Begin Date: 11/7/2024 12:01:00 AM -- **Due Date:** 11/15/2024 11:59:00 PM **End Date:** 11/15/2024 11:59:00 PM

Problem 1:

For objects near the surface of the Earth, the universal law of gravitation can be simplified to $F = mg$, where $g = 9.81$ m/s^2.

Part (a) **If the mass of the Earth were doubled while at the same time its radius remained constant, by what factor would this change its acceleration due to gravity at it's surface?**

Expanding the simplified version of Newton's universal law of gravitation

$$F = mg = \frac{GMm}{R^2}$$

solving for g

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass of the earth, and R is its radius since the earth can be treated as a point mass with all its mass at the center, thereby making the separation the radius of the earth. If you double the mass of the earth,

$$g_1 = \frac{G2M}{R^2}$$

$$\frac{g_1}{g} = \frac{\left(\frac{G2M}{R^2}\right)}{\left(\frac{GM}{R^2}\right)} = 2$$

$$\boxed{2}$$

Part (b) **If the radius of the Earth were doubled while at the same time its mass remained constant, by what factor would this change its acceleration due to gravity at the surface?**

Expanding the simplified version of Newton's universal law of gravitation

$$F = mg = \frac{GMm}{R^2}$$

solving for g

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass of the earth, and R is its radius since the earth can be treated as a point mass with all its mass at the center, thereby making the separation the radius of the earth. If you double the radius of the earth,

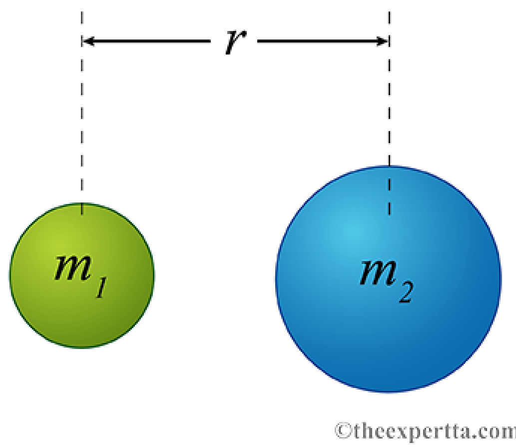
$$g_1 = \frac{GM}{(2R)^2}$$

$$\frac{g_1}{g} = \frac{\left(\frac{G2M}{(2R)^2}\right)}{\left(\frac{GM}{R^2}\right)} = \frac{1}{4}$$

$$\boxed{\frac{1}{4}}$$

Problem 2:

Object 1 with m_1 = *1.5* kg and Object 2 with m_2 = *11* kg are separated by r = *0.14* m.



Part (a) **Express the magnitude of the gravitational force F in terms of m_1, m_2, r, and the gravitational constant G.**

Newton's law of gravity gives the magnitude of the force.

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

Part (b) **Calculate the magnitude of F in N.**

Using the equation given in part a,

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.6738 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right) \frac{(1.5\ kg)(11\ kg)}{(0.14\ m)^2}$$

$$\boxed{F = 5.618 \times 10^{-8}\ N}$$

Problem 3:

A satellite is located *275* km above Earth.

Part (a) **Find the ratio of the acceleration due to gravity at the position of a satellite located *275* km above the earth's surface to the free-fall acceleration at the surface of Earth.**

The acceleration due to gravity at a given distance r from the center of the Earth can be found from Newton's second law applied to an object of mass m. For an object in free fall,

$$\Sigma F = G\frac{M_E m}{R_E^2} = ma$$

The acceleration due to gravity is then,

$$a \equiv g = G\frac{M_E}{R_E^2}$$

We can then take the ratio of the value of the acceleration due to gravity at the altitude given to that at the surface of the Earth.

$$\frac{g_d}{g} = \frac{\left(G\frac{M_E}{(R_E+d)^2}\right)}{\left(G\frac{M_E}{R_E^2}\right)}$$

$$= \left(\frac{R_E}{(R_E+d)}\right)^2$$

$$= \left(\frac{6400\ km}{(6400+275\ km)}\right)^2$$

$$\boxed{\frac{g_d}{g} = 0.9193}$$

Problem 4:

