Illumination I: The Phong Illumination Model

Components of Phong illumination or reflection model using RGB model:

OpenGL allows us to break this light's emitted intensity into 3 components: ambient L_a, diffuse L_d, and specular L_s. Each type of light component consists of 3 color components, so, for example, L_{rd} denotes the intensity of the red component of diffuse illumination.

Question: What is the amount of light that is transmitted (either by emission or reflection) from each point in the direction of the viewer.

Solution: This is achieved by first associating reflectivity or material properties to all the modelled objects in the scene, and then applying a Phong reflection calculation to determine the transmitted light intensity.

The Reflected Light Luminance/Intensity function shall be captured by:

$$I = (I_r, I_g, I_b)$$

for each of Light's components. For example,

- ambient emission

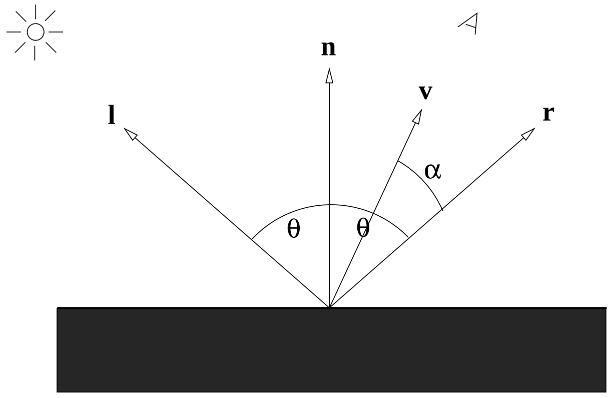
$$I_a = \begin{pmatrix} I_{ar} \\ I_{ag} \\ I_{ab} \end{pmatrix}$$

An object's material properties determines how much of a given input Light intensity is reflected. Under the Phong model, material properties are captured by reflectivity coefficient vectors $K = (k_r, k_g, k_b)$ for ambient, diffuse and specular. Thus $k_d r$ is the fraction of red diffuse light that is reflected from an object. If $k_r = 0$, then no red light is reflected.

The computation of reflected luminance/intensity function using Phong illumination, for each object and light source, shall be governed by the following four light/material interactions.

- *Emission intensity:* to model objects that glow
- *Ambient reflection:* A simple way to model indirect reflection. All surfaces in all positions and orientations are illuminated equally.
- *Diffuse reflection:* The diffuse shading produced by dull, smooth objects.
- *Specular reflection:* The bright spots appearing on smooth shiny (e.g., metallic or polished) surfaces.

Relevant Vectors for Phong Shading



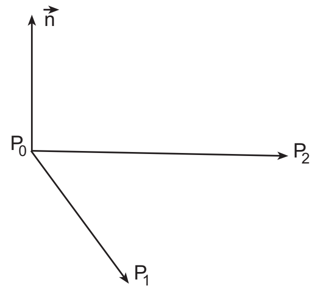
The shading of a point on a surface is a function of the relationship between the viewer, light sources, and surface. The following vectors are relevant to direct illumination. All vectors are assumed to be normalized to unit length.

- *Normal vector:* A vector $\vec{n}$ that is perpendicular to the surface and directed outwards from the surface.
- *View vector:* A vector $\vec{v}$ that points in the direction of the viewer.
- *Light vector:* A vector $\vec{l}$ that points towards the light source.
- *Reflection vector:* A vector $\vec{r}$ that indicates the direction of pure reflection of the light vector.

Normals Computation

Given any three non-collinear points, P_0, P_1, P_2, on a polygon, a normal of the polygon is given through a cross product

$$\vec{n} = (P_1 - P_0) \times (P_2 - P_0).$$



Normals by cross product.

For a polygon is given by n points P_0, P_1, . . . , P_{n-1}. If we can determine a plane equation (via least-squares fit):

$$ax + by + cz + d = 0$$

from these n points, then normalizing (a, b, c) is the unit normal vector $\vec{n}$ of the polygon.

