Begin Date: 10/18/2024 12:01:00 AM -- **Due Date:** 11/1/2024 11:59:00 PM **End Date:** 11/1/2024 11:59:00 PM

Problem 1:

An object's speed is increased by a factor of three.

Part (a) **How does the object's momentum change?**

Let's begin by writing out the equation for momentum.

$$p = mv$$

Now, let's consider a case where velocity is tripled. Let's allow the momentum in this case to be p_3 .

$$p_3 = m(3v)$$

$$p_3 = 3mv$$

$$p_3 = 3(mv)$$

Now, let's substitute for mv using the first equation we wrote.

$$p_3 = 3p$$

This shows that, as compared to the first case, the momentum tripled when the speed tripled. The correct answer is therefore:

It increases by a factor of three.

Part (b) **How does the object's kinetic energy change?**

Let's begin by writing out the equation for kinetic energy.

$$K = \frac{1}{2}mv^2$$

Now, let's consider a case where velocity is tripled. Let's allow the kinetic energy in this case to be denoted K_3 .

$$K_3 = \frac{1}{2}m(3v)^2$$

$$K_3 = \frac{1}{2}m\left(3^2 \cdot v^2\right)$$

$$K_3 = \frac{1}{2}m\left(9v^2\right)$$

$$K_3 = 9\left(\frac{1}{2}mv^2\right)$$

Looking at the first equation we wrote for kinetic energy, we see that K_3 is equal to nine times the normal equation for kinetic energy. This shows that the kinetic energy increases by a factor of nine when the speed triples. The correct answer is therefore:

Increased by a factor of nine.

Problem 2:

Two objects are dropped (with zero initial speed) from a height h.

Part (a) **If object A has twice the mass of object B, what can be said about the momentum of each just before they hit the ground? Ignore air resistance.**

Since the acceleration due to gravity is the same for all objects, both object A and object B will have the same velocity just before they hit the ground. The momentum for object B can be written as:

$$p_B = mv$$

The momentum for object A, meanwhile, can be written:

$$p_A = (2m)\,v$$

$$p_A = 2mv$$

$$p_A = 2\,(mv)$$

Substituting for mv based on the our equation for momentum m_B yields:

$$p_A = 2p_B$$

This shows that the momentum of object A is twice the momentum of object B. The correct answer is therefore:

$$\boxed{\text{object A has twice the momentum of object B.}}$$

Problem 3:

Suppose a large ship has a momentum of *1.25* $\times 10^9$ kg·m/s.

Part (a) **What is the mass of the large ship, in kilograms, if the ship is moving at a speed of *45* km/h?**

We can use the equation for momentum to relate momentum, mass, and velocity.

$$p_s = m_s v_s$$

$$\frac{p_s}{v_s} = m_s$$

Before we substitute values into this equation, we need to convert the velocity from kilometers per hour to meters per second.

$$v_s = 45\text{ km/h} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ s}}$$

$$v_s = \left(45 \cdot \frac{1000}{3600}\right)\text{ m/s}$$

Now, let's plug values into the equation that we found for the mass and solve.

$$m_s = \frac{1.25 \cdot 10^9\text{ kg} \cdot \text{m/s}}{\left(45 \cdot \frac{1000}{3600}\right)\text{ m/s}}$$

$$\boxed{m_s = 1.000 \times 10^8\text{ kg}}$$

Part (b) **How much larger is the ship's momentum than the momentum of a *1025*-kg artillery shell fired at a speed of *250* m/s?**

We can find an expression for the momentum of the shell using the equation for momentum.

$$p_a = m_a v_a$$

To solve this problem, we need to divide the momentum of the ship by the momentum of the shell.

$$p_s / p_a = \frac{1.25 \cdot 10^9 \text{ kg} \cdot \text{m/s}}{1025 \text{ kg} \cdot 250 \text{ m/s}}$$

$$\boxed{p_s / p_a = 4878}$$

Problem 4:

A railway car with a mass of 2010 kg moves along horizontal tracks at a constant speed of 4.01 m/s. It rolls under a grain terminal, which dumps grain directly down into the freight car after which the car's speed decreases by 2.01 m/s.