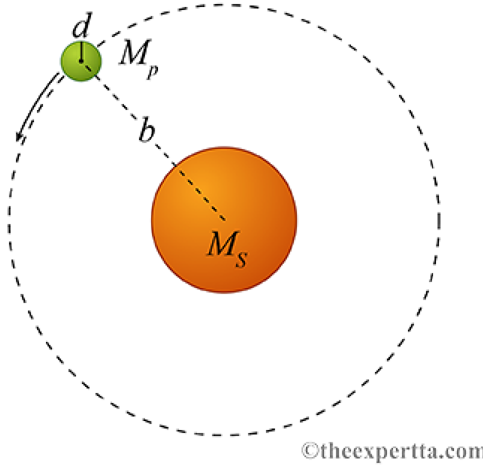
You are a scientist exploring a mysterious planet. You have performed measurements and know the following things:

•The planet has radius d.

•It is orbiting his star in a circular orbit of radius b.

•It takes time T to complete one orbit around the star.

•The free-fall acceleration on the surface of the planet is a.



Part (a) **Derive an expression for the mass M_P of the planet in terms of a, d and G the universal gravitational constant. Assume that the gravitational effect of the star at the planet's surface is negligible.**

The gravitational field of the planet is the acceleration that it gives any object of mass m. The mass of the planet is found by writing Newton's second law for an object near the surface of the planet.

$$F_g = G\frac{M_P m}{d^2} = ma$$

Solving for the mass of the planet gives

$$\boxed{M_P = \frac{ad^2}{G}}$$

Part (b) **Derive an expression for the mass M_S of the star in terms of b, T, and G the universal gravitational constant.**

The mass of the star is found by writing the expression for the gravitational force given by Newton's law of gravity and realizing that it provides the centripetal force that keeps the planet in orbit.

$$F_g = G\frac{M_S M_P}{b^2} = \frac{M_P v^2}{b}$$

Divide both sides by the mass of the planet.

$$G\frac{M_S}{b^2} = \frac{\left(2\pi\frac{b}{T}\right)^2}{b}$$

$$G\frac{M_S}{b} = \left(2\pi\frac{b}{T}\right)^2$$

Solving for the mass of the star gives

$$\boxed{M_S = 4\pi^2\frac{b^3}{\left(GT^2\right)}}$$

Problem 5:

The Sun has a mass of 1.99×10^{30} kg and a radius of 6.96×10^{8} m.

Part (a) **Calculate the acceleration due to gravity, in meters per square second, on the surface of the Sun.**

The acceleration due to gravity for a massive object is given by

$$g = \frac{GM}{R^2}$$

where G is the universal gravitation constant, M is the mass of the object, and R is the radius of the object.

$$g_{Sun} = \frac{GM}{R^2}$$

$$= \frac{\left(6.673\times10^{-11}\ \frac{N\cdot m^2}{kg^2}\right)\left(1.99\times10^{30}\ kg\right)}{\left(6.96\times10^{8}\ m\right)^2}$$

$$\boxed{g_{Sun} = 274.1\frac{m}{s^2}}$$

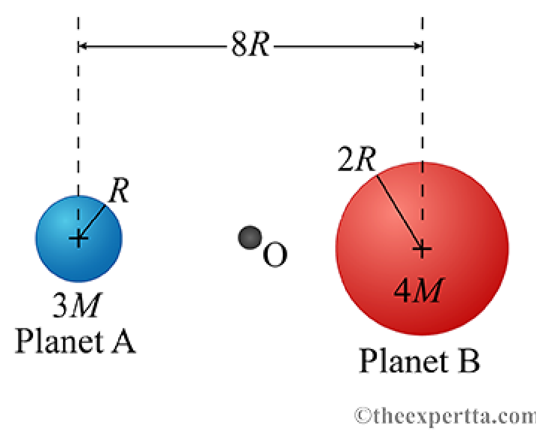
Part (b) **By what factor would your weight increase if you could stand on the Sun?**

$$\frac{g_{Sun}}{g_{Earth}} = \frac{\left(274.1\ \frac{m}{s^2}\right)}{\left(9.80\ \frac{m}{s^2}\right)}$$

$$\boxed{\frac{g_{Sun}}{g_{Earth}} = 28.0}$$

Problem 6:

Planet A has mass $3M$ and radius R, while Planet B has mass $4M$ and radius $2R$. They are separated by center-to-center distance $8R$. A rock is released halfway between the planets' centers at point O. It is released from rest. Ignore any motion of the planets.



