CS303E: Elements of Computers and Programming
Functions

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Functions

You probably remember functions from your high school math classes:

$$f(x) = x^2 + 2$$

This defines a rule for performing a computation. It has a parameter x, which doesn't have a value but stands for any number you want to put there.

Notice that the definition *doesn't perform* a computation. It only tells you how to perform it. Namely, given any specific value for x, square it and add 2.

Thus, $f(5)$ equals 27, because (substituting 5 for x in our rule),

$$f(5) = 5^2 + 2 = 27$$
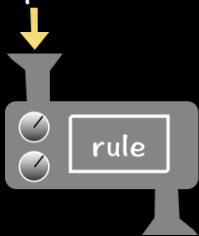
Functions

Notice several things:

- The *function definition* doesn't perform a computation; it only gives us a recipe or procedure for performing a computation.
- A *function call* (e.g., $f(5)$) does perform a computation, using the defining rule.
- To actually compute something, we need to *call the function*, supplying values for the parameters.
- The computed value is "returned" to the calling environment replacing the call with the value.

$$f(3) + f(2) = (3^2 + 2) + (2^2 + 2) = 11 + 6 = 17$$

Functions in programming languages work similarly, with a few differences.
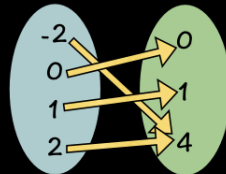
Function

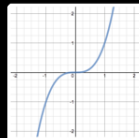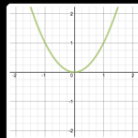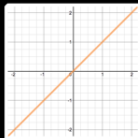We've seen lots of system-defined functions; now it's time to define our own.

General form:

```
def functionName( list of parameters ):   # header
   statement(s)                           # body
```

Meaning: a function definition defines a block of code that performs a specific task. It can reference any of the variables in the list of parameters. It may or may not return a value.

The parameters are **formal parameters**; they stand for arguments passed to the function later.

Function name

An identifier by which the function is called

Arguments

Contains a list of values passed to the function

```
def name(arguments):
    statement
    statement
    ...
    return value
```

Indentation

Function body must be indented

Function body

This is executed each time the function is called

Return value

Ends function call & sends data back to the program
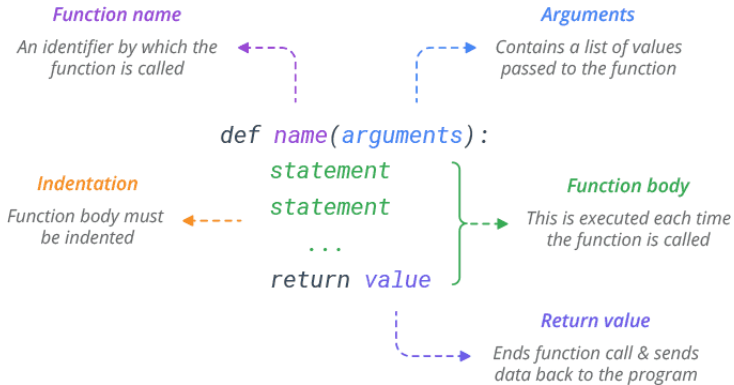
Parameters

Function Definition

```
def add(a, b):
    return a + b
```

Function Call

```
add(2, 3)
```

Arguments

getKT.com

Suppose you want to sum the integers 1 to n.

In file `functionExamples.py`:

```python
def sumToN( n ):
    """ Sum the integers from 1 to n. """
    sum = 0                          # identity element for +
    for i in range(1, n + 1 ):       # Why n+1?
        sum += i
    # Hand the answer to the calling environment.
    return sum
```

Function Example

Notice that the definition defines a *function* to perform the task, but *doesn't actually perform the task*. We still have to call/invoke the function with specific arguments.

```
>>> from functionExamples import *
>>> sumToN( 10 )
55
>>> sumToN( 1000 )
500500
>>> sumToN(1000000)
500000500000
```

You can think of the function call as being *replaced by* the value returned.

You can call a function as many times as you need.

```
def sumToN( n ):                      # function header
    ....                              # function body
```

Here n is a *formal parameter*. It is used in the definition as a place holder for an *actual parameter* (e.g., 10 or 1000) in any specific call.

sumToN(n) *returns* an int value, meaning that a call to sumToN can be used anyplace an int expression can be used.

```
>>> print( sumToN( 50 ) )
1275
>>> ans = sumToN( 30 )
>>> print( ans )
465
>>> print( "Even" if sumToN(3) % 2 == 0 else "Odd" )
Even
```

Once we've defined sumToN, we can use it almost as if were a primitive in the language without worry about the details of the definition.