Part (a) **What is the mass, in kilograms, of the grain dumped into the car?**

In the problem statement, we are told the initial mass of the train car, m_i, the initial speed of the train car, v_i, and how much it slows down by after the grain is loaded v_s. Using this information, we can write an equation for the conservation of momentum in this case and solve for the final mass of the train car plus the grain.

$$m_i v_i = m_f (v_i - v_s)$$

$$\frac{m_i v_i}{v_i - v_s} = m_f$$

Note that we are not being asked for the final mass of the train and grain together, but instead the mass of grain added, m_{grain}. We can write the following relation for the masses:

$$m_i + m_{grain} = m_f$$

Substituting this into our previous equation and solving for m_{grain}:

$$\frac{m_i v_i}{v_i - v_s} = m_i + m_{grain}$$

$$\frac{m_i v_i}{v_i - v_s} - m_i = m_{grain}$$

Now we an plug in values and solve for the mass of grain added.

$$m_{grain} = \frac{(2010 \text{ kg})(4.01 \text{ m/s})}{(4.01 \text{ m/s}) - (2.01 \text{ m/s})} - 2010 \text{ kg}$$

$$\boxed{m_g = 2020.\text{ kg}}$$

Problem 5:

A baseball of mass m = ***0.25*** kg is dropped from a height h_1 = ***2.05*** m. It bounces from the concrete below and returns to a final height of h_2 = ***1.01*** m. Neglect air resistance.

Randomized Variables

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m = ***0.25*** kg

$h_1 = \mathbf{\mathit{2.05}}$ m

$h_2 = \mathbf{\mathit{1.01}}$ m

Part (a) **Select an expression for the impulse *I* that the baseball experiences when it bounces off the concrete.**

We know that the impulse will be equal to the change in the baseball's momentum. We can therefore begin by writing the following equation:

$$I = \Delta p$$

$$I = p_2 - p_1$$

$$I = mv_2 - mv_1$$

$$I = m(v_2 - v_1)$$

To proceed, we need to find expressions for the velocity of the ball just before it hits the concrete (v_1) and just after it hits the concrete (v_2). Since we know how high the ball was before it was dropped, we can use the conservation of energy to solve for v_1. Letting the ground be the zero of gravitational potential energy, we can write the following equation for the conservation of energy before the ball hits the concrete:

$$E_{i1} = E_{f1}$$

$$mgh_1 = \frac{1}{2}mv_1^2$$

$$2gh_1 = v_1^2$$

$$\sqrt{2gh_1} = v_1$$

We can use the same methodology to get an equation for the velocity of the ball just after it hits the concrete. In this case, the initial energy of the baseball will be all kinetic energy and the final energy will be all potential once it reaches its maximum height.

$$E_{i2} = E_{f2}$$

$$mgh_2 = \frac{1}{2}mv_2^2$$

$$2gh_2 = v_2^2$$

$$\sqrt{2gh_2} = v_2$$

We now have equations for both velocities, but remember that square roots can be either positive or negative. To figure out which sign to apply to these velocities, let's consider the physics of the problem. The ball is initially falling down, so v_1 should have a negative value. When the ball bounces off the concrete, it will begin moving upward. As such, v_2 should have a positive value. With this in mind, let's plug our expressions for v_1 and v_2 into our equation for impulse and simplify it to get the desired expression.

$$I = m\left(\sqrt{2gh_2} - \left(-\sqrt{2gh_1}\right)\right)$$

$$\boxed{I = m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)}$$

Part (b) **What is this impulse, in kilogram meters per second?**

Here, we simply need to plug values into the equation we found in part (a) and solve.

$$I = m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)$$

$$I = 0.25\ \text{kg} \cdot \left(\sqrt{2 \cdot 9.8\ \text{m/s}^2 \cdot 1.01\ \text{m}} + \sqrt{2 \cdot 9.8\ \text{m/s}^2 \cdot 2.05\ \text{m}}\right)$$

$$\boxed{I = 2.697\ \text{kg} \cdot \text{m/s}}$$

Part (c) **If the baseball was in contact with the concrete for $t = 0.01$ s, what average force F_{ave} did the concrete exert on the baseball, in newtons?**

Recall that impulse is equal to force multiplied by time. To find the average force exerted, we can divide our equation for the impulse from part (a) by the given value of time.

$$I = m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)$$

$$F_{ave}\, t = m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)$$

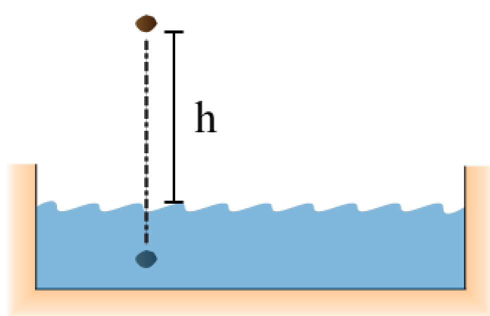
$$F_{ave} = \frac{m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)}{t}$$

$$F_{ave} = \frac{0.25\ \text{kg} \cdot \left(\sqrt{2 \cdot 9.8\ \text{m/s}^2 \cdot 1.01\ \text{m}} + \sqrt{2 \cdot 9.8\ \text{m/s}^2 \cdot 2.05\ \text{m}}\right)}{0.01\ \text{s}}$$

$$\boxed{F_{ave} = 269.7\ \text{N}}$$

Problem 6:

A rock of mass m = *1.05* kg is dropped from a height of h = *2.1* m into a pool of water. At a time of t = *1.01* s after striking the surface of the water, the rock's velocity has decreased by 50%.