Part (a) **Enter an expression for the magnitude of the acceleration of the rock immediately after it is released, in terms of M, R, and the gravitational constant, G.**

Each planet exerts a gravitational force on the rock of mass m. Apply Newton's second law, the force from Planet A is directed toward the left and that from Planet B is directed toward the right.

$$\Sigma F = -\frac{G(3M)m}{(4R)^2} + \frac{G(4M)m}{(4R)^2} = ma$$

Solving for a gives,

$$\boxed{a = \frac{GM}{16R^2}}$$

Part (b) **Calculate the magnitude of the rock's acceleration, in meters per second squared, for $M = 5.1 \times 10^{23}$ kg and $R = 4.1 \times 10^{6}$ m.**

$$a = \frac{GM}{16R^2}$$

$$= \frac{(6.67\times10^{-11}\ \text{N m}^2\text{kg}^2)(5.1\times10^{23}\ \text{kg})}{((4)(4.1\times10^6\ \text{m}))^2}$$

$$\boxed{a = 0.1265\ \text{m/s}^2}$$

Part (c) **Toward which planet is the rock's acceleration directed?**

Since the acceleration found in part b is positive, the rock moves toward the right, which is in the direction of...

Planet B

Problem 7:

Astronomical observations of our Milky Way galaxy indicate that it has a mass of about $8.0 \bullet 10^{11}$ solar masses. A star orbiting on the galaxy's periphery is about $6.0 \bullet 10^{4}$ light years from its center.

Part (a) **What should the orbital period of that star be in years?**

Kepler's third law of plantary motion provides a relationship between the orbital radius and the orbital period of a satellite.

$$\frac{R^3}{T^2} = \frac{GM}{(4\pi^2)}$$

where M is the total mass of the galaxy and R is the radius of the galaxy. Solving for the period, we find

$$T = \sqrt{4\pi^2 \frac{R^3}{GM}}$$

$$= \sqrt{4\pi^2 \frac{\left(6.0\times10^4 \text{ ly}\left(9.461\times10^{15}\frac{\text{m}}{\text{ly}}\right)\right)^3}{\left(\left(6.673\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(8.0\times10^{11})(1.99\times10^{30}\text{kg})\right)}}\left(\frac{1 \text{ y}}{(3.156\times10^7 \text{s})}\right)$$

$$\boxed{T = 2.612\times10^8 \text{ years}}$$

Part (b) **If its period is 6.0 • 10^7 years instead, what is the mass of the galaxy in solar masses? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.**

We start with Kepler's third law of planetary motion again, but now solve for the total mass.

$$\frac{R^3}{T^2} = \frac{GM}{(4\pi^2)}$$

$$M = \frac{(4\pi^2 R^3)}{GT^2}$$

$$= \frac{\dfrac{\left(4\pi^2\left(6.0\times10^4 \text{ ly}\left(9.461\times10^{15}\frac{\text{m}}{\text{ly}}\right)\right)^3\right)}{\left(\left(6.673\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(6.0\times10^7\text{y}\left(3.156\times10^7\frac{\text{s}}{1\text{ y}}\right)\right)^2\right)}}{\left(1.99\times10^{30}\frac{\text{kg}}{\text{solar mass}}\right)}$$

$$\boxed{M = 1.517\times10^{13} \text{ solar masses}}$$

Problem 8:

Jupiter has many moons orbiting it.

Parent	Satellite	Average orbital radius r(km)	Period T(y)	$r^3/T^2(km^3/y^2)$
Earth	Moon	3.84×10^5	0.07481	1.01×10^{18}
Sun	Earth	1.496×10^8	1.000	3.35×10^{24}
	Jupiter	7.783×10^8	11.86	3.35×10^{24}
Jupiter	Io	4.22×10^5	0.00485 (1.77 d)	3.19×10^{21}
	Europa	6.71×10^5	0.00972 (3.55 d)	3.20×10^{21}
	Ganymede	1.07×10^6	0.0196 (7.16 d)	3.19×10^{21}
	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Part (a) **Find the mass of Jupiter, in kg, based on data for the orbit of one of its moons.**

