SQUARECAP

Consider the Following Parable:

After the great fire of 1666 that leveled London, the world-famous architect (and physicist!), Christopher Wren, was commissioned to rebuild St Paul's Cathedral. One day in 1671, he observed **three bricklayers** on a scaffold. To each bricklayer, he asked the question, "What are you doing?"

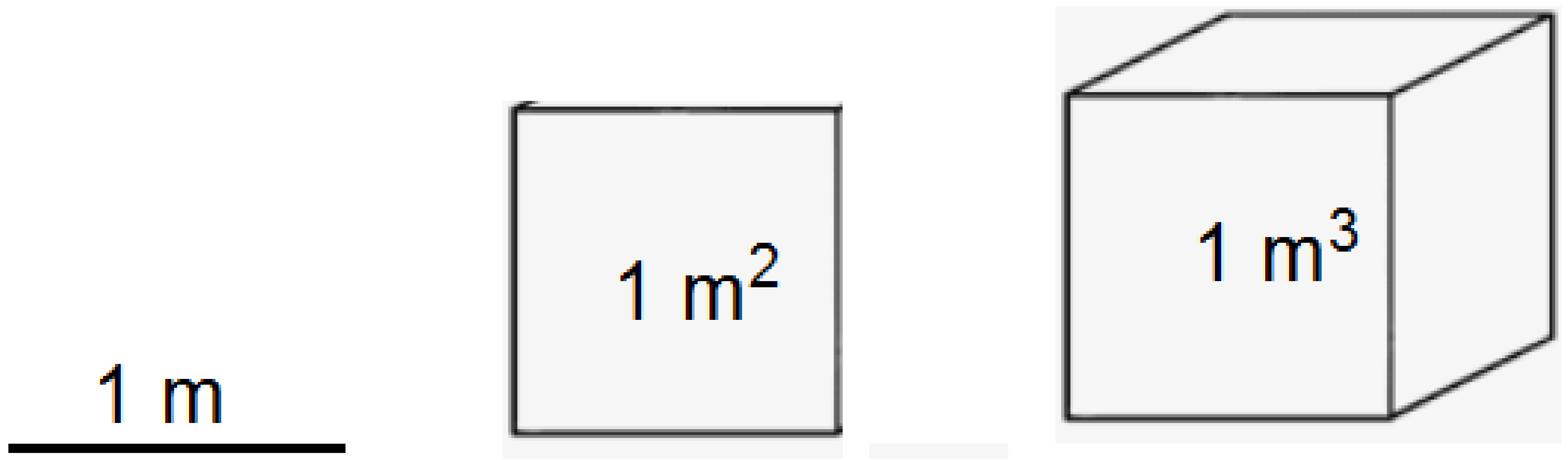
1. **The first** bricklayer replied, "I'm working hard laying bricks to feed my family."
2. **The second** bricklayer responded, "I'm a builder by trade, I'm currently building a wall."
3. **The third** brick layer replied "I'm building a great cathedral to The Almighty."

Purpose: It's worth considering why we're doing something. How does our approach align with out motivations? How do our motivations align with our approach. What is our sense of purpose?

Follow Up: We'll revisit this question towards the end of the semester and see if we feel the same way.

Recap + Elaboration: Units

- Fundamental Units: Mass (kg), Length (m), Time (s)
- Some derived units: Area (m^2), Volume (m^3)
- Some derived units: Speed (m/s), Density (kg/m^3), Flow Rate (kg/s)



Example: Density of Water

The original definition of 1 gram was the mass of 1 milliliter (=1cm^3) of water. Given this, what's the density of water in SI units?

Compare:

Air at sea level is $\sim$ 1

Checking Units!

- Suppose you are solving to find a distance, *given only a speed and time*, and you work out a formula for the answer:

$$d = vt^2$$

[L]=[L]/[T]*[T]*[T]=[L][T]

- This can’t be correct! Must’ve made a mistake!
- In fact, you can sometimes “guess” the answer up to a numerical factor.

$$d = cvt$$

(c a dimensionless constant! (just a number))

- The mathematically exact statement is the ***Buckingham π Theorem***

Concept Check: The units for force, as we'll learn, are [M][L]/[T][T]. Given this, which of the following could be an equation for force?

