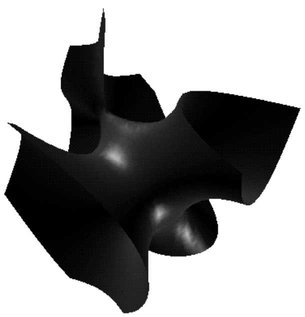
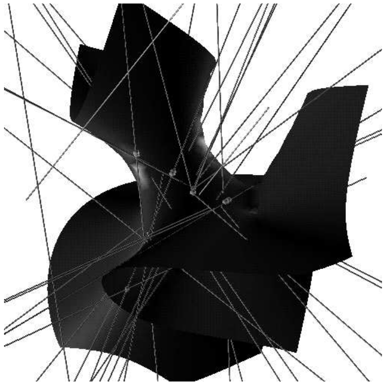
Shapes

Primitive Shapes

- equations

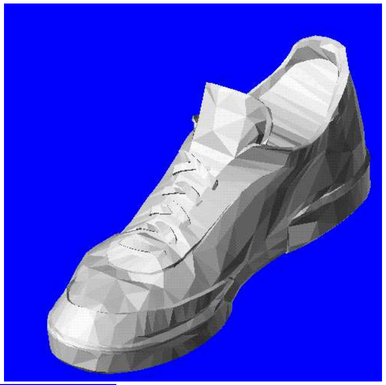


- system of equations

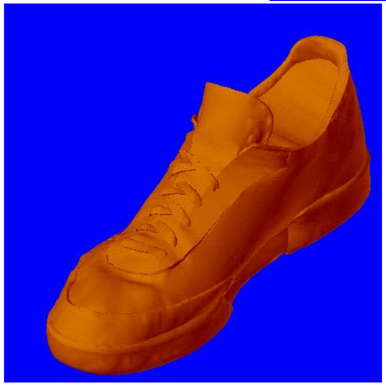


Modelling Transformations

- extrusion, revolution, swept volumes
- boolean set operations
- L-systems

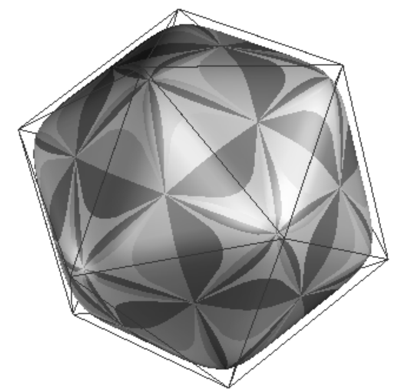


- shell surfaces

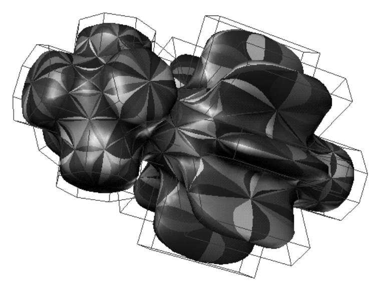


Patch Complexes

- Triangulations
- Curvilinear Complexes



- Mixed Complexes



Domains, Scenes

Nano-World

- Molecular Models (CPK, Ribbon etc)

Micron-World

- Semiconductors

Meter-World

- Human Anatomy

KiloMeter-World

- Oil Reservoirs

LightYears-World

- Cosmology

Conics and Splines

- Conics
  - Implicit form
  - Parametric form
- Implicit Bézier Forms
  - General quadratics
  - Cubics and higher degree
- Conics as A-SPLINES
  - Implicit Bernstein-Bézier (BB) and B-basis forms
- Conics as Rational Bézier Forms
  - General quadratics
  - Conversion forms for conics
- Conics as NURBS
  - Parametric rational Bézier form

Conic Curves

Conic Sections (Implicit form)

- Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a, b > 0$$

- Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad a, b > 0$$

- Parabola

$$y^2 = 4ax \quad a > 0$$

Conic Sections (Parametric form)

- Ellipse

$$x(t) = a\frac{1-t^2}{1+t^2}$$

$$y(t) = b\frac{2t}{1+t^2} \quad (-\infty < t < +\infty)$$

- Hyperbola

$$x(t) = a\frac{1+t^2}{1-t^2}$$

$$y(t) = b\frac{2t}{1-t^2} \quad (-\infty < t < +\infty)$$

- Parabola

$$
\begin{aligned}
x(t) &= at^2 \\
y(t) &= 2at \quad (-\infty < t < +\infty)
\end{aligned}
$$

Rational Quadratic Bezier Forms

Quadratic Rational Bézier Form:

- Homogeneous form

$$\begin{bmatrix} x(t) \\ y(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ w_0 \end{bmatrix} B_0^2(t) + \begin{bmatrix} x_1 \\ y_1 \\ w_1 \end{bmatrix} B_1^2(t) + \begin{bmatrix} x_2 \\ y_2 \\ w_2 \end{bmatrix} B_2^2(t)$$

$$= \begin{bmatrix} x_0 B_0^2(t) + x_1 B_1^2(t) + x_2 B_2^2(t) \\ y_0 B_0^2(t) + y_1 B_1^2(t) + y_2 B_2^2(t) \\ w_0 B_0^2(t) + w_1 B_1^2(t) + w_2 B_2^2(t) \end{bmatrix}$$

