CS429: Computer Organization and Architecture

Logic Design

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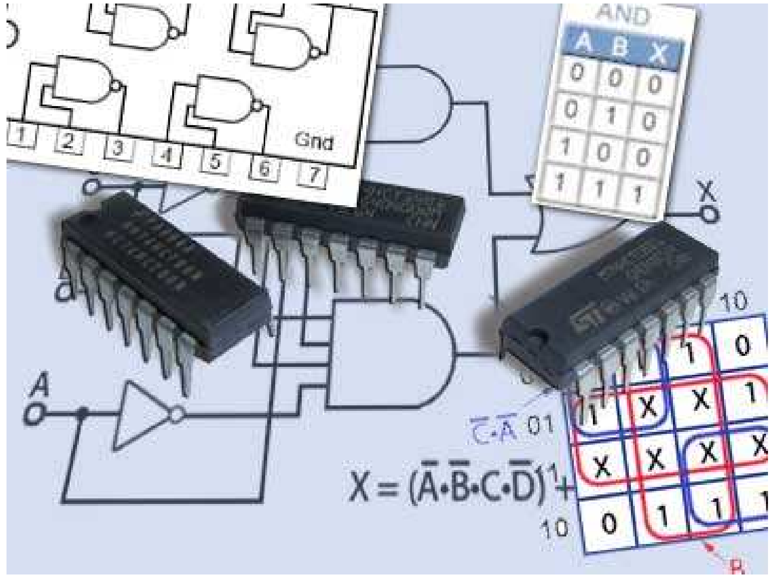


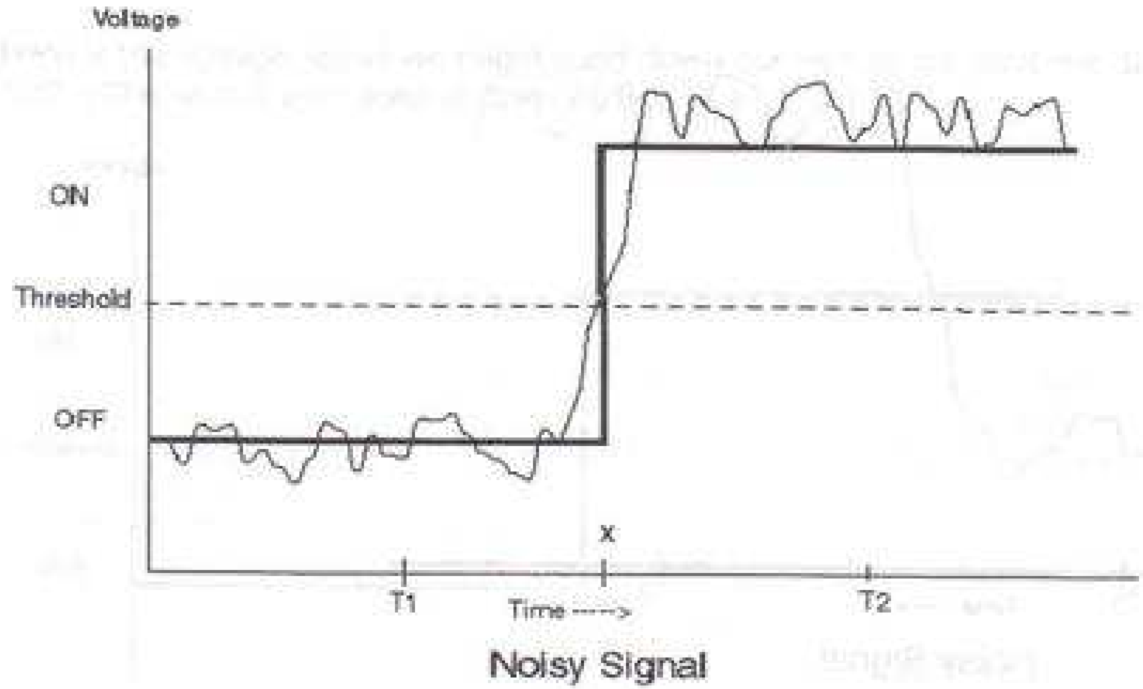
To execute a program we need:

- **Communication:** getting data from one place to another
- **Computation:** perform arithmetic or logical operations
- **Memory:** store the program, variables, results

Everything is expressed in terms of bits.

- Communication: Low or high voltage on a wire
- Computation: Compute boolean functions
- Storage: Store bits

Digital Signals



- Use voltage thesholds to extract discrete values from a continuous signal.
- Simplest version: 1-bit signal
  - Either high range (1) or low range (0)
  - With a guard range between them.
- Not strongly affected by noise or low-quality elements; circuits are simple, small and fast.

Truth Tables

And: `A & B` $= 1$ when both A $= 1$ and B $= 1$.

A	B	&
0	0	0
0	1	0
1	0	0
1	1	1

Or: `A | B` $= 1$ when either A $= 1$ or B $= 1$.

<table>
<tr><th>A</th><th>B</th><th>|</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>

Not: `~A` $= 1$ when A $= 0$.

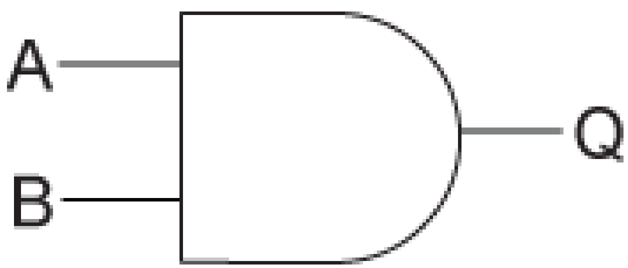
A	~
0	1
1	0

Xor: `A ^ B` $= 1$ when either A $= 1$ or B $= 1$, but not both.

A	B	^
0	0	0
0	1	1
1	0	1
1	1	0

Gates

What does it mean for a hardware device to represent a boolean function (or truth table), say **and**?