Part (a) **What is the magnitude of the average force the rock experiences, in newtons, during the time t?**

To begin, we will want to find an expression for the velocity of the rock just before it strikes the surface of the water. Applying the conservation of energy with the surface of the water as our zero of potential energy, we get the following equation:

$$E_i = E_f$$

$$mgh = \frac{1}{2}mv^2$$

$$2gh = v^2$$

$$\sqrt{2gh} = v$$

Now that we have a formula for the velocity, let's consider our next step. Since we know both the initial and final velocity of the rock as well as its mass, we know its initial and final momentum. We can therefore use the relationship between impulse and the change in momentum to solve for the average force. One important thing to note is that, since the rock is moving downwards, both its initial and final velocity will have negative values.

$$I = \Delta p$$

$$F_{ave}\, t = p_f - p_i$$

$$F_{ave}\, t = m\,(0.5 \cdot -v) - m\,(-v)$$

$$F_{ave}\, t = mv - 0.5mv$$

$$F_{ave}\, t = 0.5mv$$

$$F_{ave} = \frac{mv}{2t}$$

We can solve this problem by plugging in the value for v that we found earlier.

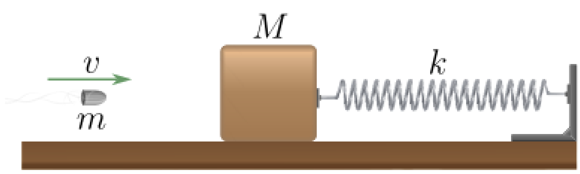
$$F_{ave} = \frac{m\sqrt{2gh}}{2t}$$

$$F_{ave} = \frac{1.05\ \text{kg} \cdot \sqrt{2 \cdot 9.8 \cdot 2.1\ \text{m}}}{2 \cdot 1.01\ \text{s}}$$

$$\boxed{F_{ave} = 3.335\ \text{N}}$$

Problem 7:

A block with mass $M = 50.2$ g rests on a frictionless wooden table. On its right side it is attached to the wall with a massless linear spring with a force constant $k = 181$ N/m. Initially the block is at rest, and the spring is in its equilibrium position. A bullet with mass $m = 5.03$ g and an unknow horizontal velocity of magnitude v strikes and becomes embedded in the block causing the spring to compress by a maximum distance $d = 0.802$ m.



Part (a) **Select the option that correctly expresses the initial speed of the bullet.**

The collision is inelastic, so we know that neither kinetic energy nor the total mechanical energy will be conserved. We should expect that a lot of energy goes into heat and sound. However, conservation of momentum may be used. With all motion along the horizontal direction, the bullet of mass m initially moves with speed v, and immediately after the collision, the combined mass of the bullet and the block, $m + M$, move with speed v_{new}.

$$mv = (m + M)v_{new}$$

Solving for this new speed,

$$v_{new} = \frac{m}{m + M} v$$

Note that the ratio of the masses indicates that this new speed is less than the initial speed of the bullet:

$$m < m + M \quad \Longrightarrow \quad \frac{m}{m + M} < 1 \quad \Longrightarrow \quad v_{new} < v$$

From here on, we may use conservation of mechanical energy. The kinetic energy of the block plus bullet will be converted entirely into spring potential energy.

$$\begin{aligned} \cancel{\tfrac{1}{2}}(m + M) v_{new}^2 &= \cancel{\tfrac{1}{2}} kd^2 \\ (m + M)\left(\frac{m}{m + M} v\right)^2 &= kd^2 \\ \frac{m^2}{m + M} v^2 &= kd^2 \\ v^2 &= \frac{k(m + M)}{m^2} d^2 \end{aligned}$$

$$\boxed{v = \frac{d}{m}\sqrt{k(m + M)}}$$

Part (b) **What is the speed, in meters per second, of the bullet just before it enters the block?**

Perform numeric substitutions into the expression from Part (a) and simplify.

$$\begin{aligned} v &= \frac{d}{m}\sqrt{k(m + M)} \\ &= \frac{0.802 \text{ m}}{5.03 \times 10^{-3} \text{ kg}} \sqrt{(181 \text{ N/m})(5.03 \times 10^{-3} \text{ kg} + 50.2 \times 10^{-3} \text{ kg})} \end{aligned}$$

$$\boxed{v = 504.1 \text{ m/s}}$$

Problem 8:

A freight train is comprised of 15 cars each with a mass *m*. The train is moving at a constant velocity of $V = 5.5$ m/s through a train yard. To add another car to the train engineers will let the car roll down a hill of vertical height $h = 5.25$ m, until it strikes the train, coupling to it. The train and the new car are moving in the same direction before the collision.

