Begin Date: 10/11/2024 12:01:00 AM -- **Due Date:** 10/18/2024 11:59:00 PM **End Date:** 10/18/2024 11:59:00 PM

Problem 1:

A car drives around a horizontal, circular track at constant speed. Consider the following three forces that act on the car: (1) The upward normal force exerted on the car by the road, (2) the downward gravitational force on the car, (3) and the frictional force that is directed toward the center of the circular path.

Part (a) **Which of these forces does zero work on the car as the car moves along the circular path?**

The work done by a constant force is given by $W = Fd\cos\theta$, where F is the magnitude of the force, d is the magnitude of the displacement, and θ is the angle between the force and the displacement. The work is zero when $\cos\theta = 0$ (which corresponds to $\theta = 90^\circ$) or when there is no displacement. The car's velocity is perpendicular to all three of these forces, so "1, 2, and 3" is the correct answer.

Problem 2:

A car is being lifted by a crane so that it moves upwards at a constant speed.

Part (a) **Which force is doing *negative* work on the car as it is being lifted?**

The car is being lifted at a constant speed upward. There are two forces acting in the vertical direction on the car, the downward force of gravity and the upward force of the tension in the crane cable. Work is positive when the force and displacement are in the same direction.

The expression for the work is

$$W = F\cos\theta$$

Since the displacement is upward and the force of gravity is directed downward, $\theta = 180^\circ$. The cosine of 180° is equal to -1. So, the work done by the force of gravity is

$$W = mg\cos 180^\circ = -mg$$

$\boxed{\text{The car's gravitational interaction with the Earth.}}$

Part (b) **Which force is doing *positive* work on the car as it is being lifted?**

The car is being lifted at a constant speed upward. There are two forces acting in the vertical direction on the car, the downward force of gravity and the upward force of the tension in the crane rope. Work is positive when the force and displacement are in the same direction.

The expression for the work is

$$W = F\cos\theta$$

Since the displacement is upward and the tension in the rope is directed upward, $\theta = 0^\circ$. The cosine of 0° is equal to $+1$. So, the work done by the tension is

$$W = mg\cos 0^\circ = +mg$$

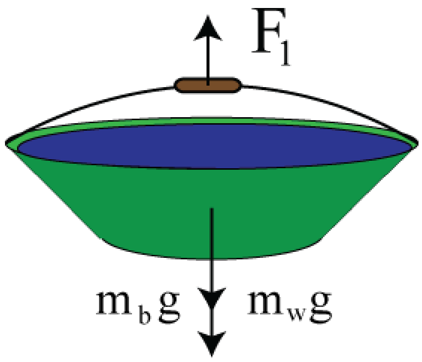
$\boxed{\text{The car's contact interaction with the rope.}}$

Problem 3:

A farmer is using a rope and pulley to lift a bucket of water from the bottom of a well that is h_y = ***8.5*** m deep. The farmer uses a force F_1 = ***48*** N to pull the bucket of water directly upwards. The total mass of the bucket of water is $m_b + m_w$ = ***3.1*** kg.

Part (a) **Select the correct free body diagram.**

The correct free-body diagram will have F_1 pulling the bucket upward and the forces of gravity on the water and the bucket pulling it downward.



Part (b) **Calculate how much work W_f in J the farmer does on the bucket of water (via the rope) to raise it to ground level.**

Work is given by the force exerted multiplied by the displacement in the direction of the force. Since the bucket is moving in the same direction as F_1, we can write the following equation for the work done by the farmer:

$$W_f = F_1 \cdot h_y$$

$$W_f = 48 \text{ N } \cdot 8.5 \text{ m}$$

$$\boxed{W_f = 408.0 \text{ J}}$$

Part (c) **Calculate how much work W_g in J gravity does on the bucket filled with water as the farmer lifts it up the well.**

For the force of gravity, the direction of the displacement is opposite the direction in which the force is exerted. That means that gravity will be doing a negative amount of work on the bucket.

$$W_g = (m_b g + m_w g) \cdot -h_y$$

$$W_g = g\,(m_b + m_w) \cdot -h_y$$

$$W_g = 9.81 \text{ m/s}^2 \cdot 3.1 \text{ kg } \cdot -8.5 \text{ m}$$

$$\boxed{W_g = -258.5 \text{ J}}$$

Part (d) **Calculate the net work W_{net} in J done on the bucket of water by the two forces F_1 and F_g.**

To find the net work, we simply need to add the work done by the farmer that we found in part (b) to the work done by gravity that we found in part (c).

$$F_{net} = W_f + W_g$$

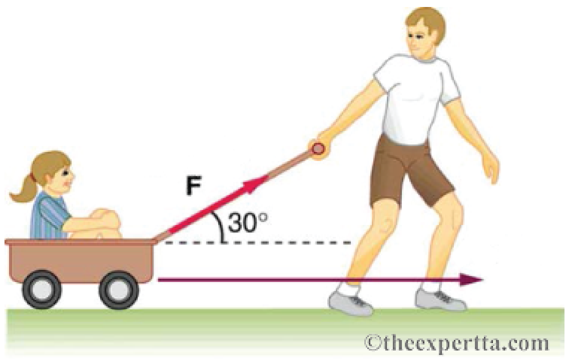
$$F_{net} = F_1 \cdot h_y + g\,(m_b + m_w) \cdot -h_y$$

$$F_{net} = 48 \text{ N } \cdot 8.5 \text{ m } + 9.81 \text{ m/s}^{2} \cdot 3.1 \text{ kg } \cdot -8.5 \text{ m}$$

$$\boxed{F_{net} = 149.5 \text{ J}}$$

Problem 4:

A boy is pulling his sister in a wagon, as shown in the figure. He exerts a force of *F* = *40.5* N at an angle of 30°.



Part (a) **How much work does the boy do pulling his sister, in joules, if he pulls her *27.5* m?**

Since the direction of motion is not the same as the direction of the force, we will need to take this into account by including a cosine term when we calculate the work so that we are only multiplying the parallel component of the force by the displacement.