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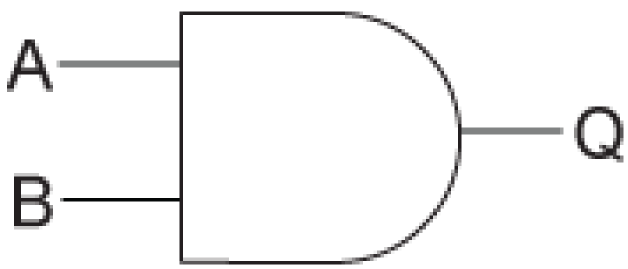
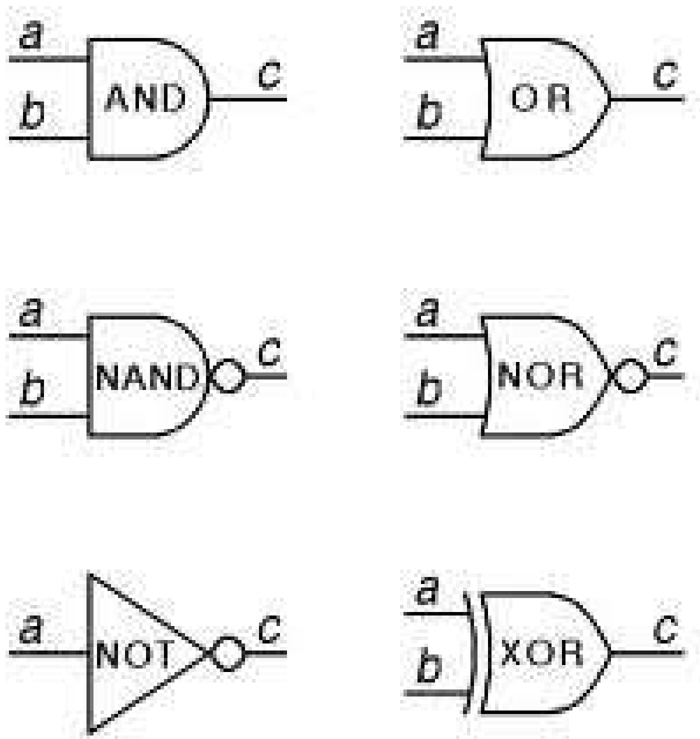


1. Place on the two input lines voltages representing logical values (T or F).
2. After a short *delay*, the output line will stabilize to a voltage representing the logical **and** of the inputs.

Computing with Logic Gates

How are these logic functions actually computed in hardware?

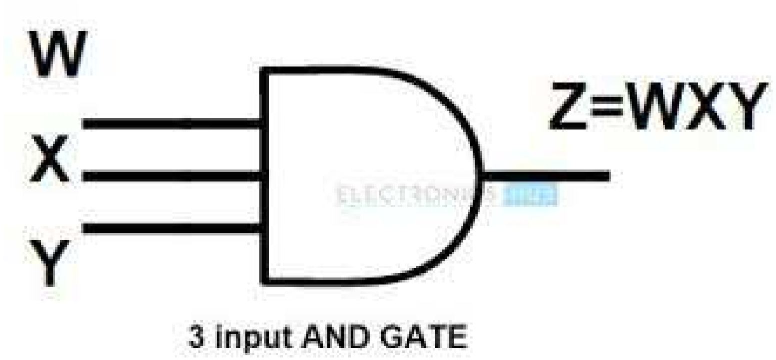
- Logic gates are constructed from transistors.
- The output is a boolean function of inputs.
- The gate responds continuously to changes in input *with a small delay.*



How many of these do you really need?

Aside: Multiple-Input Gates

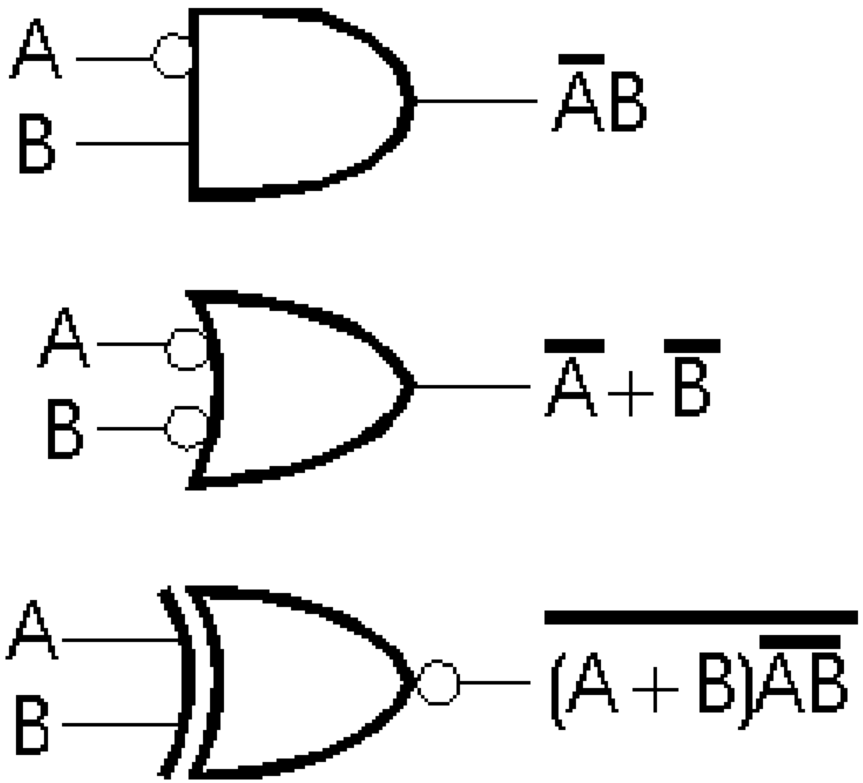
Some gates allow multiple inputs. For example, a 3-input AND is essentially just a cascade of two 2-input ANDs.



For which gates does it make sense to have extra inputs? For which doesn't it make sense?