$A) F = mvr$

$B) F = m\frac{v^2}{r^2}$

$C) F = m\frac{v^2}{r}$

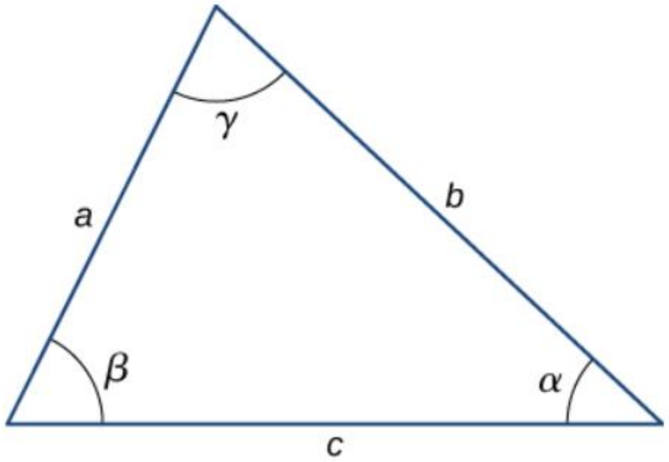
D) None of these

GIVEN THAT: m is a mass, v is a speed, and r is a radius!

Prelecture Review: The Geometry of Space

Triangles

1. Law of sines: $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$
2. Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \gamma$



Geometry

$c^2 = a^2 + b^2 - 2ab\cos\theta$ $\quad A = \frac{1}{2} lh$

$A = (\alpha + \beta + \gamma - \pi)R^2$

$\sin\theta = \frac{o}{h}$ $\quad \cos\theta = \frac{a}{h}$ $\quad \tan\theta = \frac{o}{a}$

$\sin\theta^2 + \cos\theta^2 = 1$

$C = 2\,\pi r$ $\quad A = \pi r^2$

$A = 4\,\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad y = ax^2 + bx + c$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

What are our questions and/or comments?

Comments and Questions

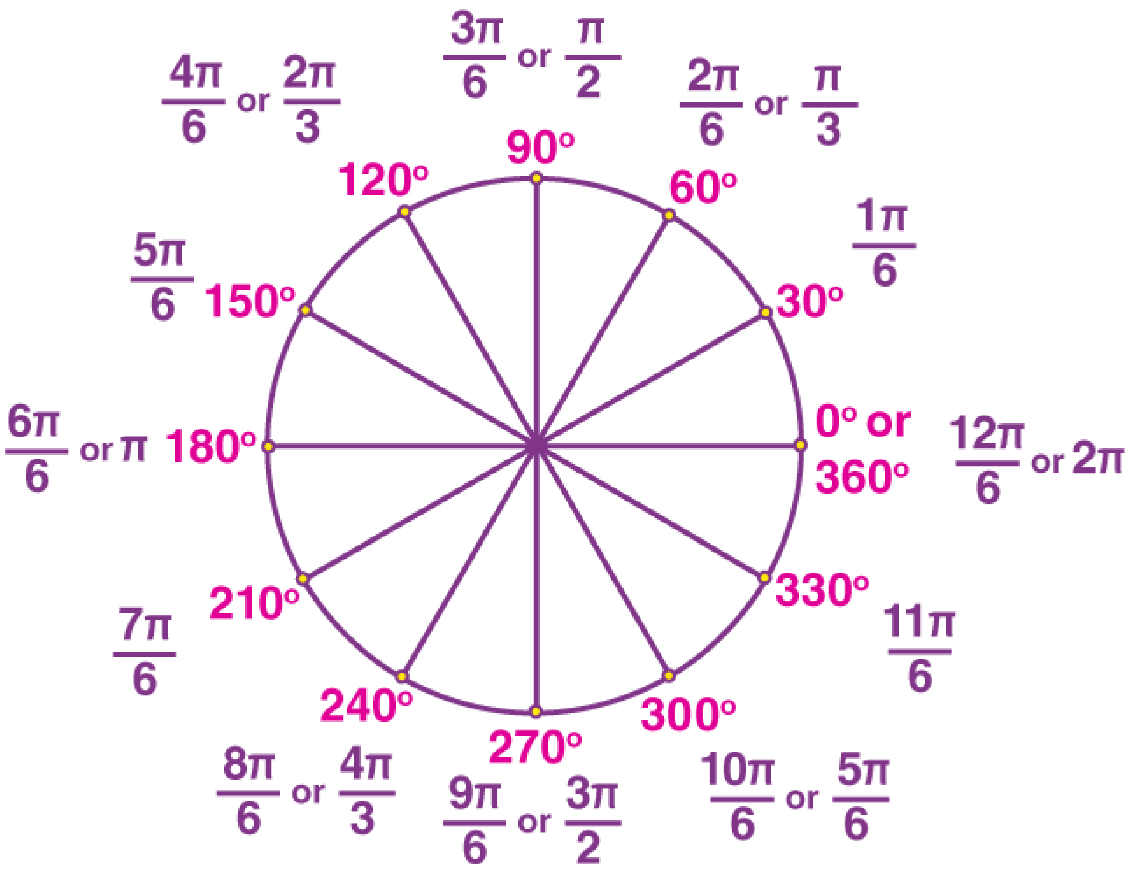
- How would someone from his time be able to figure out the circumference of the moon if at all?
- Does this have to do with dot products?
- I understand that he only used sticks, eyes and brain. But I do not certainly know how?
- What did his discovery help with in his era?
- I understand Carl Sagan's explanation of why different shadow lengths at two different locations implies a round earth, but why did Ertosthenes need to use two cities that were so far apart for his measurements? Couldn't he have used sticks that were closer together and still achieve the shadow discrepancy?
- How did the guy that measured out the distance between the two cities know that he had the correct distance. I would assume that he had to do so in a straight line, and not count extremely hills/mountains. How was this done?
- I understand that Eratosthenes was a genius, but I have trouble understanding why someone would walk 800km for him.