Randomized Variables

$V = 5.5$ m/s
$h = 5.25$ m

Part (a) **Select an expression for the train's velocity V_f after it has coupled with the new car.**

To solve this problem we first need to figure out the velocity of the train car when it strikes the train. We can use the conservation of energy to figure out the final velocity of the car.

$$E_i = E_f$$

$$mgh = \frac{1}{2} m v_c^2$$

$$2gh = v_c^2$$

$$v_c = \pm\sqrt{2gh}$$

Now, recall that square roots can be either positive or negative. Because of this, we need to look at the physics of the problem to determine the direction of this car's velocity. Since the train has a positive velocity and the problem statement notes that the train and the car are initially moving in the same direction, this means that the train car has a positive velocity.

$$v_c = \sqrt{2gh}$$

With this information, we can set up an equation using the conservation of momentum to find the final velocity of the train after the new car is coupled in.

$$15mV + mv_c = 16mV_f$$

$$15mV + m\sqrt{2gh} = 16mV_f$$

$$m\left(15V + \sqrt{2gh}\right) = 16mV_f$$

$$\frac{m\left(15V + \sqrt{2gh}\right)}{16m} = V_f$$

$$\boxed{V_f = \frac{15V + \sqrt{2gh}}{16}}$$

Part (b) **What is V_f in m/s?**

To complete this problem, we simply need to plug values into the equation we found in part (a) and solve.

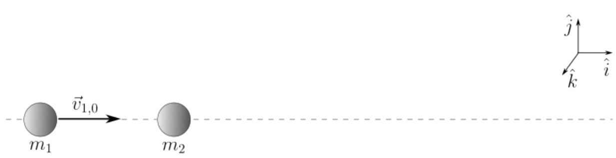
$$V_f = \frac{15V + \sqrt{2gh}}{16}$$

$$v_f = \frac{15 \cdot 5.5\ \text{m/s} + \sqrt{2 \cdot 9.8 \cdot 5.25\ \text{m}}}{16}$$

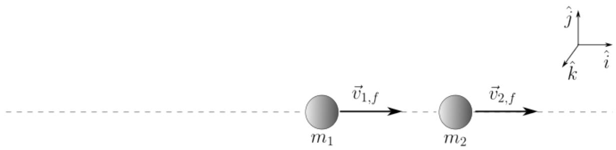
$$\boxed{v_f = 5.790\ \text{m/s}}$$

Problem 9:

The line which passes through the centers of two small masses, $m_1 = 101$ g and $m_2 = 903$ g, is parallel to the x axis. Initially mass m_2 is at rest, and mass m_1 is directed towards it with the velocity $\vec{v}_{1,0} = \left(1.75\ \text{m/s}\right)\hat{i}$, as shown.



After colliding elastically, masses m_1 and m_2 are both moving parallel to the x axis with velocities $\vec{v}_{1,f} = v_{1,f}\,\hat{i}$ and $\vec{v}_{2,f} = v_{2,f}\,\hat{i}$, respectively, as shown in the second drawing. The proportionate lengths of the vectors may differ from what is shown.



Part (a) **What is the x component, in meters per second, of the final velocity of mass m_1?**

While total energy is always conserved, with an elastic collision, kinetic energy is explicitly conserved. Conservation of momentum and conservation of kinetic energy yield two equations in two unknowns. The final velocities are obtained from standard formulas. For the first mass, m_1, the x component of the final velocity is given by

$$\begin{aligned} v_{1,f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1,0} \\ &= \frac{101\ \text{g} - 903\ \text{g}}{101\ \text{g} + 903\ \text{g}} \left(1.75\ \text{m/s}\right) \end{aligned}$$

Units of mass cancel in the ratio, so there is no need to convert to kilograms.

$$\boxed{v_{1,f} = -1.398\ \text{m/s}}$$

If $m_1 \ll m_2$, meaning that m_1 is insignificantly small as compared to m_2, then mass m_1 would bounce directly back with its original speed. The masses are quite different, but not extremely so, and hence, while the direction is reversed, the final speed is merely similar to the initial speed.