Aside: Inverted Inputs/Outputs

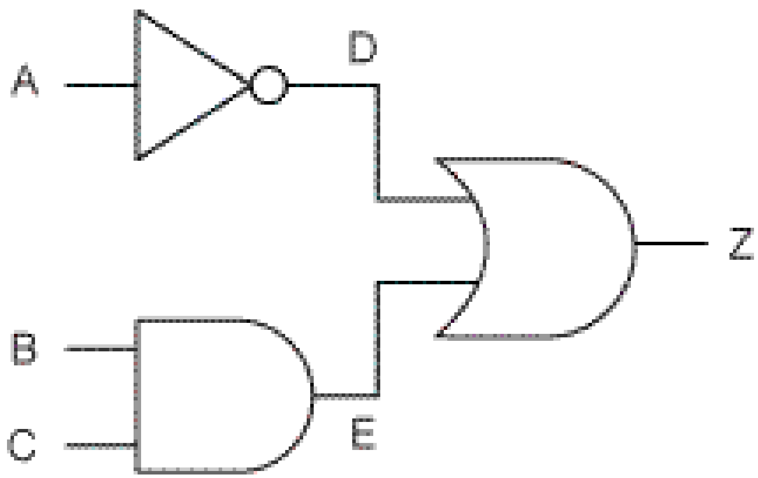
A small circle on either the input or output of a gate means that that signal is inverted. That is, it's as if there were an inverter (not) gate there.



What would an **implies** gate look like?

A Complex Function

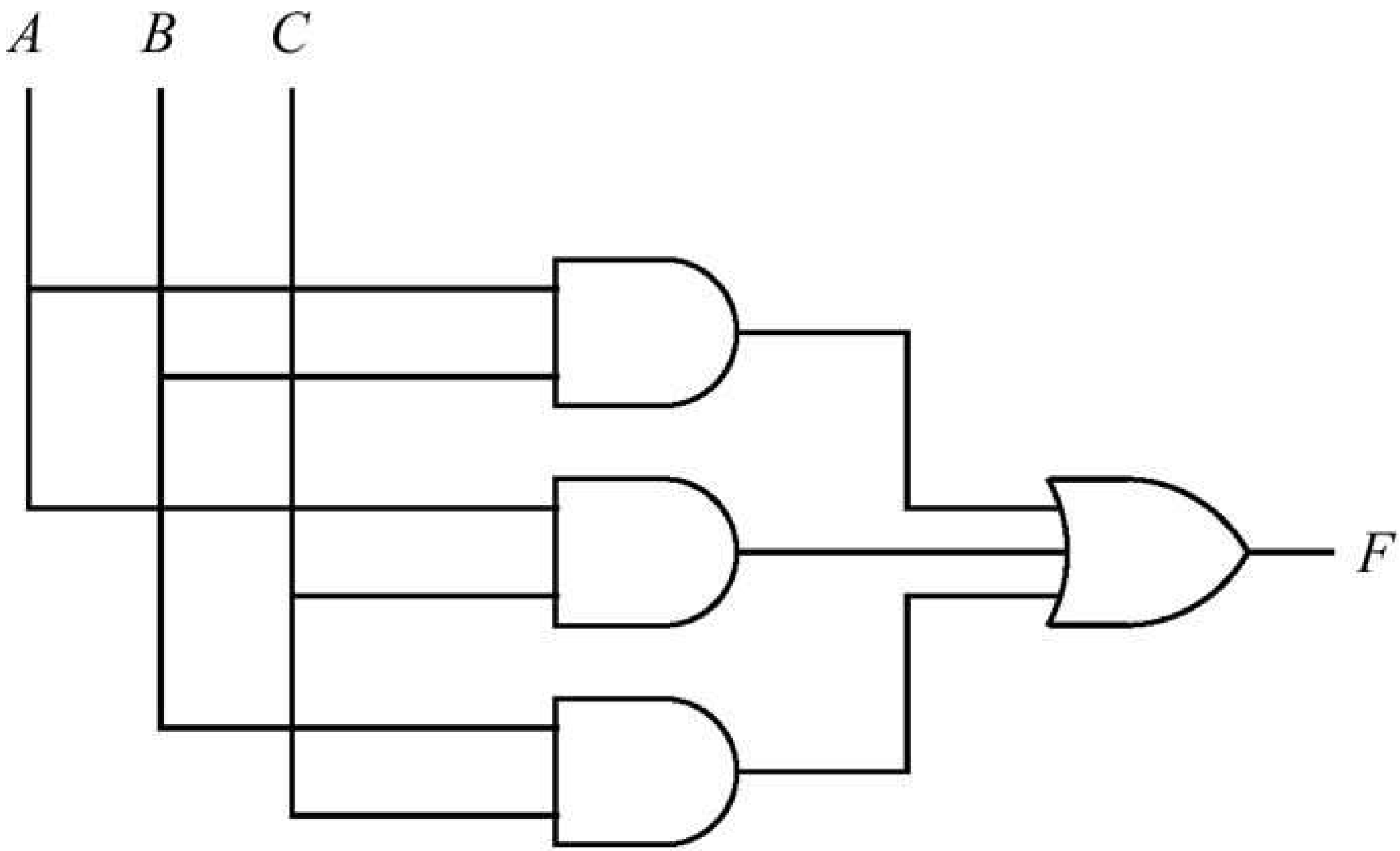
Primitive boolean functions are implemented by logic gates; more complex functions, by combinations of gates.