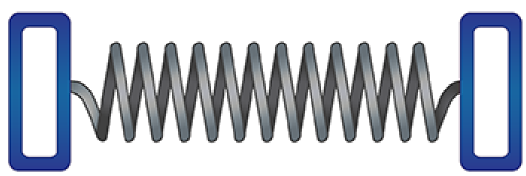
$$W = Fdcos\,(\theta)$$

$$W = 40.5 \text{ N } \cdot 27.5 \text{ m } \cdot cos\,(30^{\circ})$$

$$\boxed{W = 964.5 \text{ J}}$$

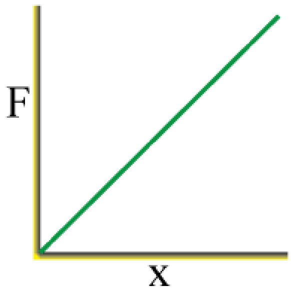
Problem 5:

As a part of an exercise routine to strengthen their pectoral muscles, a person stretches a spring which has a spring constant $k = 405$ N/m.



Part (a) **If $x = 0$ corresponds to when the springs are at their relaxed state, which image best represents the magnitude of the force applied to the springs as a function of the stretch distance.**

The spring constant tells us what force must be exerted to stretch the spring one meter. This is a linear function with twice the force stretching the spring twice as far, three times the force stretching it three times as far, etc. Therefore, the correct schematic will be the one which shows the distance and force increarcsing linearly.



Part (b) **Write an equation for the work necessary to stretch the spring from the relaxed state to a distance x_1.**

We can find the work by calculating the area under the curve of this graph. To find the area under the curve, note that since the spring constant tells us how many newtons of force are exerted for each meter the spring is stretched, it gives us the slope of the graph. We can use this information to write the force in terms of the distance the spring is stretched (as we have no option to use force as a variable for our answer.)

$$F = kx$$

Now, to find the area under the curve, notice that the graph forms a triangle with height equal to the final force and a base with a length equal to the final position of the spring. We can therefore use the formula for the area of triangle to get the following equation for the work:

$$W_1 = \frac{1}{2} F x_1$$

To get this answer in terms of the correct variables, we can plug in the equation we found for force previously.

$$W_1 = \frac{1}{2}(kx_1)\, x_1$$

$$\boxed{W_1 = \frac{kx_1^2}{2}}$$

Part (c) **Calculate the work, in joules, required to stretch the spring from its relaxed state to the position $x_1 = 45.2$ cm.**

Here, we simply need to plug the given values into the equation we found in part (b), making sure to convert the distance from centimeters to meters.

$$W_1 = \frac{kx_1^2}{2}$$

$$W_1 = \frac{405\ \mathrm{N/m}\ \cdot \left(45.2 \cdot 10^{-2}\ \mathrm{m}\right)^2}{2}$$

$$\boxed{W_1 = 41.37\ \mathrm{J}}$$

Part (d) **Write an equation for the work necessary to stretch the spring from the position x_1 to position x_2.**

To find the work necessary to stretch the spring from x_1 to x_2, we can find the work needed to stretch the spring from 0 to x_2 and then subtract out the work needed to stretch the spring from 0 to x_1, as this work will have already been done if the spring is at x_1 as a starting position. We can use the equation that we found in part (b) to find expressions for the work required to move the spring from 0 to x_1 and 0 to x_2

$$W_1 = \frac{kx_1^2}{2}$$

$$W_2 = \frac{kx_2^2}{2}$$

Now, let's employ the fact that the work to go from x_1 to x_2 will be equal to the work required to go from 0 to x_2 minus the work required to go from 0 to x_1 to write the desired equation for the work done stretching the spring from x_1 to x_2.

$$W_{1\to2} = W_2 - W_1$$

$$\boxed{W_{1\to2} = \frac{kx_2^2}{2} - \frac{kx_1^2}{2}}$$

Part (e) **Calculate the work, in joules, required to stretch the spring from $x_1 = 45.2$ cm to $x_2 = 70.3$ cm.**

Here, we simply need to plug in values to our results from part (d) in order to find the answer. In so doing, we will need to be careful to convert the distances from centimeters to meters.

$$W_{1\to2} = \frac{kx_2^2}{2} - \frac{kx_1^2}{2}$$

$$W_{1\to2} = \frac{405\ \mathrm{N/m}\ \cdot \left(70.3 \cdot 10^{-2}\ \mathrm{m}\right)^2}{2} - \frac{405\ \mathrm{N/m}\ \cdot \left(45.2 \cdot 10^{-2}\ \mathrm{m}\right)^2}{2}$$

$$\boxed{W_{1\to2} = 58.71\ \mathrm{J}}$$

Problem 6:

Consider an object with mass $m = 2.04$ kg on a frictionless table. The object moves along the positive x axis subject to a force which "repels" the object from the origin,

$$F = \frac{a}{x^2} + \frac{b}{x} \quad \text{for} \quad x > 0$$

where x is the position of the object relative to the origin.

Part (a) **Write an expression for the work done by the repulsive force on the object as it moves from an initial position x_1 to a final position x_2.**

When the object moves by an infinitesimal distance $\mathrm{d}x$, the work done by the force F is

$$\mathrm{d}W = F\,\mathrm{d}x$$

Because the force changes with position, we must sum the infinitesimal contributions to the work by performing an integral

$$
\begin{aligned}
W &= \int_{x_1}^{x_2} \mathrm{d}W \\
&= \int_{x_1}^{x_2} F\,\mathrm{d}x \\
&= \int_{x_1}^{x_2} \left(\frac{a}{x^2} + \frac{b}{x}\right) \mathrm{d}x
\end{aligned}
$$

Noting that the integral of a sum is the sum of the integrals,

$$
\begin{aligned}
W &= \int_{x_1}^{x_2} \frac{a}{x^2} \mathrm{d}x + \int_{x_1}^{x_2} \frac{b}{x} \mathrm{d}x \\
&= a \int_{x_1}^{x_2} x^{-2}\,\mathrm{d}x + b \int_{x_1}^{x_2} \frac{\mathrm{d}x}{x}
\end{aligned}
$$

where the constant multiplying the integrand has been factored out so that it multiplies the integral. The first integral may be evaluated using the formula

$$
\int x^n\,\mathrm{d}x = \frac{1}{n+1} x^{n+1}
$$

and the second integral evaluates to a logarithm.

$$
\begin{aligned}
W &= a \frac{x^{-2+1}}{-2+1}\bigg|_{x_1}^{x_2} + b \ln(x)\bigg|_{x_1}^{x_2} \\
&= -a \frac{1}{x}\bigg|_{x_1}^{x_2} + b\Big[\ln\left(x_1\right) - \ln\left(x_2\right)\Big] \\
&= -a\left(\frac{1}{x_2} - \frac{1}{x_1}\right) + b \ln\!\left(\frac{x_2}{x_1}\right)
\end{aligned}
$$

$$
\boxed{W = a\left(\frac{1}{x_1} - \frac{1}{x_2}\right) + b \ln\!\left(\frac{x_2}{x_1}\right)}
$$