Kepler's third law of plantary motion provides a relationship between the orbital radius and the orbital period of the satellite moon.

$$\frac{R^3}{T^2} = \frac{GM_J}{(4\pi^2)}$$

where M_J is the mass of the planet Jupiter. Solving for the mass and using the data for Io, we find

$$M_J = \frac{4\pi^2}{G}\left(\frac{R^3}{T^2}\right)$$

$$= \frac{4\pi^2}{\left(6.673\times10^{-11}\ \frac{N\cdot m^2}{kg^2}\right)} \frac{\left(4.22\times10^8\ m\right)^3}{\left((0.00485\ y)\left(3.16\times10^7\ \frac{s}{1\ y}\right)\right)^2}$$

$$\boxed{M_J = 1.893\times10^{27}\ kg}$$

Problem 9:

The Earth's orbit around the Sun is very close to circular.

Parent	Satellite	Average orbital radius r(km)	Period T(y)	$r^3/T^2(km^3/y^2)$
Earth	Moon	3.84×10^5	0.07481	1.01×10^{18}
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	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Part (a) **Calculate the mass of the Sun, in kg, based on data for Earth's orbit.**

Using the values given in the table for the Earth's orbit and Kepler's third law of planetary motion, we can find the mass of the Sun.

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Solving for the mass of the Sun gives,

$$M = \frac{(4\pi^2)}{G}\left(\frac{r^3}{T^2}\right)$$

$$= \frac{(4\pi^2)}{\left(6.673\times10^{-11}\ \frac{N\cdot m^2}{kg^2}\right)}\left(3.35\times10^{24}\ \frac{km^3}{y^2}\right)\left(\frac{1\ y}{(3.154\times10^7\ s)}\right)^2\left(\frac{1000\ m}{1\ km}\right)^3$$

$$\boxed{1.98\times10^{30}\ kg}$$

Problem 10:

A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation).

Parent	Satellite	Average orbital radius r(km)	Period T(y)	r^3 / T^2 (km^3 / y^2)
Earth	Moon	3.84×10^5	0.07481	1.01×10^{19}
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	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Part (a) **Calculate the radius of such an orbit based on the data for the moon in the figure in km.**

Using the values given in the table for the Moon, we are given

$$\frac{r^3}{T^2} = 1.01\times10^{19}\ \frac{km^3}{y^2}$$

from which we can find the radius.

$$r = \left(1.01\times10^{19}\ \frac{km^3}{y^2}T^2\right)^{\frac{1}{3}}$$

$$= \left(1.01\times10^{19}\ \frac{km^3}{y^2}\right)\left(\left(1\ d\frac{1\ y}{365\ d}\right)^2\right)^{\frac{1}{3}}$$

$$\boxed{4.23\times10^4\ km}$$

Problem 11:

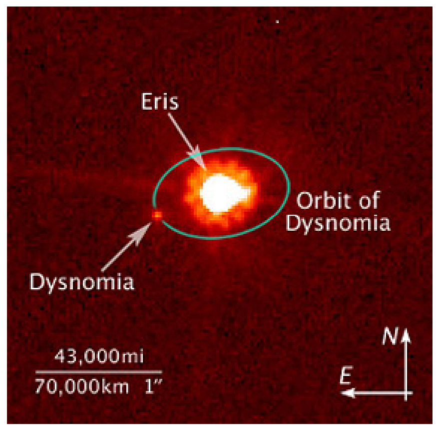
Great Problems in Physics Series

Dr. Kathleen Blackett earned her B.S. in physics from the University of Michigan-Dearborn, and her Ph.D. in experimental particle physics from the University of Tennessee. Kathy's career includes participating in searches for mesons with exotic quantum numbers as a postdoc at Brookhaven National Laboratory, working as systems/test engineer for AT&T, and taking some time to raise her family. In addition, Kathy taught physics and math for several years at the University of South Florida, St. Petersburg College and Eckerd College. Currently Kathy is a content author on the Expert TA team and volunteers for NASA as a Solar System Ambassador.

The following problem, while not especially complicated, is powerful. Like most individuals of my generation, I grew up believing that there were nine planets in our solar system. Period. This problem has convinced me otherwise. Sorry Pluto! You will always be special to me!