A	B	C	Z
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

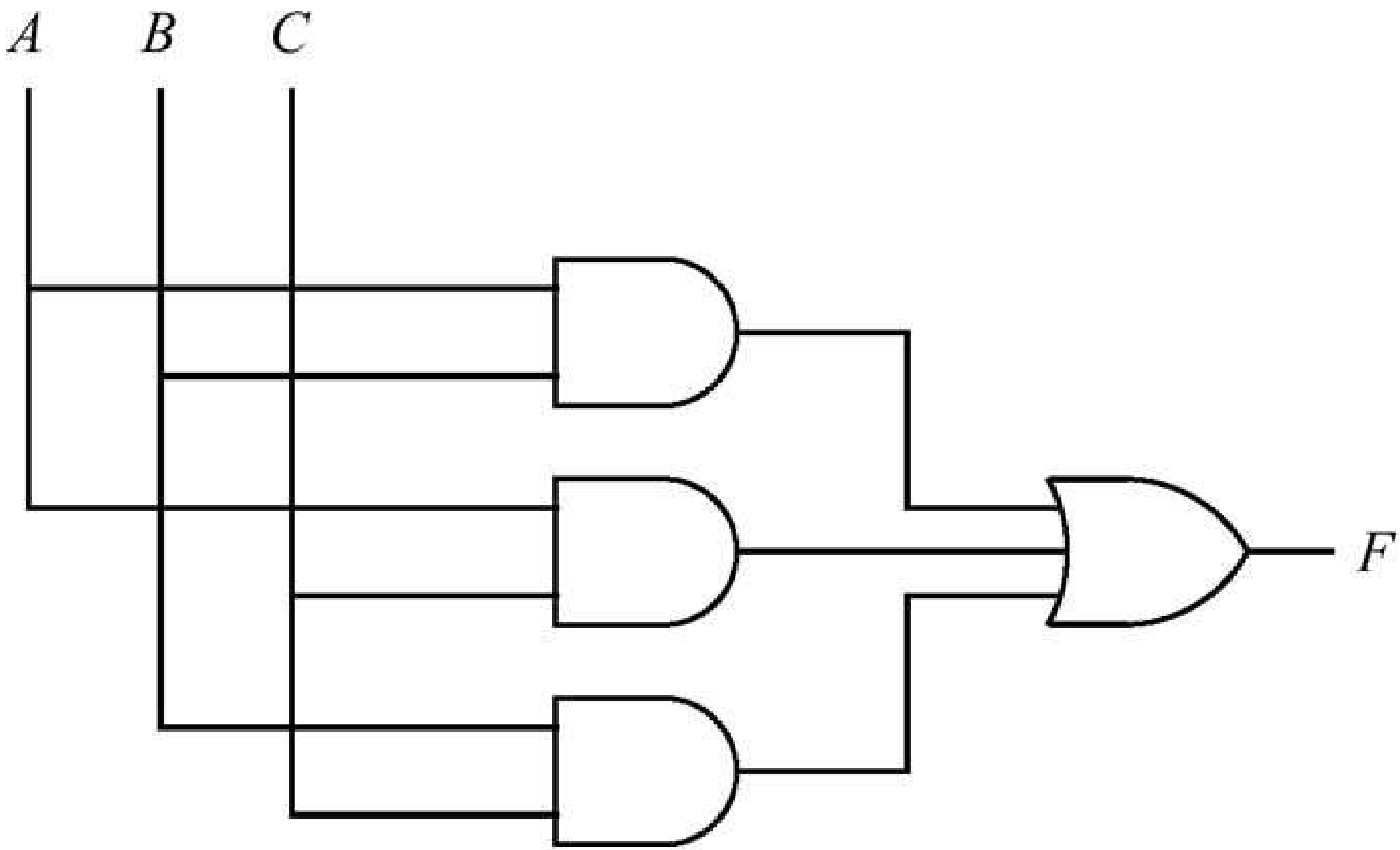
```
Z = !A || (B && C);
```

Another Circuit



Which wires are connected and which are not? Can you see what this circuit does?

Another Circuit



Which wires are connected and which are not? Can you see what this circuit does?

This is called a *majority circuit*. What function does it compute?

Sets of Logic Gates

It's pretty easy to see that any boolean function can be implemented with AND, OR and NOT. Why? We call that a *functionally complete* set of gates.

You can get by with fewer gates. How would you show each of the following?

- AND and NOT is complete.
- OR and NOT is complete.
- NAND is complete.
- NOR is complete.
- AND alone is not complete.
- OR alone is not complete.

Often circuit designers will restrict themselves to a small subset of gates (e.g., just NAND gates). Why would they do that?

Using Logic for Arithmetic

Suppose you wanted to do addition with logic. How might you go about that?

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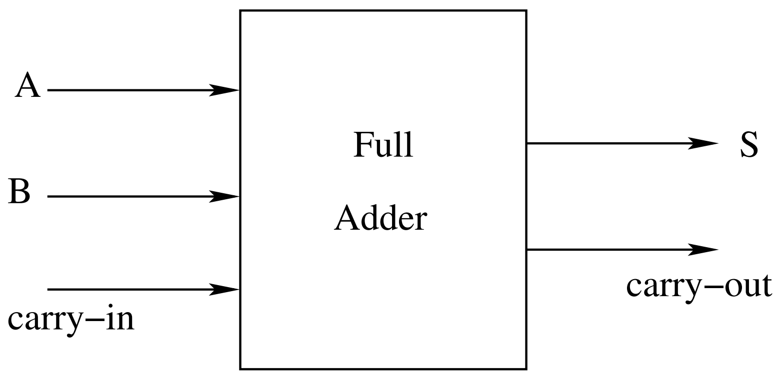
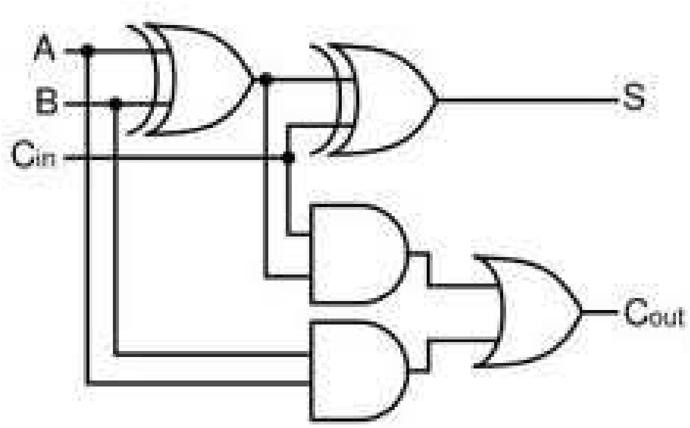
Define a circuit (full adder) that does one step in an addition:



A	B	Cin	Cout	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Full Adder

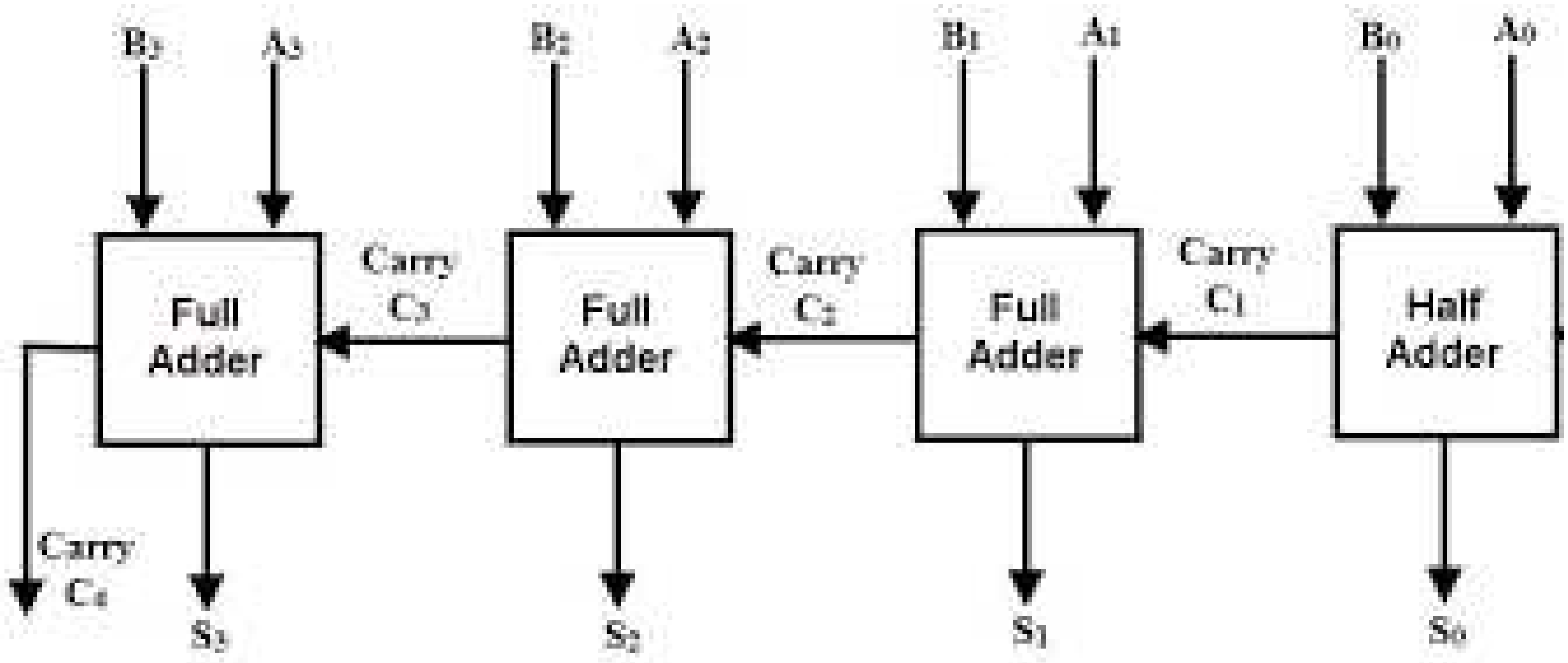
The following circuit is a full adder:



A half adder is a simpler circuit with only inputs A and B.

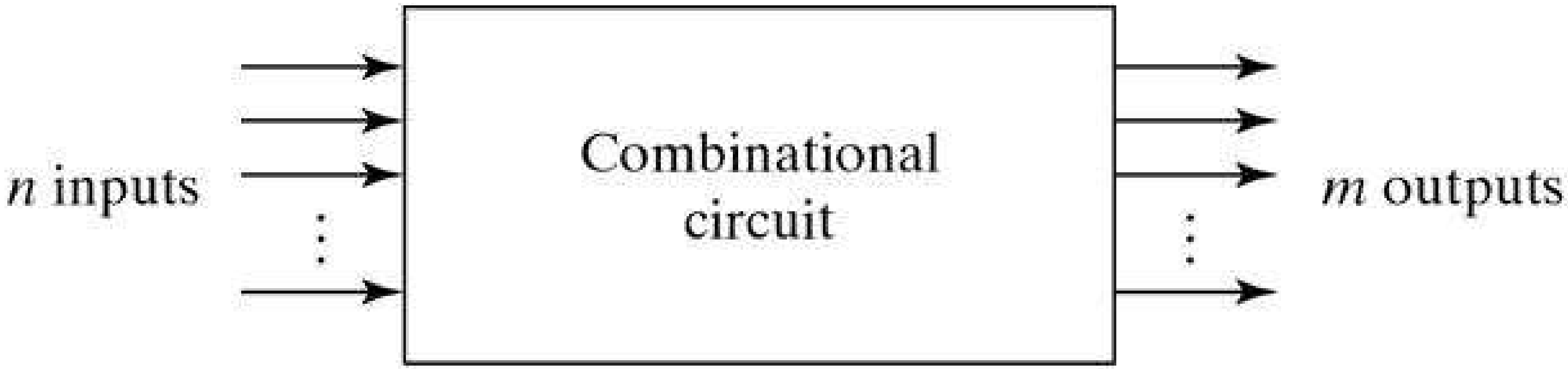
A	B	Cin	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Adding a Pair of 4-bit Ints



How do you subtract? How do you multiply?