It is also reasonable to combine the terms multiplies by a with a common denominator.

$$
\boxed{W = a\left(\frac{x_2 - x_1}{x_1 x_2}\right) + b \ln\!\left(\frac{x_2}{x_1}\right)}
$$

Part (b) **If the object starts at a position of $x_1 = 5.02$ m away from the origin, how much work, in joules, is required by an external force to bring it to a position of $x_2 = 1.01$ m away when $a = 15.2\ \mathrm{N \cdot m^2}$ and $b = 10.3\ \mathrm{N \cdot m}$?**

Modify the expression obtained in the previous step by adding the numeric coefficients.

$$
W = \left(15.2\ \mathrm{N \cdot m^2}\right)\left(\frac{x_2 - x_1}{x_1 x_2}\right) + \left(10.3\ \mathrm{N \cdot m}\right) \ln\!\left(\frac{x_2}{x_1}\right)
$$

Evaluate this expression with the initial and final positions to find the work done by the force F.

$$
\begin{aligned}
W &= \left(15.2\ \mathrm{N \cdot m^2}\right)\left(\frac{1.01\ \mathrm{m} - 1.01\ \mathrm{m}}{\left(5.02\ \mathrm{m}\right)\left(1.01\ \mathrm{m}\right)}\right) + \left(10.3\ \mathrm{N \cdot m}\right) \ln\!\left(\frac{1.01\ \mathrm{m}}{5.02\ \mathrm{m}}\right) \\
&= -28.54\ \mathrm{N \cdot m} \\
&= -28.54\ \mathrm{J}
\end{aligned}
$$

In order to move the object towards the origin, and external force must perform an amount of work that is opposite to this.

$$
W_{\mathrm{ext}} = -W
$$

$$
\boxed{W_{\mathrm{ext}} = 28.54\ \mathrm{J}}
$$

Part (c) **If the object starts at rest at a position $x_3 = 0.49$ m and is released, at what speed v, in meters per second, will the object be moving when it is at position $x_4 = 10.1$ m?**

odify the expression obtained in the previous step by adding the numeric coefficients.

$$W = (15.2\ \mathrm{N \cdot m^2})\left(\frac{x_4 - x_3}{x_3 x_4}\right) + (10.3\ \mathrm{N \cdot m})\ln\left(\frac{x_4}{x_3}\right)$$

Evaluate this expression with the initial and final positions to find the work done by the force F.

$$\begin{aligned} W &= (15.2\ \mathrm{N \cdot m^2})\left(\frac{10.1\ \mathrm{m} - 0.49\ \mathrm{m}}{(0.49\ \mathrm{m})(10.1\ \mathrm{m})}\right) + (10.3\ \mathrm{N \cdot m})\ln\left(\frac{10.1\ \mathrm{m}}{0.49\ \mathrm{m}}\right) \\ &= 60.68\ \mathrm{N \cdot m} \\ &= 60.68\ \mathrm{J} \end{aligned}$$

This is the net work on the particle, which, by the work-energy theorem, is the change in the kinatic energy of the object.

$$\begin{aligned} W &= \Delta K \\ &= K_{\text{final}} - K_{\text{initial}} \\ &= \frac{1}{2}mv^2 - 0 \end{aligned}$$

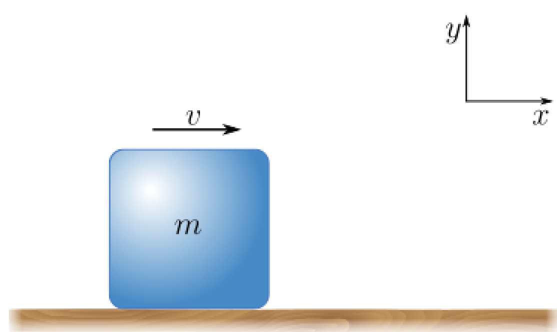
since the object began at rest. Solving for the final speed,

$$\begin{aligned} v &= \sqrt{\frac{2W}{m}} \\ &= \sqrt{\frac{2(60.68\ \mathrm{J})}{2.04\ \mathrm{kg}}} \end{aligned}$$

$$\boxed{v = 7.713\ \mathrm{m/s}}$$

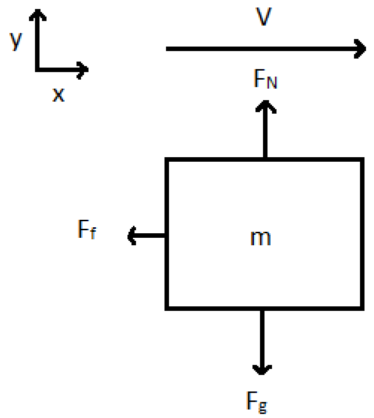
Problem 7:

A block of mass $m = 50.4$ kg slides along a horizontal surface. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.21$. The block has an initial speed of $v_0 = 10.1$ m/s in the positive x direction, as shown.



Part (a) **Write an expression for the x-component of the frictional force the block experiences, F_f, in terms of the given variables and variables available in the palette.**

Let's begin by drawing a free-body diagram of the block as it slides.



Since the block will not be moving vertically, the forces in the y-direction must be balanced. We can therefore write the following equation:

$$F_{net,y} = F_N - F_g$$

$$0 = F_N - F_g$$

$$F_g = F_N$$

$$mg = F_N$$

Now that we have an expression for the normal force in terms of the allowed variables, we can use the formula for kinetic friction to write the following expression (remembering that the force of friction is acting in what we have defined as the negative x-direction):

$$F_f = -\mu_k F_N$$

$$\boxed{F_f = -\mu_k mg}$$

Part (b) **What is the magnitude, in newtons, of the frictional force?**

To find the solution, we simply need to plug values into the equation that we found in part (a). Since we are looking for a magnitude, we will need to take the absolute value of our result.

