, and an FBD is created to the right. The net force in the x direction may be computed as

$$\begin{aligned} F_{\text{net},x} &= \sum_i F_{i,x} \\ &= F_{\text{spring}} - F_g \sin\theta \\ &= kd - mg\sin\theta \end{aligned}$$

In equilibrium, each component of the force is zero. Applied to the x component,

$$kd - mg\sin\theta = 0$$

so

$$kd = mg\sin\theta$$

Dividing by sides by the distance,

$$\boxed{k = \frac{mg}{d}\sin\theta}$$

Part (b) **Calculate the force constant of the spring in newtons per meter.**

This step is a numeric evaluation of the expression from Part (a).

$$\begin{aligned} k &= \frac{mg}{d}\sin\theta \\ &= \frac{(20.1\ \text{kg})(9.81\ \text{m/s}^2)}{4.04\times10^{-2}\ \text{m}}\sin(50.1^\circ) \end{aligned}$$

$$\boxed{k = 3744\ \text{N/m}}$$

Problem 18:

A rod is laid out along the x-axis with one end at the origin and the other end at $x = L$. The linear density is given by the following:

$\rho(x) = \rho_0+(\rho_1-\rho_0)(x/L)^2$,

where ρ_0 and ρ_1 are constant values.

Part (a) **For L = *0.15* m, ρ_0 = *1.1* kg/m, and ρ_1 = *5.1* kg/m, determine the center of mass of the rod, in meters.**

For the present case of a linear mass density $rho(x)$ extending from $x = 0$ to $x = L$, we use integration to find the center of mass X_{cm} by

$$X_{\text{cm}} = \frac{\int x\rho(x)dx}{\int \rho(x)dx},$$

where the limits of both integrals are from $x = 0$ to $x = L$.

We perform the integration in the numerator and obtain

$$\int x\rho(x)\,dx = \frac{L^2}{4}(\rho_1+\rho_0)\ .$$

The integration in the denominator gives us

$$\int \rho(x)\,dx = \frac{L}{3}(\rho_1+2\rho_0)\ .$$

We substitute the latter two expressions into the expression for X_{cm} and obtain

$$X_{\text{cm}} = \frac{3L}{4}\frac{\rho_1+\rho_0}{\rho_1+2\rho_0}\ .$$

We substitute the given values,

$$X_{\text{cm}} = \frac{3(0.15\ \text{m})}{4}\cdot\frac{(5.1\ \text{kg/m})+(1.1\ \text{kg/m})}{(5.1\ \text{kg/m})+2(1.1\ \text{kg/m})}\ .$$

$$\boxed{X_{\text{cm}} = 0.09555\ \text{m}}$$

Problem 19:

A certain carbon monoxide molecule consists of a carbon atom of mass m_c = *12* u and an oxygen atom of mass m_O = *15* u that are separated by a distance of d = *105* pm, where "u" is an atomic unit of mass.

Part (a) **Write a symbolic equation for the location of the center of mass of the carbon monoxide molecule relative to the position of the oxygen atom. This expression should be in terms of the masses of the atoms and the distance between them.**

Since we are solving for the position of the carbon atom relative to the oxygen atom, we will want to set the oxygen atom at the origin. With this in mind, we get the following result when we solve for the center of mass:

$$x_{cm} = \frac{m_o \cdot 0 + m_c d}{m_o + m_c}$$

$$\boxed{x_{cm} = \frac{m_c d}{m_o + m_c}}$$

Part (b) **Calculate the numeric value for the center of mass of carbon monoxide in units of pm.**

To get the answer, we simply need to solve the equation we found in part (a). Since we are looking for an answer in picometers and are given a distance in picometers, we actually do not have to perform any conversions to get the correct answer.

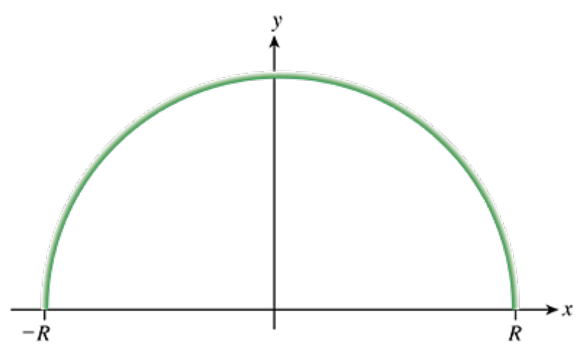
$$x_{cm} = \frac{m_c d}{m_o + m_c}$$

$$x_{cm} = \frac{12\text{ u } \cdot 105\text{ um}}{15\text{ u } + 12\text{ u}}$$

$$\boxed{x_{cm} = 46.67\text{ pm}}$$

Problem 20:

A thin wire is bent into a semicircle of radius *R*. As shown in the diagram, its center is at the origin and the two ends of the wire are on the *x* axis at *x* = R and *x* = -*R*.



Part (a) **Write an expression for the *y* coordinate of the center of mass of the wire.**

We learn from our textbook that an expression for the y-coordinate of the center of mass of a system, y_{cm}, is

$$y_{\text{cm}} = \frac{\Sigma m_i y_i}{\Sigma m_i},$$

where m_i denotes the mass of the i-th component of the system and y_i is the y-coordinate of that component's center of mass.

For a solid object, such as the wire of this problem, we replace the summations with appropriate integral expressions.

We imagine a radius extending from the origin to a point on the semicircle and consider an infinitesimal arc of the semicircle at that point. The length of that arc is $Rd\theta$ and the arc's mass is $\rho R d\theta$, where θ denotes the angle counterclockwise from the positive x-axis to the radius and ρ denotes the uniform linear density of the wire.

The mass of the whole length of the wire equals the integral expression $\rho R \int d\theta$ evaluated from $\theta = 0$ to $\theta = \pi$, which serves as the denominator of the expression for y_{cm} instead of Σm_i.

The y-coordinate of the infinitesimal arc's location equals $R\sin(\theta)$. Thus the integral expression $\rho R \int y d\theta = \rho R^2 \int \sin(\theta) d\theta$ evaluated from $\theta = 0$ to $\theta = \pi$ serves as the numerator of the expression for y_{cm} instead of $\Sigma m_i y_i$.

As a result, the expression for y_{cm}, takes the form

$$y_{\text{cm}} = \frac{R \int \sin(\theta) d\theta}{\int d\theta}$$

with the integrals evaluated from $\theta = 0$ to $\theta = \pi$. We evaluate the integrals and obtain the result ...

$$\boxed{y_{\text{cm}} = \frac{2R}{\pi}}$$