Combinational Circuits

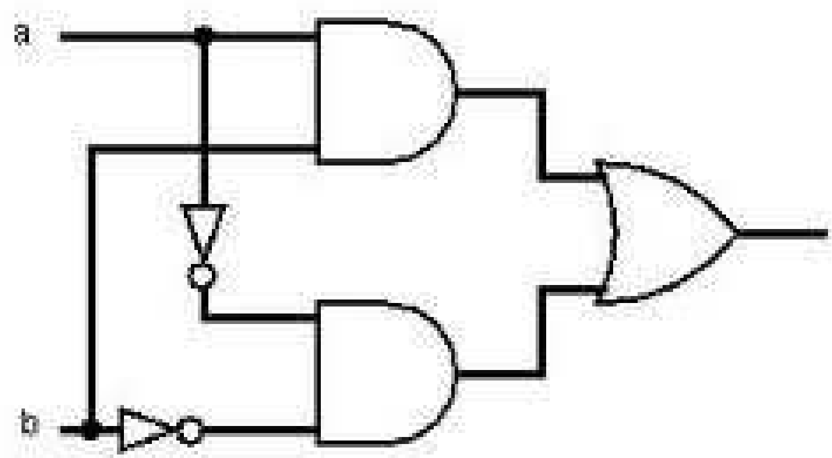


The box contains an acyclic network of logic gates.

- Continuously responds to changes in inputs.
- Outputs become (after a short delay) boolean functions of the inputs.

Bit Equality

The following circuit generates a 1 iff a and b are equal.




```
int eq = (a&&b) || (!a&&!b);
```

Can you design a simpler circuit to do this?

Hardware description languages (Verilog, VHDL)

- Describe control, data movement, ...
- "Compile" (synthesize) a hardware description into a circuit.

Verilog Example

One of the more widely used HDL's is Verilog:

```
module simp_circuit (A, B, C, x, y);
   input A, B, C;
   output x, y;
   wire e;
   and g1 (e, A, B);
   not g2 (y, C);
   or g3 (x, e, y);
endmodule
```

HCL

Hardware Control Language (HCL)

- Very simple hardware description language.
- Boolean operations have syntax similar to C logical operations.
- We'll use it to describe control logic for processors.

Data types

- bool: Boolean (a, b, c, ...)
- int: words (A, B, C, ...)
- Does not specify word size

Statements

- bool a = bool-expr;
- int A = int-expr;

HCL Operations

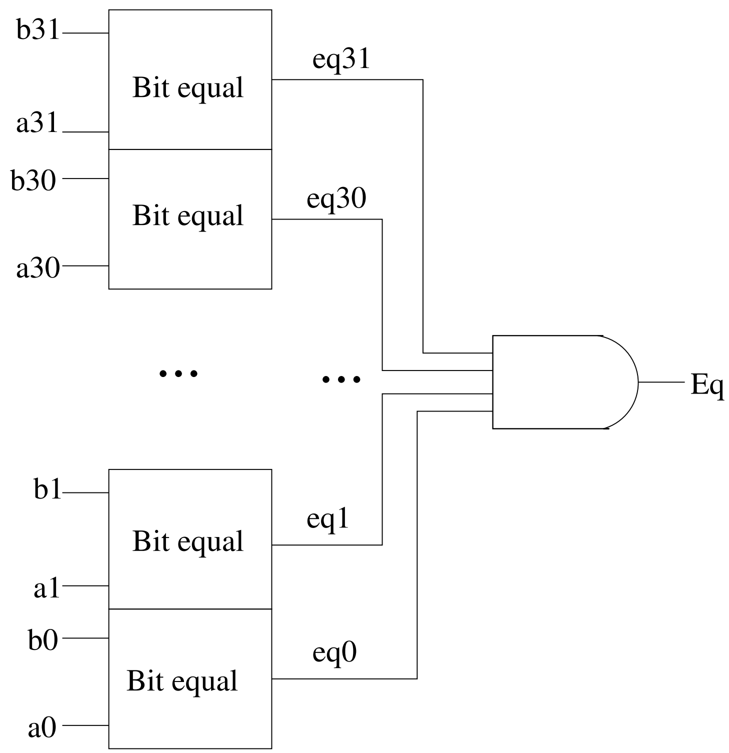
Boolean expressions

- Logic operations: `a && b`, `a || b`, `!a`
- Word comparisons: `A == B`, `A != B`, `A < B`, `A <= B`, `A >= B`, `A > B`
- Set membership: `A in {B, C, D}`

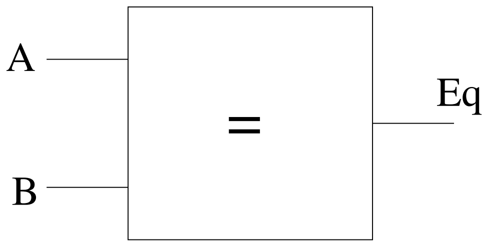
Word expressions

- Case expressions: `[a: A; b: B; c: C]`
- Evaluate Boolean expressions `a, b, c` in sequence
- Return corresponding word expression for first successful Boolean evaluation.

Word Equality



Word-level representation:



HCL Representation:

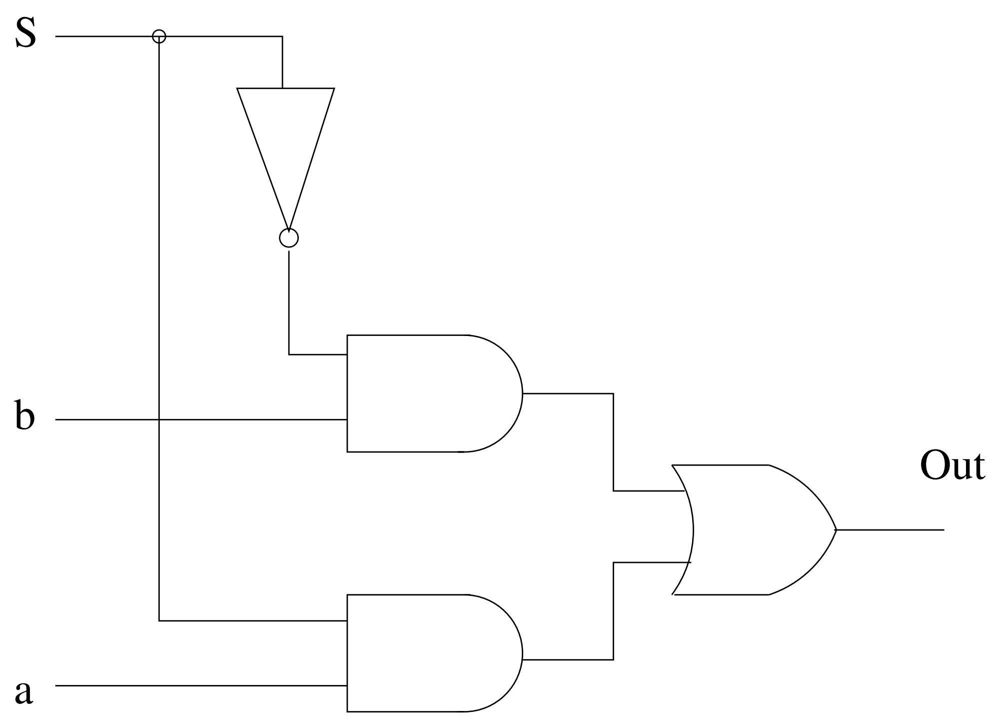
```
Eq = (A == B)
```

Assume 32-bit word size.