$$F_f = |-\mu_k mg|$$

$$F_f = |-0.21 \cdot 50.4\ \mathrm{kg} \cdot 9.8\ \mathrm{m/s}^2|$$

$$F_f = |-103.7\ \mathrm{N}|$$

$$\boxed{F_f = 103.7\ \mathrm{N}}$$

Part (c) **How far, in meters, will the block travel before coming to rest?**

The work-energy theorem tells us that the amount of work done on an object will be equal to the change in its energy. This allows us to write the following equation:

$$W = K_f - K_i$$

$$W - K_f = -K_i$$

$$-K_f = -W - K_i$$

$$K_f = W + K_i$$

The work in this case will be given by the force of friction, which we found an expression for in part a, multiplied by the distance the block travels. Combining this with the equation for kinetic energy, we can rewrite the above expression as:

$$K_f = \frac{1}{2}mv^2 + Fd$$

$$K_f = \frac{1}{2}mv^2 + (-\mu_k mg)\, d$$

$$K_f = \frac{1}{2}mv^2 - \mu_k mgd$$

When the block has stopped, the kinetic energy will be zero. We can therefore solve for the total distance that the block travels by setting the final kinetic energy equal to zero and solving for the distance the block has traveled in the above equation.

$$0 = \frac{1}{2}mv^2 - \mu_k mgd$$

$$\mu_k mgd = \frac{1}{2}mv^2$$

$$d = \frac{v^2}{2\mu_k g}$$

$$d = \frac{(10.1\ \mathrm{m/s}\ )^2}{2 \cdot 0.21 \cdot 9.81\ \mathrm{m/s}^{\ 2}}$$

$$\boxed{d = 24.76\ \mathrm{m}}$$

Problem 8:

The thrust produced by a single jet engine creates a force of F = *51000* N. It takes the jet (with a mass of m = *5100* kg) a distance of d = *0.71* km to take off.

Part (a) **What is the take-off speed of the jet v_t in m/s?**

To solve this problem, we want to find a way to relate the distance the jet travels before it takes off, its mass, and the force exerted on it to its velocity. Since we know the force acting on it and the distance it travels, we can find the work done on the jet. With this together with the mass of the plane, we can use the work-energy theorem to solve for the plane's take-off speed.

$$W = K_f - K_i$$

$$Fd = \frac{1}{2}mv_t^2 - 0$$

$$\frac{2Fd}{m} = v_t^2$$

$$\sqrt{\frac{2Fd}{m}} = v_t$$

As we plug in values and solve for the take-off speed, we must be careful to convert the distance the plane travels from kilometers to meters.

$$v_t = \sqrt{\frac{2 \cdot 51000\ \mathrm{N}\ \cdot 0.71 \cdot 10^3\ \mathrm{m}}{5100\ \mathrm{kg}}}$$

$$\boxed{v_t = 119.2\ \mathrm{m/s}}$$

Part (b) **How far in meters would you need to depress a giant spring *k* = 100,000 N/m in order to launch the jet at the same speed without help from the engine?**

For the spring to successfully launch the jet, it must be capable of doing as much work as the jet's engines did in part (a). By setting the potential energy of the spring equal to the work done by the jet's engines, we can determine exactly how far the spring must be compressed to launch the jet.

$$U_s = W$$

$$\frac{1}{2}kd_s^2 = Fd$$

$$d_s^2 = \frac{2Fd}{k}$$

$$d_s = \sqrt{\frac{2Fd}{k}}$$

As in part (a), we must be careful when we plug in values to convert the distance the plane travels from kilometers to meters.

$$d_s = \sqrt{\frac{2 \cdot 51000\ \mathrm{N}\ \cdot 0.71 \cdot 10^3\ \mathrm{m}}{100000\ \mathrm{N/m}}}$$

$$\boxed{d_s = 26.91\ \mathrm{m}}$$

Problem 9:

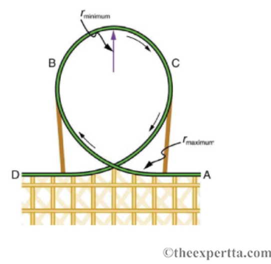
Great Problems in Physics Series

Dr. Cindy Schwarz earned a BS in Mathematical Physics at the State University of New York at Binghamton. Five years later she was awarded her doctorate in experimental particle physics from Yale University for her research done on CP-Violation at Brookhaven National Laboratory.

I really like this problem because it shows the most sensible design for modern roller coasters. The shape that is ideal is called a clothoid and this problem uses an approximation to the shape. The clothoid is a curve that is characterized by its curvature being proportional to its length. This property makes it very useful as a transition curve when designing roller coasters. To approximate, the radius of curvature at the bottom of the loop is greater than the radius of curvature at the top of the loop. This assures a smaller (safer) acceleration at the bottom of the loop.

Read more about Dr. Schwarz and her research here.

Modern roller coasters have vertical loops like the one shown in the figure. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats.



Part (a) **What is the speed, in meters per second, of the roller coaster at the top of the loop if the radius of curvature there is *11* m and the downward acceleration of the car is *1.1g*? Note that *g* here is the acceleration due to gravity.**

The centripetal acceleration and the speed are related by

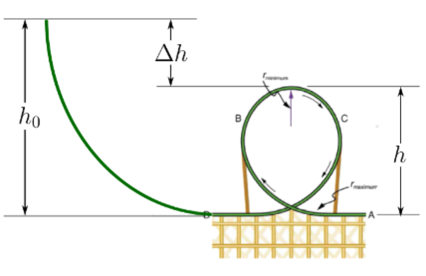
$$a_c = \frac{v^2}{r}$$

Solving for the speed and substituting numbers,

$$v = \sqrt{a_c r}$$
$$= \sqrt{1.1\,(9.81\ \mathrm{m/s^2})\,(11\ \mathrm{m})}$$
$$\boxed{v = 10.89\ \mathrm{m/s}}$$

Part (b) **The beginning of this roller coaster is at the top of a high hill. If it started from rest at the top of this hill, how high, in meters, above the top of the loop is this initial starting point? You may assume there is no friction anywhere on the track.**

The diagram from the problem statement has been modified below.



Let's choose the zero-point for potential energy at the bottom of the roller-coaster loop. When the car is at rest at the top of the hill, at height h_0, it has all gravitational potential energy with no kinetic energy. When it is at the top of the loop, at height h, there is a mixture of kinetic energy and gravitational energy, the sum of which, assuming no energy is lost, must equal the initial potential energy.

$$mgh_0 = mgh + \frac{1}{2}mv^2$$

Subtracting the potential energy at the top of the loop from both sides,

$$mg\left(h_0-h\right)=\frac{1}{2}mv^2$$

Noting that $h_0-h=\Delta h$ and dividing both sides by mg,

$$\Delta h=\frac{v^2}{2g}$$

$$=\frac{(10.89\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})}$$

$$\boxed{\Delta h=6.050\ \mathrm{m}}$$

Part (c) **If it actually starts *3.5* m higher than your answer to the previous part (yet still reaches the top of the loop with the same velocity), how much energy, in joules, did it lose to friction? Its mass is *1100* kg.**

With the initial height of the roller-coaster car increased, there is an increase in the gravitational potential energy by an amount

$$\Delta U=mgh_{\text{increase}}$$

$$=(1100\ \mathrm{kg})\left(9.81\ \mathrm{m/s^2}\right)(3.5\ \mathrm{m})$$

$$=3.777\times10^4\ \mathrm{J}$$

The problem states that the speed is the same as calculated in the first step, so the additional potential energy must all have been lost to friction and air resistance.

$$E_{\text{lost}}=\Delta U$$

$$\boxed{E_{\text{lost}}=3.777\times10^4\ \mathrm{J}}$$

Problem 10:

Suppose an elephant has a mass of *2750* kg.