Pluto was discovered in 1930, and was unquestionably considered a planet until the discoveries of many other trans-Neptunian objects (TNOs) in the 1990's and early 2000's. As more TNOs were being discovered, it was looking like Pluto had much more in common with these objects than the other eight planets. In 2006, it was decided by the International Astronomical Union (IAU) that Pluto be reclassified as a dwarf planet.

One of these trans-Neptunian objects is the dwarf planet, Eris, discovered by the team of Michael Brown, David Rabinowitz and Chad Trujillo in 2005. Later that same year, the team discovered Dysnomia, a moon of Eris. Eris and Dysnomia are pictured to the right. The discovery of Dysnomia was fortunate, because it allowed scientists to calculate the mass of Eris. After careful observation, it was determined that Dysnomia has an orbit that is approximately circular with a radius of about 37,350 km and a period of about 15.79 days.



(Credit: NASA, ESA, and M. Brown (California Institute of Technology))

Part (a) **Determine the mass, in kilograms, of Eris.**

Use the equation for the period of a satellite around the Earth, but now M_E represents the mass of Eris.

$$T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM_E}}$$

Before we apply the equation, any parameters that are not in SI units must be converted.

Converting the orbital period of Dysnomia to units of seconds:

$$T = (15.79\ \cancel{d})\left(\frac{24\ \cancel{h}}{\cancel{d}}\right)\left(\frac{60\ \cancel{min}}{\cancel{h}}\right)\left(\frac{60\ s}{\cancel{min}}\right)$$

$$= 1.364 \times 10^6\ s$$

Converting the radius of the orbit to units of meters:

$$r = (37350\ \cancel{km})\left(\frac{1000\ m}{\cancel{km}}\right)$$

$$= 3.735 \times 10^7\ m$$

Back to our equation:

$$T = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM_E}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_E}$$

$$M_E = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 (3.735 \times 10^7\ \text{m})^3}{(6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.364 \times 10^6\ \text{s})^2}$$

$$= 1.657 \times 10^{22}\ \text{kg}$$

$$\boxed{M_E = 1.657 \times 10^{22}\ \text{kg}}$$

Part (b) **Pluto has a mass of 1.309×10^{22} kg. Which is more massive, Pluto or Eris?**

Compare the mass of Eris to the mass of Pluto.

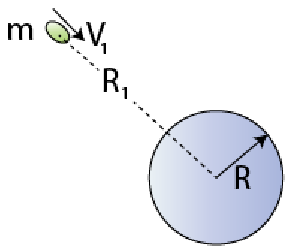
$$1.657 \times 10^{22}\ \text{kg} > 1.309 \times 10^{22}\ \text{kg}$$

The mass of Eris is greater than the mass of Pluto by about 27%. Therefore, the more massive TNO is

$\boxed{\text{Eris}}$

Problem 12:

A meteoroid is moving towards a planet. It has mass $m = 0.14 \times 10^9$ kg and speed $v_1 = 0.8 \times 10^7$ m/s at distance $R_1 = 1.1 \times 10^7$ m from the center of the planet. The radius of the planet is $R = 0.14 \times 10^7$ m. The mass of the planet is $M = 2.4 \times 10^{25}$ kg. There is no air around the planet.



Part (a) **Enter an expression for the gravitational potential energy PE_1 of the meteoroid at R_1 in terms of defined quantities and the gravitational constant G. Assume the potential energy is zero at infinity.**

The gravitational force of the Earth does work on the meteoroid to change its potential energy. The potential energy is assumed to be zero at an infinite distance from the planet, so at a distance R_1 the potential energy is negative at any finite distance from the planet.

$$\boxed{PE_1 = -G\frac{mM}{R_1}}$$

Part (b) **Calculate the value of PE_1, in joules.**

$$PE_1 = G\frac{mM}{R_1^2}$$

$$= \left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \frac{(0.14\times10^9\ \text{kg})(2.4\times10^{25}\ \text{kg})}{(1.1\times10^7\ \text{m})}$$

$$\boxed{PE_1 = -2.038 \times 10^{16}\ \text{J}}$$

Part (c) **Enter an expression for the total energy E_1 of the meteoroid at R_1 in terms of defined quantities.**

The total energy of the meteoroid is the sum of its kinetic and potential energies.