HCL representation

- Equality operation
- Generates Boolean value

Bit Multiplexor

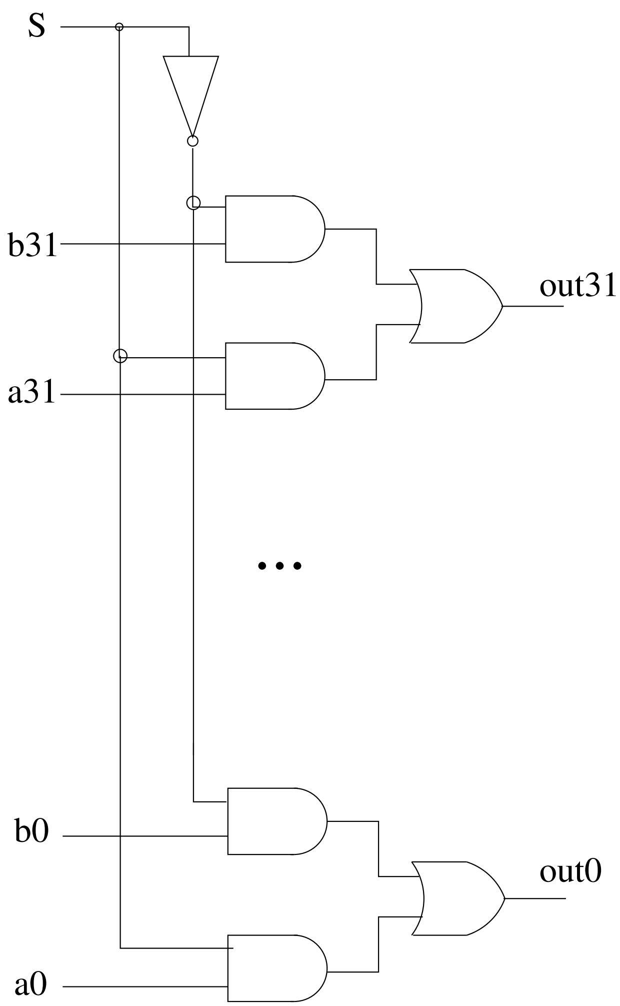


HCL Expression:

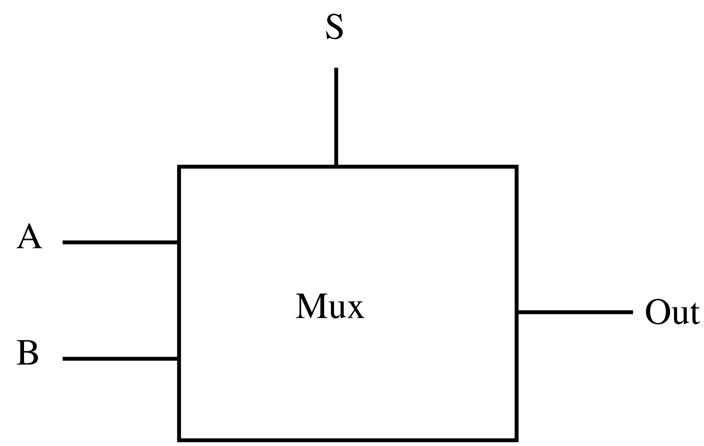
```
int out = (s && a) || (!s && b);
```

- Control signal s selects between two inputs a and b.
- Output is a when s == 1, and b otherwise.

Word Multiplexor



Word-level representation:

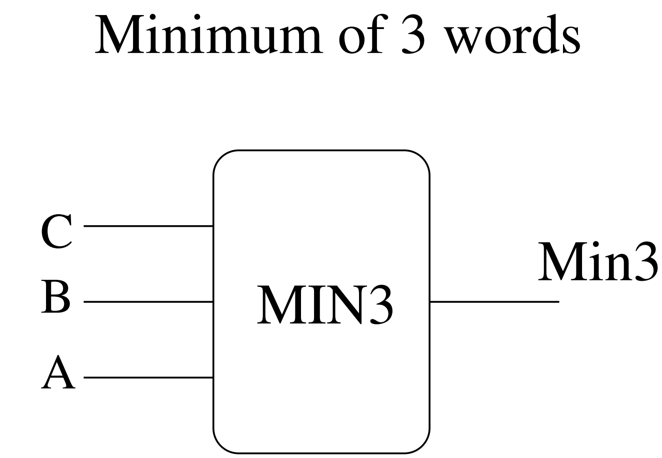


HCL Representation:

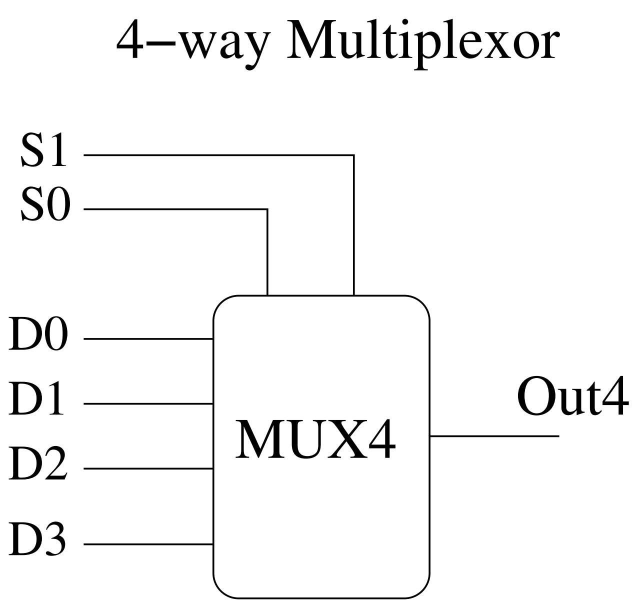
```
int Out = [
    s : A;
    1 : B;
];
```

Select input word A or B depending on control signal S.