Part (a) **How fast, in meters per second, does the elephant need to move to have the same kinetic energy as a *65*-kg sprinter running at *7.5* m/s?**

To find the required speed of the elephant, we simply need to put the kinetic energy of the elephant equal to the kinetic energy of the sprinter.

$$KE_e=KE_s$$

$$\frac{1}{2}m_ev_e^2=\frac{1}{2}m_sv_s^2$$

$$v_e^2=\frac{m_sv_s^2}{m_e}$$

$$v_e=\sqrt{\frac{m_sv_s^2}{m_e}}$$

$$v_e=\sqrt{\frac{65\ \mathrm{kg}\cdot(7.5\ \mathrm{m/s})^2}{2750\ \mathrm{kg}}}$$

$$\boxed{v_e=1.153\ \mathrm{m/s}}$$

Problem 11:

A block of mass 0.102 kg is released from rest at point A, and it slides down a curved slope to point B. The vertical height of point A is $h = 2.01$ m above point B, and the block arrives at point B with a speed $v_B = 3.04$ m/s.

Part (a) **How much work, in joules, was done by friction as the block slid from point A to point B?**

The work--kinetic-energy theorem tells us that the total work done on the block as it slides from A to B equals the block's change in kinetic energy.

The work W done on the block by friction is what is requested in this problem.

There is the downward force of gravity acting on the descending block and its work is mgh, where m denotes the block's mass, h is the height of A over B, and $g = 9.80\ \mathrm{m/s^2}$ is the acceleration due to gravity.

There is also the normal force of the curved slope on the block, but that force does no work, since at every point along the slope the normal force is perpendicular to the block's displacement.

Those are the three forces acting on the block. Their total work is $W + mgh$.

The change in the block's kinetic energy equals its final kinetic energy $1/2mv_B^2$, where v_B is its speed at B, since the block starts from rest at A.

We now apply the work--kinetic-energy theorem,

$$W + mgh = \tfrac{1}{2}mv_B^2\ ,$$

solve for W,

$$W = \tfrac{1}{2}mv_B^2 - mgh\ ,$$

and substitute the given values,

$$W = \tfrac{1}{2}(0.102\ \mathrm{kg})\,(3.04\ \mathrm{m/s})^2 - (0.102\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(2.01\ \mathrm{m})\ .$$

$$\boxed{W = -1.538\ \mathrm{J}}$$

Problem 12:

A *750*-kg car comes to rest from a speed of *75* km/h in a distance of *105* m. Assume the car is initially moving in the positive direction.

Part (a) **If the brakes are applied consistently and are the only thing making the car come to a stop, calculate the force (in newtons, in a component along the direction of motion of the car) that the brakes apply on the car.**

Since the brakes are applying force in the opposite direction of motion, they will do negative work on the car. Since the car is stopping, its final kinetic energy will be zero. With both of these facts in mind, we can use the work-energy theorem to find the force that the brakes must apply.

$$W = E_i - E_f$$

$$-F_b d = \frac{1}{2}mv^2 + 0$$

$$F_b = -\frac{mv^2}{2d}$$

Before we substitute in values, we need to convert the velocity from kilometers per hour to meters per second.

$$v = 75\ \mathrm{km/h}\ \cdot \frac{1000\ \mathrm{m}}{1\ \mathrm{km}} \cdot \frac{1\ \mathrm{h}}{60\ \mathrm{min}} \cdot \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = 75 \cdot \frac{1000}{3600}\ \mathrm{m/s}$$

Now, let's plug values into our equation and solve.

$$F_b = -\frac{750\ \mathrm{kg}\ \cdot \left(75 \cdot \frac{1000}{3600}\ \mathrm{m/s}\right)^2}{2 \cdot 105\ \mathrm{m}}$$

$$\boxed{F_b = -1550.\ \mathrm{N}}$$

Part (b) **Suppose instead of braking that the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force, in newtons, exerted on the car in this case.**

As compared to part (a), all that has changed is the distance that the car travels before it stops. Therefore, we can actually use the same equation that we used in part (a) to solve this problem if we replace the distance that the car took to brake with the new distance that it takes the car to stop.

$$F_c = -\frac{mv^2}{2d_c}$$

Next, we need to convert the velocity from kilometers per hour to meters per second before we can plug values into this equation and solve.

$$v = 75\ \mathrm{km/h}\ \cdot \frac{1000\ \mathrm{m}}{1\ \mathrm{km}} \cdot \frac{1\ \mathrm{h}}{60\ \mathrm{min}} \cdot \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = 75 \cdot \frac{1000}{3600}\ \mathrm{m/s}$$

We can now plug in values and solve for the force exerted on the car to stop it.

$$F_c = -\frac{750\ \mathrm{kg}\ \cdot \left(75 \cdot \frac{1000}{3600}\ \mathrm{m/s}\right)^2}{2 \cdot 2.00\ \mathrm{m}}$$

$$\boxed{F_c = -8.138 \times 10^4\ \mathrm{N}}$$

Part (c) **What is the ratio of the force on the car from the concrete to the braking force?**

To find the ratio, we need to divide our answer from part (b) by our answer from part (a).

$$F_c/\ F_b = \frac{\left(-\frac{mv^2}{2d_c}\right)}{\left(-\frac{mv^2}{2d}\right)}$$

$$F_c/\ F_b = \frac{d}{d_c}$$

$$F_c/\ F_b = \frac{105\ \mathrm{m}}{2.00\ \mathrm{m}}$$

$$\boxed{F_c/\ F_b = 52.50}$$

Problem 13:

A pitcher supplies a power of P = ***201*** W to a baseball of mass m = ***0.08*** kg for a period of t = ***0.23*** s.

