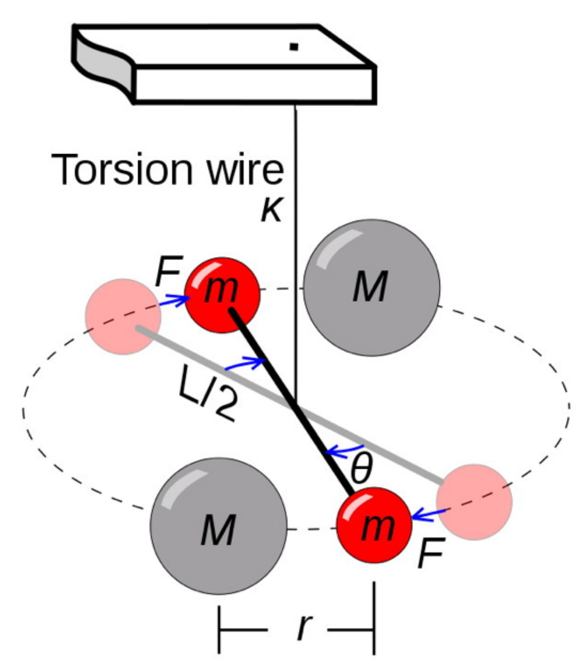
The Cavendish Experiment - weighing the earth

Prerequisite

- Newton's Universal gravitation



By Chris Burks (Chetvorno) - Own work
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Testing universal gravitation

In 1797, British scientist Henry Cavendish set up a precise experiment to measure gravity. Conceptually, the experiment looked like the figure at the right.

The two large gray balls were lead spheres about a foot in diameter. Lead is very dense; these balls weighed 348 pounds each and were fixed in place.

The smaller red balls were also lead, but just 2 inches across. They weighed 1.61 pounds each. The balls were balanced on a wooden rod hanging from a wire. If you were to rotate the rod a bit, it would twist the wire, and the wire would exert a force to twist the balls back. Almost anything hanging from a string will behave this way. For example, plug your phone into its charger and hold the charging cable a foot or so above the phone, with the phone dangling underneath. Twist your phone a bit and let go. The phone will twist right back to where it was. The experiment was like that, but with heavy lead balls. The more force you put on those small red balls, the more the wire would twist.

Cavendish found that the wire would twist even when he didn't put any force on it at all via pushing or pulling. The only unbalanced force on the red balls was the gravity from the big gray balls. Cavendish's experiment was sensitive enough that could measure the strength of the force by seeing just how much the rod and red balls twisted. Since he had measured all the masses and the distances, Cavendish was able to infer the value of G. It's quite small!

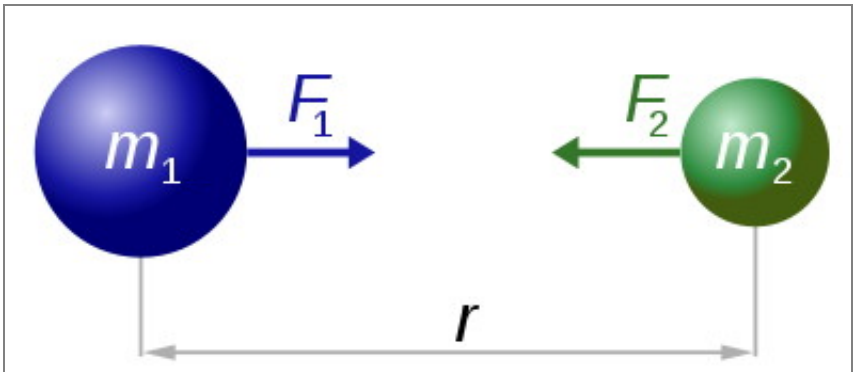
$$G = \frac{2}{3} \times 10^{-10} \text{ N-m}^2/\text{kg}^2. \text{ (to better than 1\%)}$$

When Cavendish moved the gray balls further away, the force got smaller and the rod twisted less. When he moved the gray balls closer, the opposite occurred. So he learned that the closer something is, the more the gravitational force. The gravitational forces in this experiment were very small - tenths of a microNewton, so Cavendish had to control lots of sources of error to get good results. But eventually, he confirmed an equation for gravity that goes back to Isaac Newton. This says that the force between an object of mass m_1 and an object of mass m_2 is

$$\overrightarrow{F}^{grav}_{1\to 2} = \frac{G m_1 m_2}{r_{12}^2} \hat{r}_{2\to 1} \tag{1}$$

This equation says that the gravitational force of any object 1 on any other object 2 is some constant G times the two masses of the objects divided by the square of the distance between them. This is called an "inverse square law", and it means that if Cavendish were to move his big gray spheres three times as far out as before, the gravitational force on the small spheres would be only one ninth as much.

The $\hat{r}_{2\to 1}$ part is a unit vector pointing from 2 to 1. That is, an arrow that points in the direction of 1 from the point of view of 2, and that has unit magnitude and no units. This says that the force that a exerts on 2 points back towards 1. Gravity attracts. This image from Wikipedia illustrates the gravitational force between two spheres, which could be spheres from the Cavendish experiment, or a planet and a star, or any other two objects.



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$$F_1 = F_2 = \frac{G m_1 m_2}{r^2} \tag{2}$$

Newton's third law is built in to the universal gravitational law. If you switch the roles of *a* and *b*, the magnitude of the force comes out the same, but you get $\hat{r}_{1\to 2}$ instead of $\hat{r}_{2\to 1}$. These vectors point opposite directions, just like Newton's third law says.

In everyday life, the gravitational force between most objects is too small to notice. It becomes important when at least one of the objects is really big, like an entire planet. Newton used universal gravity to explain the orbits of planets in the solar system, and since then it has also explained things like the motion and formation of galaxies or the collapse of dust clouds into stars.

Measuring the mass of the earth

Cavendish actually thought of his experiment as a way of measuring the mass of the Earth for the first time.

Why? Try setting object 1 to be Earth and object 2 to be an apple. Because he knew how much force the Earth would exert on an apple and he knew the size of the Earth (which is r in this equation — for spheres you have to measure to the center of the sphere, not the edge), Cavendish had most of the variables in the gravitational law pinned down. Once he measured G with his experiment, the only variable left in the equation was m_1 , the mass of the earth. So he could solve for it and find the mass of the Earth for the first time.

Let's see how we can do it. We know that the weight of an object of mass m on the surface of the earth is mg where $g\tilde{1}0$ N/kg. If we equate this to Newton's universal gravitation we get

$$mg = \frac{GmM_E}{R_E^2} \tag{3}$$

where M_E is the mass of the earth and R_E is the radius of the earth. In this equation we easily know m, g, and R_E. What's missing is G and M_E. So if we measure G, we can solve for the mass of the earth:

$$M_E = \frac{gR_E^2}{G} \tag{4}$$

In meters, kg, and seconds, g is about 10 N/kg, the radius of the earth is about 10^7 m ($2/\pi \times 10^7$ m), and G is about 10^{-10} N-m^2/kg^2, we get about 10^{25} kg! Rather difficult to weigh by putting on a scale!

Note that although both Newton and Cavendish understood universal gravitation mathematically, neither of them wrote their equations in the same form we do today. In fact, Cavendish didn't think about the constant G at all. His analysis was equivalent to what we just said, but the actual details were different in terms of algebra. There are lots of ways to use math to express the same fundamental knowledge!

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