Part (b) **What is the x component, in meters per second, of the final velocity of mass m_2?**

While total energy is always conserved, with an elastic collision, kinetic energy is explicitly conserved. Conservation of momentum and conservation of kinetic energy yield two equations in two unknowns. The final velocities are obtained from standard formulas. For the second mass, m_2, the x component of the final velocity is given by

$$\begin{aligned} v_{2,f} &= \frac{2m_1}{m_1 + m_2} v_{1,0} \\ &= \frac{2\left(101\ \text{g}\right)}{101\ \text{g} + 903\ \text{g}} \left(1.75\ \text{m/s}\right) \end{aligned}$$

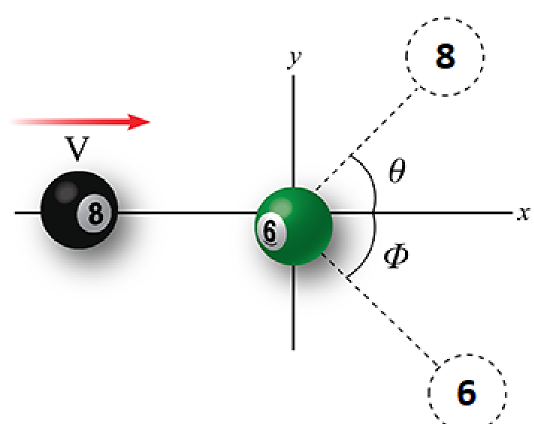
Units of mass cancel in the ratio, so there is no need to convert to kilograms.

$$\boxed{v_{2,f} = 0.3521\ \text{m/s}}$$

If $m_1 \ll m_2$, meaning that m_1 is insignificantly small as compared to m_2, then mass m_2 would remain at rest. The masses are quite different, but not extremely so, and hence, the the final speed mass m_2 is notably smaller than the initial speed of mass m_1.

Problem 10:

The eight ball, which has a mass of $m = 0.500$ kg, is initially moving with a velocity $v = \left(2.01\ \text{m/s}\right)\hat{i}$. The six ball has an identical mass and is initially at rest. After the two collide in an inelastic collision, the eight ball is deflected by an angle of $\theta = 15.2^\circ$, and the six ball is deflected by an angle of $\Phi = 10.3^\circ$, as shown in the figure.



Part (a) **Write an expression for the magnitude of six ball's velocity, in terms of the angles given in the problem and the magnitude of the eight ball's initial velocity, *v*.**

Decompose the velocities into Cartesian unit-vector components. Let the initial velocity of the eight ball be

$$\vec{v}_0 = v_0\,\hat{i} = (2.01\ \text{m/s})\,\hat{i}$$

If the magnitude of the velocity of the eight ball after the collision is v_1, we note that it is moving towards positive x and positive y, and its velocity may be expressed as

$$\vec{v}_1 = v_1 \cos\theta\,\hat{i} + v_1 \sin\theta\,\hat{j}$$

If the magnitude of the velocity of the six ball after the collision is v_2, we note that it is moving towards positive x and negative y, and its velocity may be expressed as

$$\vec{v}_2 = v_2 \cos\Phi\,\hat{i} - v_2 \sin\Phi\,\hat{j}$$

Noting that the initial momentum of the six ball is zero, conservation of momentum may be expressed as

$$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2$$

or

$$mv_0\,\hat{i} = m\left(v_1 \cos\theta\,\hat{i} + v_1 \sin\theta\,\hat{j}\right) + m\left(v_2 \cos\Phi\,\hat{i} - v_2 \sin\Phi\,\hat{j}\right)$$

We may divide through by the mass. We obtain one equation from the coefficients of $\hat{i}$, the x components, and we obtain a second equation from the coefficients of $\hat{j}$, the y components.

$$\begin{aligned} v_0 &= v_1 \cos\theta + v_2 \cos\Phi \\ 0 &= v_1 \sin\theta - v_2 \sin\Phi \end{aligned}$$

Solving the second equation as for the speed of the eight ball,

$$v_1 = \frac{\sin\Phi}{\sin\theta} v_2$$

This expression may be substituted into the first of the equations to produce

$$\begin{aligned} v_0 &= \left(\frac{\sin\Phi}{\sin\theta} v_2\right)\cos\theta + v_2\cos\Phi \\ &= v_2\left(\frac{\sin\Phi}{\tan\theta} + \cos\Phi\right) \end{aligned}$$

The template uses v instead of v_0,

$$\boxed{v_2 = \frac{v}{\cos\Phi + \sin\Phi/\tan\theta}}$$

Part (b) **What is the magnitude, in meters per second, of the velocity of the six ball?**

Numerically evaluate the expression from the previous part.

$$\begin{aligned} v_2 &= \frac{v}{\cos\Phi + \sin\Phi/\tan\theta} \\ &= \frac{2.01\ \text{m/s}}{\cos\left(10.3^\circ\right) + \sin(10.3^\circ)/\tan(15.2^\circ)} \end{aligned}$$

$$\boxed{v_2 = 1.224\ \text{m/s}}$$

Part (c) **What is the magnitude, in meters per second, of the velocity of the eight ball after the collision?**