$$\boxed{E_1 = KE_1 + PE_1 = \tfrac{1}{2}mv_1^2 - G\tfrac{mM}{R_1}}$$

Part (d) **Calculate the value of E_1, in joules.**

$$
\begin{aligned}
E_1 &= KE_1 + PE_1 \\
&= \tfrac{1}{2}mv_1^2 + G\tfrac{mM}{R_1^2} \\
&= \tfrac{1}{2}\left(0.14\times10^9\text{ kg}\right)\left(0.8\times10^7\,\tfrac{\text{m}}{\text{s}}\right)^2 + \left(6.673\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(0.14\times10^9\text{ kg}\right)\left(2.4\times10^{25}\text{ kg}\right)}{\left(1.1\times10^7\text{ m}\right)^2}
\end{aligned}
$$

$$\boxed{E_1 = 4.480\times10^{21}\text{ J}}$$

Part (e) **Enter an expression for the total energy E of the meteoroid at R, the surface of the planet, in terms of defined quantities and v, the meteoroid's speed when it reaches the planet's surface.**

The total energy at a given location from the planet is the sum of its kinetic and potential energies.

$$\boxed{E = \frac{1}{2}mv^2 - G\frac{mM}{R}}$$

Part (f) **Enter an expression for v, the meteoroid's speed at the planet's surface, in terms of G, M, v_1, R_1, and R.**

In part c, the total energy of the meteoroid was found at a particular location. Assuming that there are no non-conservative forces acting, the total energy of the meteoroid remains constant. The potential energy decreases as the kinetic energy increases. Therefore,

$$E = E_1$$

$$\tfrac{1}{2}mv^2 - G\tfrac{mM}{R} = \tfrac{1}{2}mv_1^2 - G\tfrac{mM}{R_1}$$

Solving for v gives

$$\boxed{v = \sqrt{v_1^2 - 2G\tfrac{M}{R_1} + 2G\tfrac{M}{R}}}$$

Part (g) **Calculate the value of v in meters per second.**

$$
\begin{aligned}
v &= \sqrt{v_1^2 - 2G\tfrac{M}{R_1} + 2G\tfrac{M}{R}} \\
&= \sqrt{v_1^2 - 2GM\left(\tfrac{1}{R_1} - \tfrac{1}{R}\right)} \\
&= \sqrt{\left(0.8\times10^7\text{ m/s}\right)^2 - 2\left(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.4\times10^{25}\text{ kg}\right)\left(\frac{1}{\left(1.1\times10^7\text{ m}\right)} - \frac{1}{\left(0.14\times10^7\text{ m}\right)}\right)}
\end{aligned}
$$

$$\boxed{v = 8.000\times10^6\text{ m/s}}$$

Problem 13:

Consider a spherical planet of radius *R* and mass *M*. The planet has uniform density.

Part (a) **Someone has drilled a hole straight through the center of this planet to the other side and is about to drop a small object of mass *m* into the hole. We can show that the object will experience simple harmonic motion in the hole by showing that the gravitational force on the object will obey Hooke's law, $F_{grav} = -kx$, where *k* is the force constant and *x* denotes the displacement from equilibrium position, which is the planet's center. Enter an expression for *k*, in terms of *R*, *M*, *m*, and the gravitational constant, *G*.**

The gravitational force experienced between twospherical/point mass bodies in general is

$$F_g = -\frac{GM(x)\,m}{x^2}\text{ N}$$

where G is the gravitational constant in $N(m/kg)^2$, M(x) is the mass interior to radius x in kg, m is the mass of the other object in kg, and x is the separation between the two masses' centers. Assuming the planet has a constant density, the mass interior to a radius x is

$$M(x) = \frac{4}{3}\pi x^3\varrho\text{ kg}$$

where

$$\varrho = \frac{M}{\left(\frac{4}{3}\pi R^3\right)}\text{ kg/m}^3$$

where M is the mass of the planet in kg and R is the radius of the planet in m. Substituting in,

$$F_g = -\frac{G\left(\frac{4}{3}\pi x^3 \varrho\right) m}{x^2} = -\frac{4}{3}\pi G \varrho m x = -\frac{4}{3}\pi G \frac{M}{\left(\frac{4}{3}\pi R^3\right)} m x = -\frac{GMm}{R^3} x = -kx$$

$$k = \left(G\ \mathrm{N/kg^2\,m^2}\right) \cdot (M\ \mathrm{kg}) \cdot \frac{(m\ \mathrm{kg})}{(R\ \mathrm{m})^3}$$

$$\boxed{k = \frac{GMm}{R^3}\ \mathrm{N/m}}$$

Part (b) **When the object is dropped into the hole, what will be its period of oscillation, in seconds, if R = 4×10^6 m and M = 6×10^{24} kg?**