My Comments and Questions

- Comment: People often think physics started with Newton and others around that time, and that e.g. ancient "physicists" were bad (looking at you, Aristotle). But geometry (and statics) are physical theories.

- Question: I understand that we can use geometry to determine the shape of the Earth. But what is the shape/geometry of our observable universe? How can we figure it out?

Radians and the Small Angle Approximation



$$\sin\theta \approx \tan\theta \approx \theta$$

In radians!

$$\cos\theta \approx 1$$

How to Solve a Physics Problem

(An example of *Structured Problem Solving*)

V	S / P
C	R

1. Visualize: Draw/imagine a picture or diagram, label it
2. Collect Info: Relevant concepts, relations, equations
3. Solve: Work out *symbolically*, then plug in at the end
4. Review/check the answer: Units, size, relationships

Concept Check: Which of the following is a valid trig identity? How do you know?

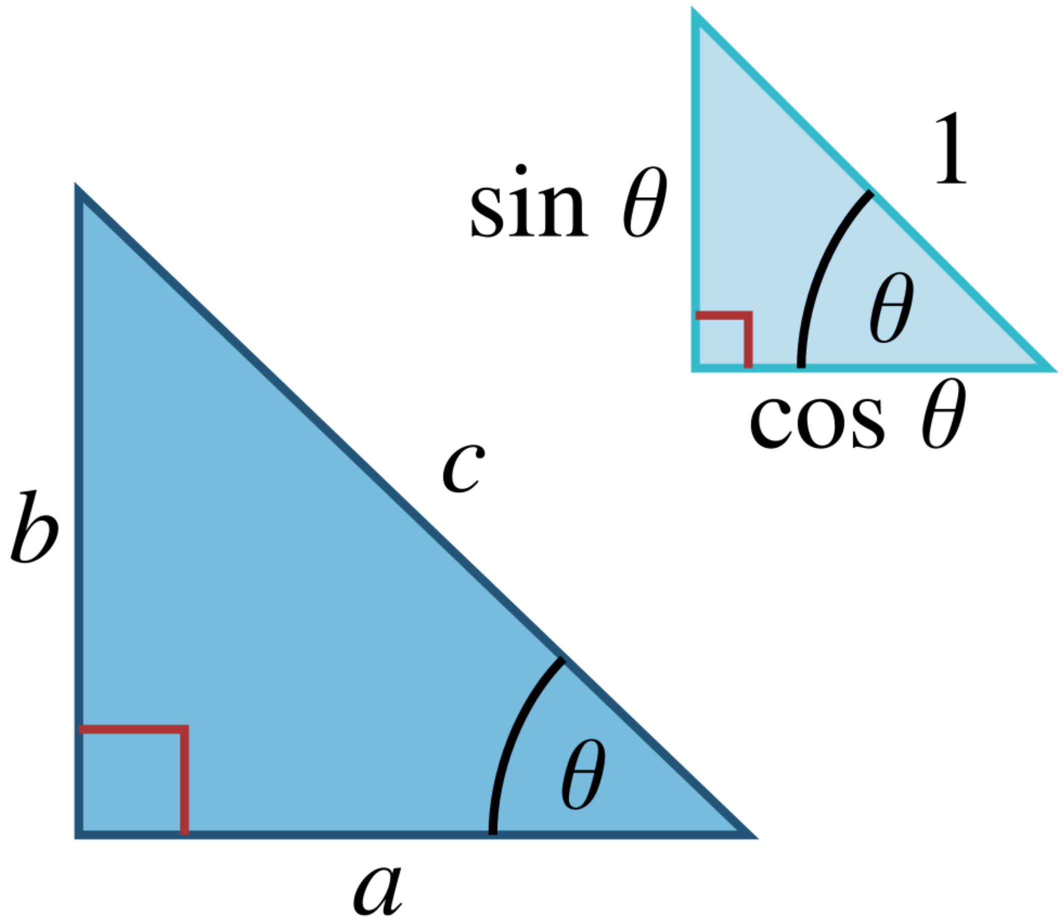
A) $\sin^2\theta - \cos^2\theta = 1$

B) $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$

C) $1 + \tan^2\theta = \dfrac{1}{\sin^2\theta}$

D) $1 - \tan^2\theta = \dfrac{1}{\sin^2\theta}$

E) None of these



Trigonometry Example: Parallax