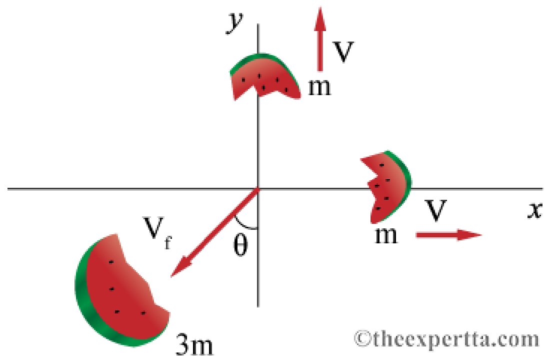
As a part of the solution to Part (a), we found an expression for v_1, and we may now back-substitute the value of v_2.

$$v_1 = \frac{\sin \Phi}{\sin \theta} v_2 = \frac{\sin(10.3^{\circ})}{\sin(15.2^{\circ})}(1.224 \text{ m/s})$$

$$\boxed{v_1 = 0.8348 \text{ m/s}}$$

Problem 11:

A watermelon is blown into three pieces by a large firecracker. Two pieces of equal mass *m* fly away perpendicular to one another, one in the x direction another in the y direction. Both of these pieces fly away with a speed of $V = 21$ m/s. The third piece has three times the mass of the other two pieces.



Randomized Variables

$V = 21$ m/s

Part (a) **Write an expression for the speed of the larger piece, that is in terms of only the variable *V*.**

$$V = 21 \frac{\text{m}}{\text{s}}$$

Write an expression for the speed of the larger watermelon piece.

Begin with conservation of momentum. We can assume that the watermelon begins at rest, so it's initial momentum is zero.

$$\mathbf{p}_i = \mathbf{p}_f$$

$$0 = \mathbf{p}_f$$

This is a vector equation. Break this into two component equations for the x and y directions.

$$0 = p_{fx}$$

$$0 = p_{fy}$$

Let's first concentrate on the x direction. The total momentum in the x direction is the sum of the individual momenta.

$$0 = p_{fx} = \Sigma m v_{fx}$$

$$0 = \Sigma m v_{fx}$$

The momentum components in the x direction consist of one of the small pieces of velocity V and a component of the momentum of the large piece.

$$0 = \Sigma m v_{fx}$$

$$0 = mV + (3m) v_{fx}$$

Solve for the component of velocity v_{fx}.

$$0 = mV + (3m) v_{fx}$$

$$-(3m)\,v_{fx} = mV$$

$$v_{fx} = \frac{mV}{-(3m)}$$

$$v_{fx} = -\frac{V}{3}$$

Now for the y direction. The work will be identical.

The total momentum in the y direction is the sum of the individual momenta.

$$0 = p_{fy} = \Sigma m v_{fy}$$

$$0 = \Sigma m v_{fy}$$

The momentum components in the y direction consist of one of the small pieces of velocity V and a component of the momentum of the large piece.

$$0 = \Sigma m v_{fy}$$

$$0 = mV + (3m)\,v_{fy}$$

Solve for the component of velocity v_{fy}.

$$0 = mV + (3m)\,v_{fy}$$

$$-(3m)\,v_{fy} = mV$$

$$v_{fy} = \frac{mV}{-(3m)}$$

$$v_{fy} = -\frac{V}{3}$$

You now have both components of the velocity of the large piece. Find the magnitude of the velocity.

$$v_{fx} = -\frac{V}{3}$$

$$v_{fy} = -\frac{V}{3}$$

$$|v_f| = \sqrt{v_{fx}^2 + v_{fx}^2}$$

$$|v_f| = \sqrt{\left(-\frac{V}{3}\right)^2 + \left(-\frac{V}{3}\right)^2}$$

$$|v_f| = \sqrt{\frac{V^2}{9} + \frac{V^2}{9}}$$

$$|v_f| = \sqrt{\frac{2V^2}{9}}$$

$$|v_f| = \sqrt{\frac{2}{9}}V$$

$$\boxed{|v_f| = \frac{\sqrt{2}}{3}V}$$

Part (b) **What is the numeric value for the speed of the larger piece, in meters per second?**

$$V = 21\frac{m}{s}$$

What is the value of the speed of the larger watermelon piece?

In part a, you derived an expression for the speed. Now plug in numbers.

$$|v_f| = \frac{\sqrt{2}}{3}V$$

$$|v_f| = \frac{\sqrt{2}}{3}\left(\frac{21m}{s}\right)$$

$$|v_f| = \frac{1.414}{3}\left(21\frac{m}{s}\right)$$

$$|v_f| = 0.4714\left(21\frac{m}{s}\right)$$

$$\boxed{|v_f| = 9.899\frac{m}{s}}$$

Part (c) **At what angle does the largest piece travel with respect to the -y axis, in degrees?**

$$V = 21\frac{m}{s}$$

What is the direction of the velocity for the large piece with respect to the -y axis?