Part (a) **What is the ball's speed in m/s when it leaves his hand?**

To begin this problem, let's find an expression for the work done to the ball using the relationship between power, work, and time.

$$P = \frac{W}{t}$$

$$Pt = W$$

Now, let's consider what happens when the ball is thrown. The ball initially has no kinetic energy, but work is done to it to throw the ball and thereby gives it kinetic energy. We can therefore use the work-energy theorem to solve for the speed of the ball.

$$W = E_f - E_i$$

$$W = \frac{1}{2}mv^2 - 0$$

$$Pt = \frac{1}{2}mv^2$$

$$\frac{2Pt}{m} = v^2$$

$$\sqrt{\frac{2Pt}{m}} = v$$

$$v = \sqrt{\frac{2 \cdot 201 \text{ W } \cdot 0.23 \text{ s}}{0.08 \text{ kg}}}$$

$$\boxed{v = 34.00 \text{ m/s}}$$

Problem 14:

A car m = ***1500*** kg is traveling at a constant speed of v = ***21*** m/s. The car experiences a drag force (air resistance) with magnitude F_d = ***210*** N.

Part (a) **Write an expression for the power the car must produce P_i to maintain its speed.**

To solve this problem, we need to use the relationship between power, force, and velocity. As the car is moving at a constant velocity, it must be producing enough force to counteract the drag force. This tells us that the force the car exerts is equal in magnitude to the drag force. Since the force and velocity are acting in the same direction, the angle between them is zero degrees and the cosine term in the relationship between power, force, and velocity will disappear.

$$P_1 = F_d v cos\,(\theta)$$

$$P_1 = F_d v cos\left(0^\circ\right)$$

$$\boxed{P_1 = F_d v}$$

Part (b) **What is the power in horsepower (hp)?**

To solve this problem, we first need to plug values into the equation we found in part (a) to solve it.

$$P_1 = F_d v$$

$$P_1 = 210 \text{ N } \cdot 21 \text{ m/s}$$

$$P_1 = 4410. \text{ W}$$

This gives us an answer in watts. However, we were asked for an answer in horsepower. To get the correct answer, we must therefore convert this result from watts to horsepower.

$$P_1 = 4410. \text{ W } \cdot \frac{1 \text{ hp}}{745 \text{ W}}$$

$$\boxed{P_i = 5.919 \text{ hp}}$$

Part (c) **The car encounters an incline which makes an angle of θ = 6 degrees with respect to the horizontal. The cruise control kicks in and increases the power to maintain the speed of the car. What is the new power (in hp) required to maintain a constant speed?**

In most respects, this problem is the same as the one in part (a). As in part (a), we will need to use the relationship between power, force, and velocity to find a solution. Also like part (a) is the fact that the force will be in the same direction as the car's motion meaning that the cosine term resulting from the dot product will be equal to one. In fact, the only thing that has changed is that the car will experience a component of the gravitational force resisting its motion in addition to the drag force, meaning that the force exerted by the car must be enough to cancel both these forces. With this in mind, we can write an equation for the power the car must exert.

$$P_2 = F v cos\left(0^\circ\right)$$

$$P_2 = F v$$

$$P_2 = \left(F_g sin\left(\theta\right) + F_d\right) v$$

$$P_2 = \left(mg sin\left(\theta\right) + F_d\right) v$$

$$P_2 = \left(1500 \text{ kg } \cdot 9.81 \text{ m/s}^2 \cdot sin\left(6^\circ\right) + 210 \text{ N}\right) \cdot 21 \text{ m/s}$$

$$P_2 = 3.671 \times 10^4 \text{ W}$$

Now we have an answer in watts. However, we were asked for an answer in horsepower. As such, we must convert our result from watts to horsepower to get the correct answer.

$$P_2 = 3.671 \times 10^4 \text{ W} \cdot \frac{1 \text{ hp}}{745.7 \text{ W}}$$

$$\boxed{P_2 = 49.23 \text{ hp}}$$

Problem 15:

A hiker of mass *41* kg is going to climb to the top of Mount Tam, which has an elevation of 2,574 ft.

Part (a) **If the hiker starts climbing at an elevation of *305* ft, what will their change in gravitational potential energy be, in joules, once they reach the top? Assume the zero of gravitational potential energy is at sea level.**

A hiker of mass 41 kg is going to climb Mount Tam, which has an elevation of 2,574 ft.

If the hiker starts climbing at 305 ft, what will their change in gravitational potential be once they reach the top? Assume the zero of potential energy is at sea level.

The change in potential energy is the final energy minus the initial energy.

$$\Delta U_g = U_{gF} - U_{gI}$$

$$\Delta U_g = mgh_F - mgh_I$$

$$\Delta U_g = mg\left(h_F - h_I\right)$$

If we set the zero of height at sea level, the initial and final heights are simply the elevations.

Notice the additional factor to convert feet to meters $(1m = 3.28084ft)$.

$$\Delta U_g = mg\left(h_F - h_I\right)$$

$$\Delta U_g = (41kg) \cdot \left(9.81\frac{m}{s^2}\right) \cdot (2574ft - 305ft) \cdot \left(\frac{1m}{3.28084ft}\right)$$

$$\Delta U_g = (41kg) \cdot \left(9.81\frac{m}{s^2}\right) \cdot (2269ft) \cdot \left(\frac{1m}{3.28084ft}\right)$$

$$\Delta U_g = (41kg) \cdot \left(9.81\frac{m}{s^2}\right) \cdot (691.6m)$$

$$\Delta U_g = 2.782 \times 10^5 \frac{(kg \cdot m^2)}{s^2}$$

$$\boxed{\Delta U_g = 2.782 \times 10^5 J}$$

Part (b) **Repeat the above calculation for the change in gravitational potential energy, but assume the gravitational potential is zero at the top of the mountain.**

A hiker of mass 41 kg is going to climb Mount Tam, which has an elevation of 2,574 ft.

If the hiker starts climbing at 305 ft, what will their change in gravitational potential be once they reach the top? Assume the zero of potential energy is at the top of the mountain.

The change in potential energy is the final energy minus the initial energy.

$$\Delta U_g = U_{gF} - U_{gI}$$

$$\Delta U_g = mgh_F - mgh_I$$

$$\Delta U_g = mg\left(h_F - h_I\right)$$

The change in gravitational potential energy is the same regardless of where you choose to call height zero.