The period of oscillation is related to the angular frequency by the relation

$$T = \frac{2\pi}{\omega}\ \mathrm{s}$$

where

$$\omega = \left(\frac{k}{m}\right)^{0.5}\ \mathrm{rad/s}$$

where k is the force constant in N/m and m is the mass in kg. Substituting in for the force constant,

$$T = \frac{2\pi}{\left(\frac{\frac{GMm}{R^3}}{m}\right)^{0.5}} = \frac{2\pi}{\left(\frac{GM}{R^3}\right)^{0.5}} = 2\pi \cdot \left(\frac{R^3}{GM}\right)^{0.5}$$

Plugging in numbers and converting units as needed,

$$T = 2 \cdot \pi \cdot \left(\frac{\left(4 \cdot 10^6\ \mathrm{m}\right)^3}{\left(6.67 \cdot 10^{-11}\ \mathrm{N/kg^2 m^2} \cdot 6 \cdot 10^{24}\ \mathrm{kg}\right)}\right)^{0.5}$$

$$\boxed{T = 2513\ \mathrm{s}}$$

Part (c) **What would be the period of oscillation, in seconds, if the planet were Earth? The radius of Earth is 6.38×10^6 m and its mass is 5.9×10^{24} kg. Assume the mass is distributed uniformly.**

The period of oscillation is related to the angular frequency by the relation

$$T = \frac{2\pi}{\omega}\ \mathrm{s}$$

where

$$\omega = \left(\frac{k}{m}\right)^{0.5}\ \mathrm{rad/s}$$

where k is the force constant in N/m and m is the mass in kg. Substituting in for the force constant,

$$T = \frac{2\pi}{\left(\frac{\frac{GMm}{R^3}}{m}\right)^{0.5}} = \frac{2\pi}{\left(\frac{GM}{R^3}\right)^{0.5}} = 2\pi \cdot \left(\frac{R^3}{GM}\right)^{0.5}$$

Plugging in numbers for the earth and converting units as needed,

$$T = 2 \cdot \pi \cdot \left(\frac{(6.38 \cdot 10^6 \text{ m})^3}{(6.67 \cdot 10^{-11} \text{ N/kg}^2\text{m}^2 \cdot 5.9 \cdot 10^{24} \text{ kg})} \right)^{0.5}$$

$$\boxed{T = 5104 \text{ s}}$$

Problem 14:

In this problem you will answer a few questions about "escape speed".

Part (a) **True or false: Planets with low escape speeds, such as Mercury, generally don't have atmospheres because the average speed of gas molecules is close to the escape speed.**

Atmospheric gases can have speeds that are dependent on the Kelvin temperature and the mass of the molecules. Molecules that are lighter move faster than more massive ones, and of course, the higher the temperature the greater the speed.

The escape velocity is determined by the mass of the planet and the distance the molecule is from the center of the planet, which can be approximated by the radius of the planet. The radius of Mercury is about 38 % that of Earth, and its mass is only about 6 % that of Earth. So, the escape velocity for Mercury is about one-third that of Earth. So, the speed of gas molecules would easily exceed the escape velocity.

True

Part (b) **True or false: A spacecraft would have the same escape speed as a gas molecule.**

The escape velocity is determined by the mass of the planet and the distance the object is from the center of the planet, which can be approximated by the radius of the planet. It is not dependent on the mass of the object. Therefore, all particles or objects have the same escape speed.

True

Part (c) **Which of the following best describes escape speed?**

The escape velocity is derived by setting the kinetic energy of a particle equal to its gravitational potential energy, assuming no frictional forces are acting on the particle. The potential energy is chosen to be zero at infinity. At infinity, the kinetic energy of the particle is also zero.

The speed reached of an object falling from infinity to the Earth, but in the opposite direction.

Problem 15:

An object of mass m is launched from a planet of mass M and radius R.