From the symmetry of the problem, it is apparent that $\theta = 45°$. The following steps will show this.

$$v_{fx} = -\frac{V}{3}$$

$$v_{fy} = -\frac{V}{3}$$

$$\tan(\theta) = \frac{v_{fx}}{v_{fy}}$$

$$\tan(\theta) = \frac{\left(-\frac{V}{3}\right)}{\left(-\frac{V}{3}\right)}$$

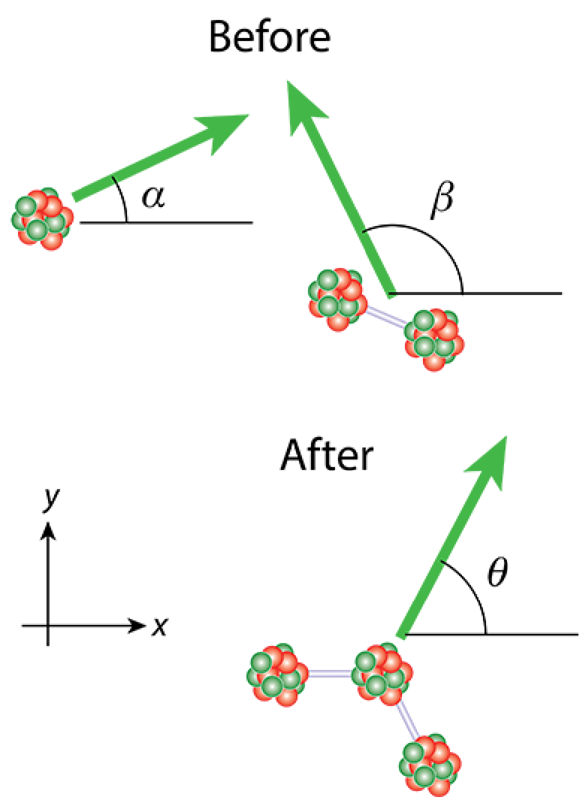
$$\tan(\theta) = 1$$

$\theta = \arctan(1)$

$\boxed{\theta = 45°}$

Problem 12:

An oxygen atom (mass 16 u) is moving with speed $v_1 = 1010$ m/s at an angle of $\alpha = 15°$. An oxygen molecule (mass 32 u) is moving with speed $v_2 = 510$ m/s at an angle of $\beta = 101°$. A collision forms an ozone molecule (mass 48 u) which moves with velocity $\vec{v}$ at angle θ. All angles are measured from positive x, as shown in the diagram.



Part (a) **Write an expression for the x component of the velocity of the ozone molecule.**

The linear momentum $\vec{p}$ of an object with mass m and velocity $\vec{v}$ is $\vec{p} = m\vec{v}$. If m is the mass of the oxygen atom, then 2m is the mass of the oxygen molecule, and 3m is the mass of the ozone molecule. The initial momentum is

$$\vec{p}_i = (mv_1 \cos\alpha + 2mv_2 \cos\beta)\,\hat{i} + (mv_1 \sin\alpha + 2mv_2 \sin\beta)\,\hat{j}$$

If v_x and v_y are the components of the final velocity of the system, then the final momentum is

$$\vec{p}_f = (3mv_x)\,\hat{i} + (3mv_y)\,\hat{j}$$

Conservation of the x component of momentum reads as follows:

$$mv_1 \cos\alpha + 2mv_2 \cos\beta = 3mv_x$$

Solving for v_x,

$$\boxed{v_x = \frac{v_1 \cos\alpha + 2v_2 \cos\beta}{3}}$$

Part (b) **Write an expression for the y component of the velocity of the ozone molecule.**

Conservation of the y component of momentum reads as follows:

$$mv_1 \sin\alpha + 2mv_2 \sin\beta = 3mv_y$$

Solving for v_y,

$$\boxed{v_y = \frac{v_1 \sin\alpha + 2v_2 \sin\beta}{3}}$$

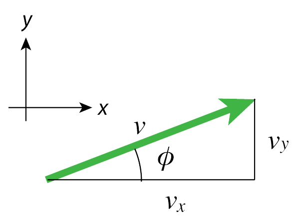
Part (c) **What is the speed, in meters per second, of the ozone molecule?**

Calculating v_x and v_y,

$$v_x = \frac{(1010 \text{ m/s}) \cos(15^\circ) + 2\,(510 \text{ m/s}) \cos(101^\circ)}{3} = 260.3 \text{ m/s}$$

$$v_y = \frac{(1010 \text{ m/s}) \sin(15^\circ) + 2\,(510 \text{ m/s}) \sin(101^\circ)}{3} = 420.9 \text{ m/s}$$

The diagram shows the right triangle that relates the components of the velocity to its magnitude.