We need to know what it does, but don't care anymore how it does it!

This is called **information hiding** or **functional abstraction**.

Suppose later we discover that we could have coded `sumToN` more efficiently (as discovered by the 8-year old C.F. Gauss in 1785):

```python
def sumToN( n ):
    """ Sum the integers from 1 to n. """
    return ( n * (n+1) ) // 2
```

Because we defined `sumToN` *as a function, we can just swap in this definition without changing any other code.* If we'd done the implementation in-line, we'd have had to go find every instance and change it.

```
>>> sumToN(10)
55
>>> sumToN( 1000000000000 )
500000000000500000000000
```

Return Statements

When you execute a `return` statement, you go back to the calling environment. You may or may not hand a value back to the caller.

General forms:

```
return
return expression
```

A `return` that doesn't specify a value actually returns the constant `None`.

Every function has an *implicit* return at the end.

```python
def printTest ( x ):
  print( x )
  # implicit return here
```

A `return` statement can only appear within a function, but you can have as many as you need.

Return vs. Print

You can think of a Python function as a recipe for performing some computation. The computation *may* return a value:

```python
def cube ( x ):
    return x ** 3
```

If it does, you can think of the value returned as replacing the call to the function.

```python
    cube ( 3 ) + cube ( 2 )
```

is the same as

```python
    27 + 8
```

Return vs. Print

But often a Python function doesn't return a value (actually returns the constant None) either always or sometimes.

```python
def cubeNonnegative( x ):
    if x >= 0:
        return x ** 3
    else:
        print( "Negative argument supplied." )
```

Notice that this returns an int value for nonnegative arguments but prints an error message for negative arguments.

Almost always print an error message; don't return it. A caller expecting an int value to be returned probably can't handle a string.

The point of an error message is to inform the user that something went wrong.

Define your program in file
`Filename.py`:

```python
def main ():

    Python statement
    Python statement
    Python statement
        ...
    Python statement
    Python statement
    Python statement

main ()
```

This defines a function `main`; could have a different name.

If you wanted to end the program, you could include a `return` statement.

You couldn't use `return` if you just had the statements at the top level.

To run it:

```
> python Filename.py
```

In file returnExamples.py:

```python
def printSquares ():
    """ Compute and print squares until 0 is entered
        by the user. """
    while True:
        num = int (input ("Enter an integer or 0 to exit: "))
        if ( num != 0 ):          # "if num:" works
            print ( "The square of", num, "is:", num ** 2 )
        else:
            return                # no value is returned

printSquares ()
```

This doesn't return a value, but accomplishes it's purpose by the "side effect" of printing.

```
> python returnExamples.py
Enter an integer or 0 to exit: 7
The square of 7 is: 49
Enter an integer or 0 to exit: -12
The square of -12 is: 144
Enter an integer or 0 to exit: 0
>
```

A function that "doesn't return a value" actually returns the
constant `None`.

Some More Function Examples

Suppose we want to multiply the integers from 1 to n:

```python
def multToN( n ):
    """ Compute the product of the numbers from 1 to n. """
    prod = 1                    # identity element for *
    for i in range(1, n+1):
        prod *= i
    return prod
```

Convert fahrenheit to celsius:

```python
def fahrToCelsius( f ):
""" Convert fahrenheit temperature value to celsius
    using formula: C = 5/9(F-32). """
    return 5 / 9 * ( f - 32 )
```

Or celsius to fahrenheit:

```python
def celsiusToFahr( c ):
    """ Convert celsius temperature value to fahrenheit
        using formula: F = 9/5 * C + 32. """
    return 9 / 5 * c + 32
```

Fahr to Celsius Table

In slideset 1, we showed the C version of a program to print a table of Fahrenheit to Celsius values. Here's a Python version:

In file `FahrToCelsius.py`:

```python
from functionExamples import fahrToCelsius

def printFahrToCelsius():
    """ Print table fahrenheit to celsius for temp
        in [0, 20, 40, ... 300]."""
    lower = 0
    upper = 300
    step  = 20
    print("Fahr\tCelsius")
    for fahr in range( lower, upper + 1, step ):
        # Use an already defined function.
        celsius = fahrToCelsius( fahr )
        print( format( fahr, "3d"), "\t", \
               format( celsius, "6.2f") )
    return           # not actually necessary

printFahrToCelsius()
```
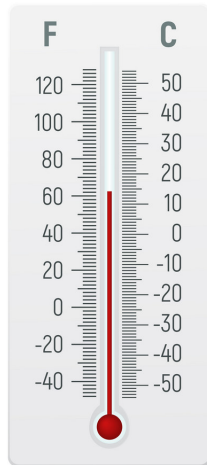
Notice that `printFahrToCelsius` returns None.

```
> python FahrToCelsius.py
Fahr    Celsius
  0       -17.78
 20        -6.67
 40         4.44
 60        15.56
 80        26.67
100        37.78
120        48.89
140        60.00
160        71.11
180        82.22
200        93.33
220       104.44
240       115.56
260       126.67
280       137.78
300       148.89
```

Exercise: Do a similar problem converting Celsius to Fahrenheit.

Functions Inside Functions

There are occasionally reasons to define one function within another function. *But it's generally a bad idea, unless you know what you're doing.*

For this class, always have your functions at the top level of your .py file.

Suppose you want to print out a table of the first 100 primes, 10 per line.

You could sit down and write this program from scratch, without using functions. But it would be a complicated mess (section 5.8 of the book).

Better to use **functional abstraction**: find parts of the algorithm that can be coded separately and "packaged" as functions.

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541

Here's some Python-like pseudocode to print 100 primes:

```
def print100Primes():
    primeCount = 0
    num = 0
    while (primeCount < 100):
        if (we've already printed 10 on the current line):
            go to a new line
        nextPrime = ( the next prime > num )
        print nextPrime on the current line
        num = nextPrime
        primeCount += 1
```

Note that most of this is just straightforward Python programming! The only "new" part is how to find the next prime. So we'll make that a *function*.

So let's *assume* we can define a function:

```
def findNextPrime( num ):
    """ Return the first prime greater
        than num. """
    < body >
```

in such a way that it returns the first prime larger than `num`.

Is that even possible? Is there always a "next" prime larger than num?

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        than num. """
    < body >
```

in such a way that it returns the first prime larger than num.

Is that even possible? Is there always a "next" prime larger than num?

Yes! There are an infinite number of primes. So if we keep testing successive numbers starting at num + 1, we'll eventually find the next prime. *That may not be the most efficient way!*

Notice we're following a "divide and conquer" approach: Reduce the solution of our bigger problem into one or more subproblems which we can tackle independently.

It's also an instance of "information hiding." We don't want to think about how to find the next prime, while we're worrying about printing 100 primes. Put that off!

Now solve the original problem, *assuming* we can write
findNextPrime.
In file IsPrime3.py:

```python
def print100Primes ():
    """ Print a table of the first 100 primes ,
        10 primes per line. """
    primeCount = 0                    # primes we've found
    onLine = 0                        # primes printed on line
    num = 0                           # need next prime > num
    while (primeCount < 100):
        # Do we stay on current line?
        if ( onLine >= 10 ):
            print()
            onLine = 0
        # This is the only thing left to define:
        nextPrime = findNextPrime( num )
        num = nextPrime
        primeCount += 1
        print( format( nextPrime , "3d"), end = " " )
        onLine += 1
    print()
```

Looking Ahead

Here's what the output should look like.

```
>>> from IsPrime3 import print100Primes
>>> print100Primes ()
  2   3   5   7  11  13  17  19  23  29
 31  37  41  43  47  53  59  61  67  71
 73  79  83  89  97 101 103 107 109 113
127 131 137 139 149 151 157 163 167 173
179 181 191 193 197 199 211 223 227 229
233 239 241 251 257 263 269 271 277 281
283 293 307 311 313 317 331 337 347 349
353 359 367 373 379 383 389 397 401 409
419 421 431 433 439 443 449 457 461 463
467 479 487 491 499 503 509 521 523 541
```

Of course, we couldn't do this if we really hadn't defined
findNextPrime. So let's see what that looks like.

How to Find the Next Prime

The next prime ($>$ num) can be found as indicated in the
following pseudocode:

```
def findNextPrime( num ):
    if num < 2:
        return 2 as the answer
    else:
        guess = num + 1
        while ( guess is not prime )
            guess += 1
        return guess as the answer
```

Again we solved one problem by assuming the solution to another
problem: deciding whether a number is prime.

Can you think of ways to improve this algorithm?

Note that we're assuming we can write:

```
def isPrime( num ):
    """ Boolean test for primality. """
    < body >
```

```
def findNextPrime( num ):
    """ Find the first prime > num. """
    if ( num < 2 ):
        return 2
    guess = num + 1
    while ( not isPrime( guess ) ):
        guess += 1
    return guess
```

This works (assuming we can define isPrime), but it's pretty inefficient. How could you fix it?

When looking for the next prime, we don't have to test every number, just the odd numbers (after 2).