Normals for Implicitly Defined Surfaces

Given a surface defined by an *implicit representation*, i.e., defined by some equation

$$f(x, y, z) = 0$$

then the normal at some point is given by *gradient vector*

$$\vec{n} = \begin{pmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{pmatrix}$$

Normals for Parametric Surfaces

Surfaces in computer graphics are most often represented parametrically. The *parametric representation* of a surface is defined by three functions of 2 variables or *parameters*:

$$x = \phi_x(u, v),$$
$$y = \phi_y(u, v),$$
$$z = \phi_z(u, v).$$

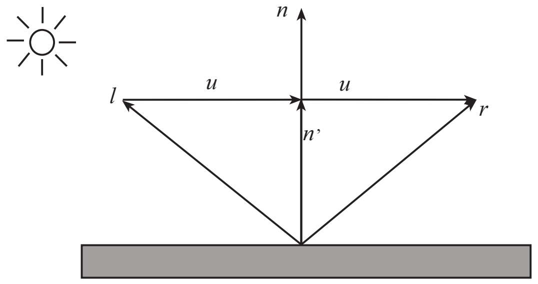
Then the normal of the surface at a point is defined as the

$$\vec{n} = \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v}$$

where

$$\frac{\partial \phi}{\partial u} = \begin{pmatrix} \partial \phi_x / \partial u \\ \partial \phi_y / \partial u \\ \partial \phi_z / \partial u \end{pmatrix} \qquad \frac{\partial \phi}{\partial v} = \begin{pmatrix} \partial \phi_x / \partial v \\ \partial \phi_y / \partial v \\ \partial \phi_z / \partial v \end{pmatrix}$$

The Reflection Vector

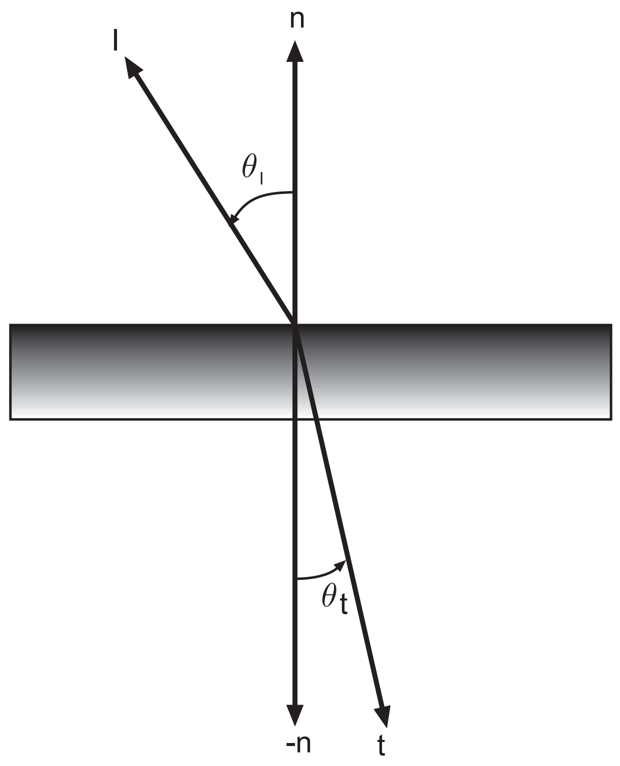


$$\vec{n}' = (\vec{n} \cdot \vec{l})\vec{n}$$

$$\vec{u} = \vec{n}' - \vec{l}$$

$$\vec{r} = \vec{l} + 2\vec{u} = \vec{l} + 2(\vec{n}' - \vec{l}) = 2(\vec{n} \cdot \vec{l})\vec{n} - \vec{l}$$

The Refraction Vector



If η_l and η_t are the refractive indices of the materials on the two sides of the surface, then Snell's law states that

$$\eta_l Sin(\theta_l) = \eta_t Sin(\theta_t)$$

Using this and the fact that $\vec{l}$, $\vec{n}$, and $\vec{t}$ are assumed coplanar, we can calculate the unit transmitted light vector $\vec{t}$, as follows. let $\eta = \frac{\theta_t}{\theta_l}$, we have

$$Cos(\theta_t) = ((1 - \frac{1}{\eta^2}(1 - Cos^2(\theta_l)))^{\frac{1}{2}}$$

and

$$t = -\frac{1}{\eta}\vec{l} - (Cos(\theta_t) - \frac{1}{\eta}Cos(\theta_l))\vec{n}$$