You see a lightpost in the distance directly in front of you, but cannot discern how far away it is. You take a few steps sideways exactly to your right, a distance of *1 meter*. Facing the same direction as before, you now see the lightpost is not directly in front of you, but *5 degrees* to your left.

How far away is the lightpost from your original position?

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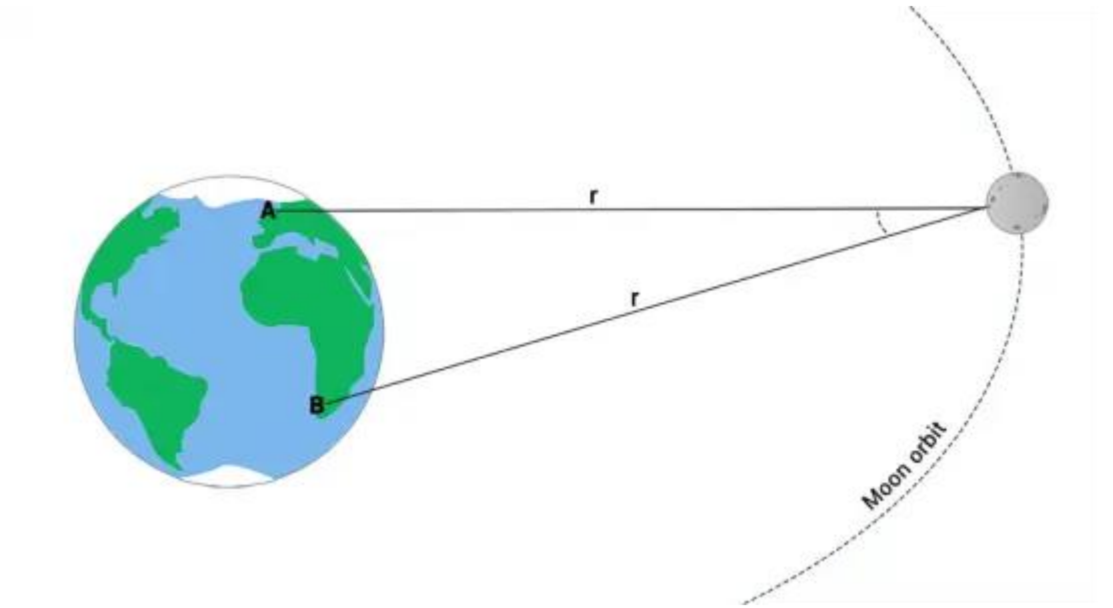
How far away is the lightpost from your original position? About 11.4m

The Distance to the Moon

- Parallax Distance:

$$D = \frac{b}{2 \times \tan(\theta/2)}$$

- Hipparchus and later Ptolemy were among the first to use this method. They estimated the distance to the Moon to be about 60 times the Earth's radius, which is fairly close to the actual value of approximately 60.3 times.



Non-Euclidean Geometry?

- Texas Triangle Area

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First Homework Due: Friday Sept 6
First Discussions: Wednesday Sept 4
(so none this week!)

Example: Area of a right triangle

Let’s say we want the area of a right triangle, but know only the hypotenuse? What can we say?

$$A = kh^2$$

The quantity k is some dimensionless constant that depends on the kind of right triangle → So it’s the same for ones that are similar!