- Rational (projected) form

$$\begin{bmatrix} \bar{x}(t) \\ \bar{y}(t) \end{bmatrix} = \begin{bmatrix} \frac{x_0 B_0^2(t)+x_1 B_1^2(t)+x_2 B_2^2(t)}{w_0 B_0^2(t)+w_1 B_1^2(t)+w_2 B_2^2(t)} \\ \frac{y_0 B_0^2(t)+y_1 B_1^2(t)+y_2 B_2^2(t)}{w_0 B_0^2(t)+w_1 B_1^2(t)+w_2 B_2^2(t)} \end{bmatrix}$$

$$= \frac{\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} B_0^2(t) + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} B_1^2(t) + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} B_2^2(t)}{w_0 B_0^2(t) + w_1 B_1^2(t) + w_2 B_2^2(t)}$$

Conversions:

- Conic parameterization elements in Bézier form

$$
\begin{aligned}
2t &= B_1^2(t) + 2B_2^2(t) \\
1 - t^2 &= B_0^2(t) + B_1^2(t) \\
1 + t^2 &= B_0^2(t) + B_1^2(t) + 2B_2^2(t) \\
t^2 &= B_2^2(t)
\end{aligned}
$$

Conics as Rational Bézier Curves

Conics as NURBS (Ellipse)

- Rational Bézier

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{\begin{bmatrix} a(1-t^2) \\ b(2t) \end{bmatrix}}{1+t^2}$$

$$= \frac{\begin{bmatrix} aB_0^2(t) + aB_1^2(t) + 0B_2^2(t) \\ 0B_0^2(t) + bB_1^2(t) + b2B_2^2(t) \end{bmatrix}}{B_0^2(t) + B_1^2(t) + 2B_2^2(t)}$$

$$= \frac{\begin{bmatrix} a \\ 0 \end{bmatrix} B_0^2(t) \begin{bmatrix} a \\ b \end{bmatrix} B_1^2(t) \begin{bmatrix} 0 \\ 2b \end{bmatrix} B_2^2(t)}{B_0^2(t) + B_1^2(t) + 2B_2^2(t)}$$

which implies

$$\begin{array}{lll} w_0 = 1 & x_0 = a & y_0 = 0 \\ w_1 = 1 & x_1 = a & y_1 = b \\ w_2 = 2 & x_2 = 0 & y_2 = 2b \end{array}$$

Conics as NURBS (Hyperbola)

- Rational Bézier

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{\begin{bmatrix} a(1+t^2) \\ b(2t) \end{bmatrix}}{1-t^2}$$

$$= \frac{\begin{bmatrix} aB_0^2(t) + aB_1^2(t) + a2B_2^2(t) \\ 0B_0^2(t) + bB_1^2(t) + b2B_2^2(t) \end{bmatrix}}{B_0^2(t) + B_1^2(t)}$$

$$= \frac{\begin{bmatrix} a \\ 0 \end{bmatrix} B_0^2(t) \begin{bmatrix} a \\ b \end{bmatrix} B_1^2(t) \begin{bmatrix} 2a \\ 2b \end{bmatrix} B_2^2(t)}{B_0^2(t) + B_1^2(t) + 0B_2^2(t)}$$

which implies

$$\begin{array}{lll} w_0 = 1 & x_0 = a & y_0 = 0 \\ w_1 = 1 & x_1 = a & y_1 = b \\ w_2 = 0 & x_2 = 2a & y_2 = 2b \end{array}$$

Conics as NURBS (Parabola)

- Rational Bézier

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{\begin{bmatrix} a(t^2) \\ a(2t) \end{bmatrix}}{1}$$

$$= \frac{\begin{bmatrix} 0B_0^2(t) + 0B_1^2(t) + aB_2^2(t) \\ 0B_0^2(t) + aB_1^2(t) + a2B_2^2(t) \end{bmatrix}}{B_0^2(t) + B_1^2(t) + B_2^2(t)}$$

$$= \frac{\begin{bmatrix} 0 \\ 0 \end{bmatrix} B_0^2(t) \begin{bmatrix} 0 \\ a \end{bmatrix} B_1^2(t) \begin{bmatrix} a \\ 2a \end{bmatrix} B_2^2(t)}{B_0^2(t) + B_1^2(t) + B_2^2(t)}$$

which implies

$$\begin{array}{lll} w_0 = 1 & x_0 = 0 & y_0 = 0 \\ w_1 = 1 & x_1 = 0 & y_1 = a \\ w_2 = 1 & x_2 = a & y_2 = 2a \end{array}$$

Not Unique

- x, y, w are not unique
  - Numerator and denominator can be multiplied by a common (positive) factor
- The following example is a common alternative form:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} w_0 B_0^2(t) + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} w_1 B_1^2(t) + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} w_2 B_2^2(t)}{w_0 B_0^2(t) + w_1 B_1^2(t) + w_2 B_2^2(t)}$$

which derives from rewriting

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \longrightarrow w \begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix}$$

Reading Assignment and News

Chapter 10 pages 516 - 526, of Recommended Text.

Please also track the News section of the Course Web Pages for the most recent Announcements related to this course.

(http://www.cs.utexas.edu/users/bajaj/graphics23/cs354/)