```python
def findNextPrime ( num ):
    """ Find the first prime > num. """
    if ( num < 2 ):
        return 2

    # If (num >= 2 and num is even), the
    # next prime after num is at least
    # (num - 1) + 2, which is odd.
    if ( num % 2 == 0 ):
        num -= 1
    guess = num + 2

    while ( not isPrime( guess )):
        guess += 2
    return guess
```

Now all that remains is to write isPrime.

Is a Number Prime?

We already solved a version of this in slideset 5. Let's rewrite that code as a Boolean-valued function:

```python
def isPrime( num ):
    """ Test whether num is prime. """

    # Deal with evens and numbers < 2.
    if (num < 2 or num % 2 == 0 ):
        return ( num == 2 )

    # See if there are any odd divisors
    # up to the square root of num.
    divisor = 3
    while (divisor <= math.sqrt( num )):
        if ( num % divisor == 0 ):
            return False
        else:
            divisor += 2
    return True
```

By the way, a Boolean-valued function is often called a **predicate**.

Testing Our Code

```
>>> from IsPrime3 import findNextPrime , isPrime
>>> findNextPrime ( -10 )
2
>>> findNextPrime ( 2 )
3
>>> findNextPrime ( 1000 )
1009
>>> findNextPrime ( 100000000 )
100000007
>>> isPrime ( 100000007 )
True
>>> isPrime ( 1001 )
False
>>> isPrime ( 1003 )
False
>>> isPrime ( 1007 )
False
>>> isPrime ( 1009 )
True
```

One More Example

Suppose we want to find and print k primes, starting from a given number:

In file `IsPrime3.py`:

```python
def findKPrimesStartingFrom ( k, num ):
    """ Find the next k primes bigger than num. """
    if (k < 1):
        print( "You asked for zero primes!" )
    else:
        for i in range( k ):
            nextPrime = findNextPrime( num )
            print( nextPrime, end=" " )
            num = nextPrime
        print()
```

Notice that we can use functions we've defined such as `findNextPrime` and `isPrime` (almost) *as if* they were Python primitives.

```
>>> from IsPrime3 import findKPrimesStartingFrom
>>> findKPrimesStartingFrom( -10, 100000000 )
You asked for zero primes!
>>> findKPrimesStartingFrom( 5, -10 )
2 3 5 7 11
>>> findKPrimesStartingFrom( 10, 100000000 )
100000007 100000037 100000039 100000049 100000073 100000081
    100000123 100000127 100000193 100000213
```

Functions and Return Values

Functions can return a value or not. A function that doesn't return a value is sometimes called a **procedure**.

Of the functions defined earlier:

- sumToInt, multToN, findNextPrime all return int values
- farhToCelsius and celsiusToFahr return float values
- isPrime returns a bool value
- printSquares, printFahrToCelsius, print100Primes, and findKPrimesStartingFrom don't return a value (return None).

This function has four formal parameters:

```
def functionName ( x1, x2, x3, x4 ):
    < body >
```

Any call to this function should have exactly four actual arguments, which are matched to the corresponding formal parameters:

```
    functionName( 9, 12, -3, 10 )
    functionName( 'a', 'b', 'c', 'd' )
    functionName( 2, "xyz", 2.5, [3, 4, 5] )
```

This is called using **positional** arguments; it's by far the most common approach.

Keyword Arguments

It is also possible to use the formal parameters as **keywords**.

```
def functionName ( x1, x2, x3, x4 ):
    functionBody
```

These two calls are equivalent:

```
functionName ( 'a', 'b', 'c', 'd' )
functionName ( x3 = 'c', x1 = 'a', x2 = 'b', x4 = 'd' )
```

You can list the keyword arguments in any order, but all must still be specified.

You can mix keyword and positional arguments, but *must* have positional arguments first in order.

```python
def functionName ( x1, x2, x3, x4 ):
    functionBody

functionName( 'a', 'b', x4 = 'd', x3 = 'c' )    # OK
functionName( x2 = 'b', x1 = 'a', 'c', 'd' )    # illegal
```

Why do you think they make this rule?

Default Parameters

You can also specify **default arguments** for a function. If you don't specify a corresponding actual argument, the default is used.