Ambient Light Reflection

Ambient light is simplest to deal with. Let I_a denote the intensity of ambient light. For each surface, let

$$0 \leq k_a \leq 1$$

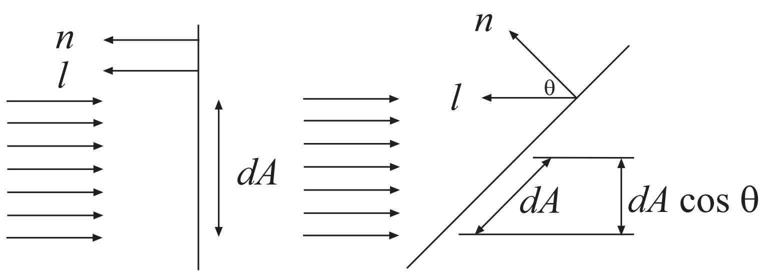
denote the surface's *coefficient of ambient reflection*, that is, the fraction of the ambient light that is reflected from the surface. The ambient component of illumination is

$$I_a = k_a L_a$$

Note that this is a vector equation (whose components are RGB).

Diffuse Reflection

Diffuse reflection arises from the assumption that light from any direction is reflected uniformly in all direction. Such a reflector is called a pure *Lambertian reflector*.



The key parameter of surface that controls diffuse reflection is k_d, the surface's *coefficient of diffuse reflection*. Let I_d denote the diffuse reflection component of the light source. Assume $\vec{l}$ and $\vec{n}$ are normalized, then $\cos\theta = (\vec{n} \cdot \vec{l})$. If $(\vec{n} \cdot \vec{l}) < 0$, then the point is on the dark side of the object.

The diffuse component to illumination is

$$I_d = k_d \max(0, \vec{n} \cdot \vec{l}) L_d$$

Specular Reflection I

Most objects are not perfect Lambertian reflector. One of the most common deviation is for smooth metallic or highly polished objects. They tend to have *specular highlights* (or "shiny spots").

The parameters of surface that control specular reflection under Phong model, are k_s, the surface's *coefficient of specular reflection*, and s, *shininess*.

The formula for the specular component is

$$I_s = k_s(\vec{r} \cdot \vec{v})^s L_s$$

Specular Reflection II

Another way of calculating specular reflection under the Phong model, is via the *halfway vector* (OpenGL).

Define $\vec{h}$ to be the *halfway vector*, the normalized vector which is the halfway of $\vec{l}$ and $\vec{v}$. Define $\vec{h} =$ Normalize $(\vec{l} + \vec{v})$.

The formula for the specular component can then be written as

$$I_s = k_s(\vec{n} \cdot \vec{h})^s L_s$$

The Phong Model Illumination Equation

The total illumination of a point in OpenGL is computed for the supported Light sources and is calculated

$$\begin{aligned} I &= I_e + I_a + \frac{1}{a + bd + cd^2}(I_d + I_s) \\ &= I_e + k_a L_a + \frac{1}{a + bd + cd^2}(k_d \max(0, \vec{n} \cdot \vec{l}) L_d + k_s (\vec{n} \cdot \vec{h})^s L_s), \end{aligned}$$

where d is the distance from the object to the light source.

The reflection material properties for front/back of each surface is specified by OpenGL using for example,

GLfloat ambient[]=0.1,0.25,0.0,1.0

GLfloat diffuse[]=0.1,0.25,0.0,1.0

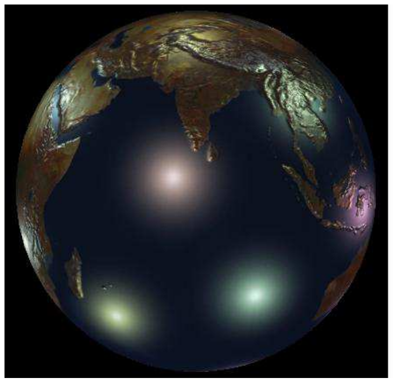
GLfloat specular[]=1.0,0.0,1.0,1.0

GLfloat emission[]=0.0,0.8,0.0,1.0

glMaterialfv(GL-front-and-back,GL-specular,specular)

glMaterialf(GL-front-and-back,GL-shininess, 100.0)

For multiple light sources, we add up the ambient, diffuse, and specular components for each light source.



Reading Assignment and News

Chapter 6 pages 293 - 303, of Recommended Text.

Please also track the News section of the Course Web Pages for the most recent Announcements related to this course.

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