Applying the Pythagorean theorem to the components:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(260.3 \text{ m/s})^2 + (420.9 \text{ m/s})^2}$$

Calculating,

$$\boxed{v = 494.9 \text{ m/s}}$$

Part (d) **What is the angle, in degrees, between the velocity vector of the ozone molecule and the positive x axis?**

The angle is given by

$$\theta = \tan^{-1}\left|\frac{v_y}{v_x}\right| = \tan^{-1}\left|\frac{420.9 \text{ m/s}}{260.3 \text{ m/s}}\right|$$

Calculating,

$$\boxed{\theta = 58.26^\circ}$$

Problem 13:

Rockets achieve high speeds by using the momentum from their exhaust. The exhaust is a small amount of mass moving backwards at some velocity -**v**, so the rocket gains forward velocity to conserve momentum.

Part (a) **Can the speed of the rocket exceed the exhaust velocity of the fuel?**

By momentum conservation the rocket gains speed whenever it expels exhaust backwards. Note that the exhaust velocity is relative to the rocket, while the rocket's velocity is relative to, say, Earth.

Let's consider the case when the rocket is moving at a speed equal to its exhaust speed and it expels additional exhaust. (The exhaust will appear stationary to an observer on Earth, but that makes no difference to our considerations.) As a result, the rocket will gain additional speed, thus exceeding its exhaust speed. The correct choice is "Yes. The rocket continually gains speed as long as the fuel is available."

Problem 14:

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available.

Part (a) **What is the takeoff acceleration, in meters per square second, of a *12000*-kg ABM that expels 196 kg of gas per second at an exhaust speed of *2.25* × 10^3 m/s?**

To solve this problem, we need to use the fact that the force of thrust on the rocket is equal to the velocity with which the propellant is expelled multiplied by the rate at which it is expelled. From there, we can use Newton's Second Law to solve for the acceleration due to thrust.

$$F_t = vR$$

$$ma_t = vR$$

$$a_t = \frac{vR}{m}$$

To get the net acceleration of the rocket, we just need to subtract the acceleration due to gravity from the acceleration due to thrust.

$$a = a_t - g$$

$$a = \frac{vR}{m} - g$$

$$a = \frac{2.25 \cdot 10^3 \text{ m/s} \cdot 196 \text{ kg/s}}{12000 \text{ kg}} - 9.8 \text{ m/s}^2$$

$$\boxed{a = 26.95 \text{ m/s}^2}$$

Problem 15:

Suppose a space probe of mass m_1 = *3750* kg expels m_2 = *3250* kg of its mass at a constant rate with an exhaust speed of v_{ex} = *1.75* × 10^3 m/s.

Part (a) **What is the increase in speed, of the space probe, in terms of the variables given in the introduction? You may assume the gravitational force is negligible at the probe's location.**

To find the increase in speed of the probe, we can multiply the velocity with which propellant is expelled by the natural log of the initial mass of the probe divided by its final mass. In this case, we don't know the final mass of the probe directly, but we can find it by subtracting the mass of propellant used from the initial mass of the probe.

$$\Delta v = v_{ex} \ln\left(\frac{m_I}{m_f}\right)$$

$$\Delta v = v_{ex} \ln\left(\frac{m_1}{m_2 - m_2}\right)$$

Part (b) **Calculate the increase in speed, in meters per second, of the space probe.**

Here, we simply need to solve the equation that we found in part (a) to get the answer.

$$\Delta v = v_{ex} \ln\left(\frac{m_1}{m_1 - m_2}\right)$$

$$\Delta v = 1.75 \cdot 10^3 \text{ m/s} \cdot \ln\left(\frac{3750 \text{ kg}}{3750 \text{ kg} - 3250 \text{ kg}}\right)$$

$$\Delta v = 3526 \text{ m/s}$$

Problem 16:
Full solution not currently available at this time.

A rocket fires its thrusters in deep space. From rest, it reaches a speed of 50.1 m/s in 7.01 s. The speed of the exhaust, relative to the rocket, is 1200 m/s and the mass of fuel burned is 101 kg.

Part (a) **What was the initial mass, in kilograms, of the rocket?**

m_0 **= dm*exp(v/u)/(exp(v/u)-1)**
m_0 **= 101*exp(50.1/1200)/(exp(50.1/1200)-1)**
m_0 **=** 2470.
Tolerance: ± 74