```python
def printRectangleArea( width = 1, height = 2 ):
    area = width * height
    print("width: ", width, "\theight: ", height, \
          "\tarea:", area)

printRectangleArea()                       # use defaults
printRectangleArea(4, 2.5)                 # positional args
printRectangleArea(height = 5, width = 3)  # keyword args
printRectangleArea(width = 1.2)            # default height
printRectangleArea(height = 6.2)           # default width
```

Using Defaults

```
> python RectangleArea.py
width:  1        height:  2       area: 2
width:  4        height:  2.5     area: 10.0
width:  3        height:  5       area: 15
width:  1.2      height:  2       area: 2.4
width:  1        height:  6.2     area: 6.2
```

Notice that you can mix default and non-default arguments, but must define the non-default arguments first.

```
def email (address, message = ""):
```

All values in Python are objects, including numbers, strings, etc.

When you pass an argument to a function, you're actually passing a **reference** (pointer) to the object, not the object itself.

There are two kinds of objects in Python:

mutable: you can change them in your program.

immutable: you can't change them in your program.

If you pass a reference to a mutable object, it can be changed by your function. If you pass a reference to an immutable object, it can't be changed by your function.

Python Types

Class	Description	Syntax example
int	An immutable fixed precision number of unlimited magnitude	42
float	An immutable floating point number (system-defined precision)	3.1415927
str	An immutable sequence of characters.	'Wikipedia' "Wikipedia" """Spanning multiple lines"""
bool	An immutable truth value	True, False
tuple	Immutable, can contain mixed types	(4.0, 'string', True)
bytes	An immutable sequence of bytes	b'Some ASCII' b"Some ASCII"
list	Mutable, can contain mixed types	[4.0, 'string', True]
set	Mutable, unordered, no duplicates	{4.0, 'string', True}
dict	A mutable group of key and value pairs	{'key1': 1.0, 3: False}

Passing an Immutable Object

Consider the following code:

```python
def increment (x):
  x += 1
  print ( "Within the call x is: ", x )

x = 3
print ( "Before the call x is: ", x )
increment ( x )
print ( "After the call x is: ", x )

def revList (lst):
  lst.reverse ()
  print ( "Within the call lst is: ", lst )

lst = [1, 2, 3]
print ( "Before the call lst is: ", lst )
revList ( lst )
print ( "After the call lst is: ", lst )
```

Invoking this code:

```
>python Test.py
Before the call x is:  3
Within the call x is:  4
After the call x is:  3

Before the call lst is:  [1, 2, 3]
Within the call lst is:  [3, 2, 1]
After the call lst is:  [3, 2, 1]
```

Notice that the immutable integer parameter to `increment` was unchanged, while the mutable list parameter to `revList` was changed.

Scope of Variables

Variables defined in a Python program have an associated **scope**, meaning the portion of the program in which they are defined.

A **global variable** is defined outside of a function and is visible after it is defined. *Use of global variables is generally considered bad programming practice.*

Python Variable Scope

A **local variable** is defined within a function and is visible from the definition until the end of the function.

A local definition overrides a global definition.

Overriding

A local definition (locally) overrides the global definition.

```
x = 1                      # x is global

def func ():
    x = 2                  # this x is local
    print ( x )            # will print 2

func ()
print ( x )                # will print 1
```

Running the program:

```
> python funcy.py
2
1
```

Global Variables

```python
callCount = 0              # global variable

def caller ():
  global callCount    # needed to access
  callCount += 1

caller()
print( "callCount = ", callCount )
caller()
print( "callCount = ", callCount )
caller()
print( "callCount = ", callCount )
```

```
> python Test.py
callCount =  1
callCount =  2
callCount =  3
```

What would happen if you took out the line containing `global`?

The Python `return` statement can also return multiple values. In fact it returns a *tuple* of values.

```python
def multipleValues ( x , y ):
  return x + 1, y + 1

print ( "Values returned are: ",  multipleValues ( 4, 5.2 ))

x1 , x2 = multipleValues ( 4, 5.2 )
print ( "x1: ", x1, "\tx2: ", x2 )
```

```
Values returned are:   (5, 6.2)
x1:   5   x2:   6.2
```

You can operate on this using tuple functions, which we'll cover later in the semester, or assign them to variables.

Order of Things in a .py File

Python is pretty permissive about the order of things in your .py file. The following is the order I prefer:

- Header / Extended comment explaining what's in the file
- Any imports required
- Program constants
- Function definitions
- `main` function definition
- Call to `main` function

Include comments throughout.

Next stop: Objects and Classes.