Part (a) **Derive and enter an expression for the minimum launch speed needed for the object to escape gravity, *i.e.* to be able to just reach $r = \infty$.**

This is a conservation of energy problem. Because the object will not remain near the surface of the planet, it is essential to use the the full expression for the gravitational potential energy,

$$U_G = -G\frac{Mm}{r}$$

where the radial distance is measured from the center of the spherically symmetric planet. Note that

$$\lim_{r \to \infty} U_G = 0$$

The escape velocity has the minimum magnitude such that an object leaves the surface and does not return, so the kinetic energy decreases to zero as the radius approaches infinity. Hence, the total mechanical energy is zero.

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0$$

The mass of the object, m, cancels. If the object begins with the escape velocity at the surface of the planet, then,

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0$$

or

$$\frac{1}{2}v^2 = G\frac{M}{R}$$

and

$$\boxed{v = \sqrt{\frac{2GM}{R}}}$$

Part (b) **Calculate this minimum launch speed (called the *escape speed*), in meters per second, for a planet of mass** $M = 1.03 \times 10^{22}$ kg **and** $R = 75.2 \times 10^3$ km**.**

Substitute the numeric values into the expression obtained in Part (a) noting that the radius was given in kilometers.

$$v = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2\left(6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(1.03 \times 10^{22}\ \text{kg}\right)}{75.2 \times 10^{3+3}\ \text{m}}}$$

$$\boxed{v = 135.2\ \text{m/s}}$$

Problem 16:

Full solution not currently available at this time.

A *4000*-kg spaceship is in a circular orbit *160* km above the surface of Earth. It needs to be moved into a higher circular orbit of *360* km to link up with the space station at that altitude. In this problem you can take the mass of the Earth to be 5.97×10^{24} kg.

Part (a) **How much work, in joules, do the spaceship's engines have to perform to move to the higher orbit? Ignore any change of mass due to fuel consumption.**

***W* = (6.67*10^-11)*(5.97*10^24)*m/2*(1/((6378+hi)*1000)-1/((6378+hf)*1000))**
***W* = (6.67*10^-11)*(5.97*10^24)*4000/2*(1/((6378+160)*1000)-1/((6378+360)*1000))**
***W* =** 3.616×10^9
Tolerance: ± 1.1×10^8

Problem 17:

When exploring a small planet with a radius of 394 km and a mass of 1.23×10^{19} kg, a projectile with a mass of 2.12 kg is launched from its surface at escape velocity. Unfortunately, the projectile collides with the spacecraft that is orbiting at an altitude of 1005 km above the surface.

Part (a) **With what speed, in meters per second, does the projectile strike the spacecraft?**

This is a conservation of energy problem. Because the object will not remain near the surface of the small planet, it is essential to use the full expression for the gravitational potential energy that is derived from the universal law of gravitation,

$$U_G = -G\frac{Mm}{r}$$

where the radial distance is measured from the center of the spherically symmetric small planet. (Spherical symmetry is generally only an approximation for real heavenly bodies.) M is the mass of the small planet, and m is the mass of the projectile. Note that

$$\lim_{r\to\infty} U_G = 0$$

The escape velocity has the minimum magnitude such that an object leaves the surface and does not return, so the kinetic energy decreases to zero as the radius approaches infinity. Hence, the total mechanical energy is zero.

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0$$

The mass of the object, m, cancels.

$$\frac{1}{2}v^2 - G\frac{M}{r} = 0$$

It's not actually necessary to calculate the escape velocity from the surface. The total mechanical energy is zero at all points after the launch including at the instant of impact. The velocity at impact is equal to the escape velocity from that orbital position. Hence, if R is the radius of the small planet, and h is the altitude of the spacecraft, then

$$\frac{1}{2}v^2 - G\frac{M}{R+h} = 0$$

or

$$\begin{aligned}\frac{1}{2}v^2 &= G\frac{M}{R+h}\\ v &= \sqrt{\frac{2GM}{R+h}}\\ &= \sqrt{\frac{2\left(6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.23\times10^{19}\ \text{kg}\right)}{(394+1005)\times10^3\ \text{m}}}\end{aligned}$$

Note that kilometers have been converted into meters.

$$\boxed{v = 34.26\ \text{m/s}}$$