If we set the zero of height the top of the mountain, the final height is zero

The initial height will be negative, but notice that you are subtracting this negative number. The total change in height will be the same as calculated in part a.

$$\Delta U_g = mg\left(h_F - h_I\right)$$

$$\Delta U_g = (41kg)\cdot\left(9.81\frac{m}{s^2}\right)\cdot\left(0ft - (305 - 2574)\ ft\right)\cdot\left(\frac{1m}{3.28084ft}\right)$$

$$\Delta U_g = (41kg)\cdot\left(9.81\frac{m}{s^2}\right)\cdot(2269ft)\cdot\left(\frac{1m}{3.28084ft}\right)$$

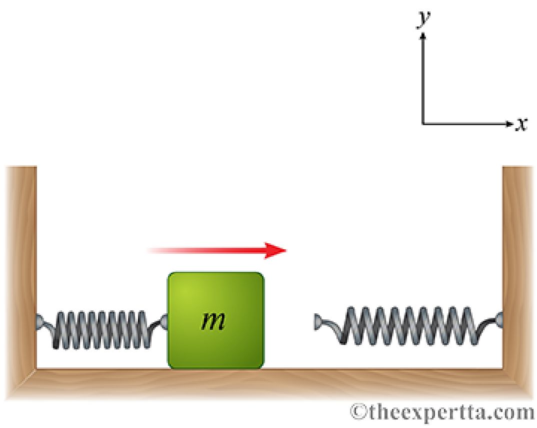
$$\Delta U_g = (41kg)\cdot\left(9.81\frac{m}{s^2}\right)\cdot(691.6m)$$

$$\Delta U_g = 2.782\times10^5\frac{\left(kg\cdot m^2\right)}{s^2}$$

$$\boxed{\Delta U_g = 2.782\times10^5 J}$$

Problem 16:

A block of mass m = ***0.11*** kg is set against a spring with a spring constant of k_1 = ***501*** N/m which has been compressed by a distance of 0.1 m. Some distance in front of it, along a frictionless surface, is another spring with a spring constant of k_2 = ***101*** N/m. The block is not connected to the first spring and may slide freely.



Part (a) **How far d_2, in meters, will the second spring compress when the block runs into it?**

To find how far the second spring is compressed, we can use the conservation of energy. Initially, the block has spring potential energy based on the first spring and no kinetic energy. When the second spring is compressed as far as it will go, the block has spring potential energy based on the second spring and, as the maximum compression will happen at the moment when the block's velocity is changing direction, will have no kinetic energy. We can therefore write the following equation for the conservation of energy in this case:

$$E_i = E_f$$

$$\frac{1}{2}k_1 d_1^2 = \frac{1}{2}k_2 d_2^2$$

$$\frac{k_1 d_1^2}{k_2} = d_2^2$$

$$\sqrt{\frac{k_1 d_1^2}{k_2}} = d_2$$

$$d_2 = \sqrt{\frac{501\ \mathrm{N/m}\ \cdot (0.1\ \mathrm{m}\ )^2}{101\ \mathrm{N/m}}}$$

$$\boxed{d_2 = 0.2227\ \mathrm{m}}$$

Part (b) **How fast *v*, in meters per second, will the block be moving when it strikes the second spring?**

To find the speed of the block before it hits the second spring, we can use the conservation of energy as we did in part (a). The block will initially have only spring potential energy when it is compressing the first spring. When the block is just about to hit the second spring, all its energy will be kinetic. As such, we can write the following equation for the conservation of energy.

$$E_i = E_f$$

$$\frac{1}{2}k_1 d_1^2 = \frac{1}{2}mv^2$$

$$\frac{k_1 d_1^2}{m} = v^2$$

$$\sqrt{\frac{k_1 d_1^2}{m}} = v$$

$$v = \sqrt{\frac{501\ \mathrm{N/m}\ \cdot (0.1\ \mathrm{m}\ )^2}{0.11\ \mathrm{kg}}}$$

$$\boxed{v = 6.749\ \mathrm{m/s}}$$

Part (c) **Now assume friction is present on the surface in between the ends of the springs at their equilibrium lengths, and the coefficient of kinetic friction is μ_k = 0.5. If the distance between the springs is x = 1 m, how far d_2, in meters, will the second spring now compress?**

For the most part, we can use the same method that we used in part (a) to solve this problem. The only thing that has changed is that there is now a nonconservative force of friction acting on the block. Before we proceed, we need to find an expression for the force of friction. Since the block is not moving up or down, the normal force must be equal to the force of gravity experienced by the block. The force of friction can therefore be written as:

$$F_f = \mu_k F_N$$

$$F_f = \mu_k F_g$$

$$F_f = \mu_k mg$$

Next, we need to find the work done by friction. We can do this using the equation for work noting that the force is acting in the opposite direction of the block's motion.

$$W_f = F_f x cos\left(180^{\circ}\right)$$

$$W_f = -F_f x$$

$$W_f = -\mu_k mgx$$

Now that we know the work done by friction, we can set up an equation for the conservation of energy.

$$E_i = E_f - W_f$$

$$\frac{1}{2}k_1 d_1^2 = \frac{1}{2}k_2 d_2^2 - (-\mu_k mgx)$$

$$\frac{1}{2}k_1 d_1^2 = \frac{1}{2}k_2 d_2^2 + \mu_k mgx$$

$$\frac{1}{2}k_1 d_1^2 - \mu_k mgx = \frac{1}{2}k_2 d_2^2$$

$$\frac{k_1 d_1^2 - 2\mu_k mgx}{k_2} = d_2^2$$

$$\sqrt{\frac{k_1 d_1^2 - 2\mu_k mgx}{k_2}} = d_2$$

$$d_2 = \sqrt{\frac{501\ \mathrm{N/m}\ \cdot (0.1\ \mathrm{m}\ )^2 - 2 \cdot 0.5 \cdot 0.11\ \mathrm{kg}\ \cdot 9.81\ \mathrm{m/s}^{\ 2} \cdot 1\ \mathrm{m}}{101\ \mathrm{N/m}}}$$

$$\boxed{d_2 = 0.1973\ \mathrm{m}}$$

Problem 17:

Answer the following question concerning non-conservative forces.

Part (a) **From the given list please select all forces that are classified as non-conservative forces.**

Recall that nonconservative forces are those that do varying amounts of work depending on the path taken to reach the final position, whereas the work done by conservative forces only depend on the final position irregardless of the path. As a consequence, gravitational, electrostatic, and spring forces are conservative. Drag force and kinetic friction, meanwhile, are both nonconservative as they will do more work if the path is longer. The last option regarding a collision between cars requires a bit of consideration. Since the bumpers of both cars are dented, this means that the energy was not all used strictly to bounce the cars off one another This is to say that the collision was not perfectly elastic. As energy is only conserved within a system in the case of a perfectly elastic collision, we can conclude that this scenario is an example of a nonconservative force. The list of nonconservative forces is therefore:

"Kinetic friction", "Drag force (air resistance)", and "The force created between one car bumping into another, where the bumpers of both cars are dented."

Problem 18:

A hydroelectric power facility (see figure) converts the gravitational potential energy of water behind a dam to electric energy.

Part (a) **What is the gravitational potential energy relative to the generators of a lake of volume 50.0 km^3 (mass = 5.00 × 10^{13} kg) , given that the lake has an average height of *37.5* m above the generators?**

We can use the equation for gravitational potential energy to find the potential energy of the lake.

$$\Delta U_g = mgh$$

$$\Delta U_g = 5.00 \cdot 10^{13}\ \mathrm{kg}\ \cdot 9.8\ \mathrm{m/s}^{\,2} \cdot 37.5\ \mathrm{m}$$

$$\boxed{\Delta U = 1.838 \times 10^{16}\ \mathrm{J}}$$

Part (b) **A 9 megaton fusion bomb (which is around 1000 times bigger than the WWII-era nuclear bombs) contains 3.8 × 10^{16} J of energy. What fraction of this energy is contained in the potential energy of the entire lake?**

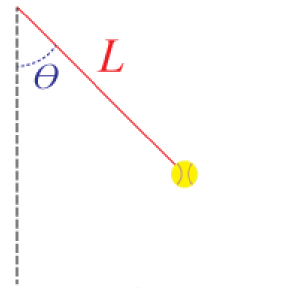
To find the desired ratio, we simply need to divide our answer from part (a) by the energy in a fusion bomb.

$$U_{g,lake}/\ E_{bomb} = \frac{5.00 \cdot 10^{13}\ \mathrm{kg}\ \cdot 9.8\ \mathrm{m/s}^{\,2} \cdot 37.5\ \mathrm{m}}{3.8 \cdot 10^{16}\ \mathrm{J}}$$

$$\boxed{U_{g,lake}/\ E_{bomb} = 0.4836}$$

Problem 19:

A 0.25 kg ball is suspended from a light *0.6* m string as shown. The string makes an angle of *21*° with the vertical. Let $U = 0$ when the ball is at its lowest point ($\theta = 0$).



Part (a) **What is the gravitational potential energy, in joules, of the ball before it is released?**

Consider the following drawing. The vertical dashed line represents the equilibrium location for the pendulum. When the ball is at it's lowest point, it is the length L from the ceiling. When the ball is displaced, it is at a height h above that position. Therefore, the ball has a graviational potential energy

$$U = mgh$$

We do not know the value of h, but it can be found using trigonometry. We can see a right triangle is formed in the drawing between the vertical line, the ball's position, and the point where the string is attached to the ceiling. In this triangle, we know the angle and the hypoteneuse. When the ball is displaced, it is $L - h$ from the ceiling.

$$cos\theta = \frac{(L-h)}{L}$$

which gives

$$h = L\left(1 - cos\theta\right)$$

Therefore,

$$\begin{aligned} U &= mgh \\ &= mgL\left(1 - cos\theta\right) \\ &= (0.25\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(0.6\ \mathrm{m})\left(1 - cos\left(21^\circ\right)\right) \end{aligned}$$

$$\boxed{U = 0.09764\ \mathrm{J}}$$

Part (b) **What will be the speed of the ball, in meters per second, when it reaches the bottom?**

Assuming that there are no frictional forces acting on the ball, the total energy of the ball is conserved. It starts from rest with its total energy equal to its gravitational potential energy, $E = U = mgh$. At the lowest point of the motion, we choose the height to be zero for convenience, so the total energy of the ball is equal to its kinetic energy and its potential energy is equal to zero J at that location. Therefore, applying the principle of the conservation of energy, we have

$$mgh = \frac{1}{2}mv^2$$

from which we can solve for the speed v at the ball's lowest position.

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2gL\left(1 - cos\theta\right)} \end{aligned}$$

$$= \sqrt{2\left(9.80\ \mathrm{m/s^2}\right)(0.6\ \mathrm{m})\left(1 - \cos\left(21^\circ\right)\right)}$$

$$\boxed{v = 0.8838\ \mathrm{m/s}}$$

Problem 20:

A mysterious force has been discovered in nature that acts on all particles in the three-dimensional space. You determine that the force is always pointed away from a definite point in space, which we can call the force center. The magnitude of the force has the following functionality: $F = Ae^{-br}$, where r is the distance from the force center to any other point.

Part (a) **Write an expression for the potential energy U_D of a particle when it is at a distance D from the force center, assuming the potential energy to be zero when the particle is at in?nity.**

The potential energy function is the negative integral of the given force function with respect to position, taking account of the given condition that the potential energy is zero at infinity. We integrate the given force function and obtain

$$U = \frac{A}{b} e^{-br} + C.$$

The condition at infinity gives us

$$C = 0,$$

so the potential energy at distance $r = D$ from the center is

$$\boxed{U_D = \frac{A}{b} e^{-bD}}$$

Part (b) **If D is measured in meters, then what must the units be for the proportionality constant A in order that the energy be in Joules?**

For the potential energy to be in joules and D in meters, the force must be in newtons. So A must also be in newtons.

However, we know from Newton's second law that

$$1\ \mathrm{N} = 1\ \mathrm{kg\ m/s^2}.$$

Therefore,

$$\boxed{\mathrm{units} = \mathrm{kg\ m/s^2}}$$

Part (c) **If D is measured in meters, then what must the units be for the constant b in order that the energy be in Joules?**

Whatever the units of potential energy are, if D is in meters, b must be in inverse meters, since the exponent is dimensionless and has no unit. Therefore the units of b are

$$\boxed{\mathrm{units} = 1/\mathrm{m}}$$

Part (d) **If the particle is a distance *0.11* m from the force center and the constants are A = *11* and b = *2*, then what is the potential energy of that particle?**

We substitute the given values into the expression for the potential energy that we obtained in part (a).

$$PE = \frac{11\ \mathrm{kg\ m/s^2}}{2\ 1/\mathrm{m}} e^{-(2\ 1/\mathrm{m})(0.11\ \mathrm{m})}.$$

$$\boxed{PE = 4.414\ \mathrm{J}}$$