Word Examples




```
int Min3 = [
  A <= B && A <= C : A;
  B <= A && B <= C : B;
  1                : C;
]
```




```
int Out4 = [
  !s1 && !s0 : D0;
  !s1        : D1;
  !s0        : D2;
  1          : D3;
]
```

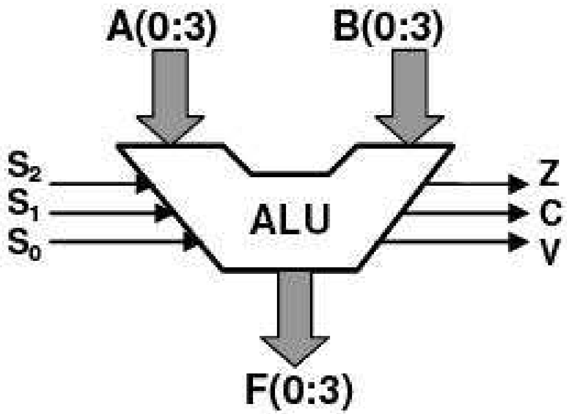
What do these do?

Constructing an ALU

An ALU is an Arithmetic Logic Unit

- Multiple functions: `add`, `subtract`, `and`, `xor`, others
- Combinational logic to perform functions.
- Control signals select function to be performed.
- Modular: multiple instances of 1-bit ALU

A 4-bit ALU



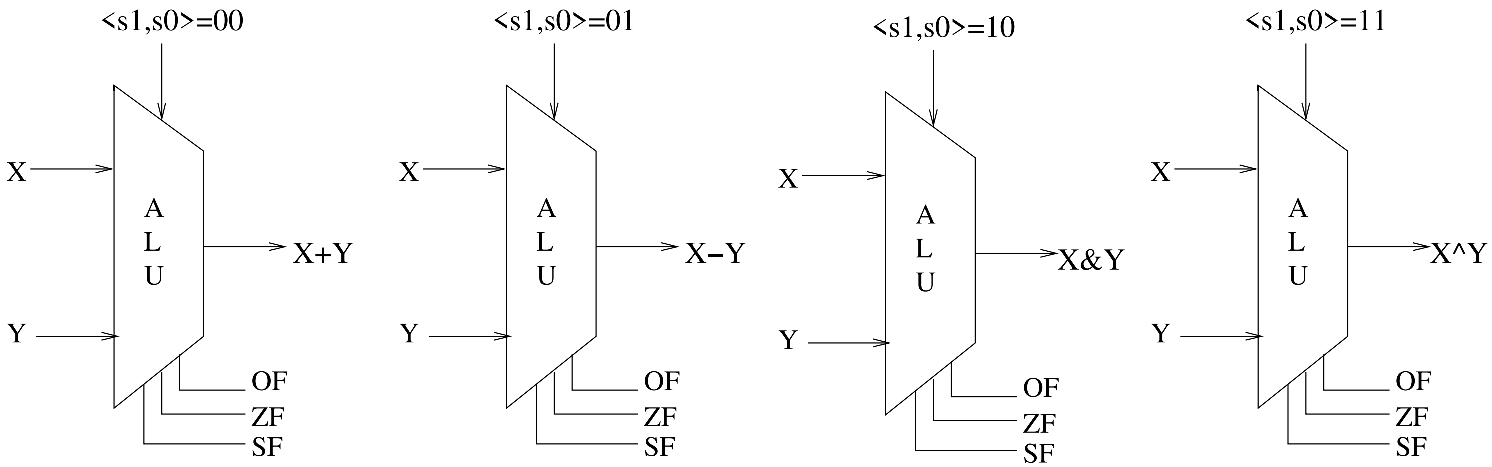
Z, C and V are status flags

Z = 1 if F=0
C = Carry or Borrow
V = Overflow

S_2	S_1	S_0	Function (F)
0	0	0	A+B
0	0	1	A-B
0	1	0	A-1
0	1	1	A+1
1	0	0	$A \wedge B$
1	0	1	$A \vee B$
1	1	0	NOT A
1	1	1	$A \oplus B$

- Combinational logic: continuously responding to inputs.
- Control signal selects function computed; Y86 ALU has only 4 arithmetic/logical operations.
- Also computes values of condition codes. Note these are not the same as the three Y86 flags:
    - OF: overflow flag
    - ZF: zero flag
    - SF: sign flag

The Y86 ALU in HCL




```
int Out = [
    !s1 && !s0: X+Y;
    !s1 && s0 : X-Y;
    s1  && !s0: X&Y;
    1         : X^Y;
];
```

Sequential Logic

How would you design a circuit that records a bit? What does that even mean?

Sequential Logic

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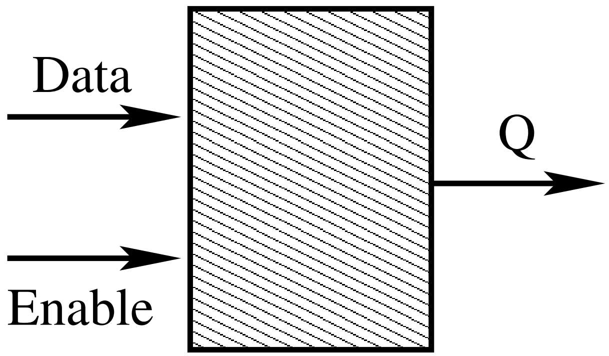
Ideally, you'd like a *bi-stable* device (latch) as follows:



The value on line `Q` is the current stored value.

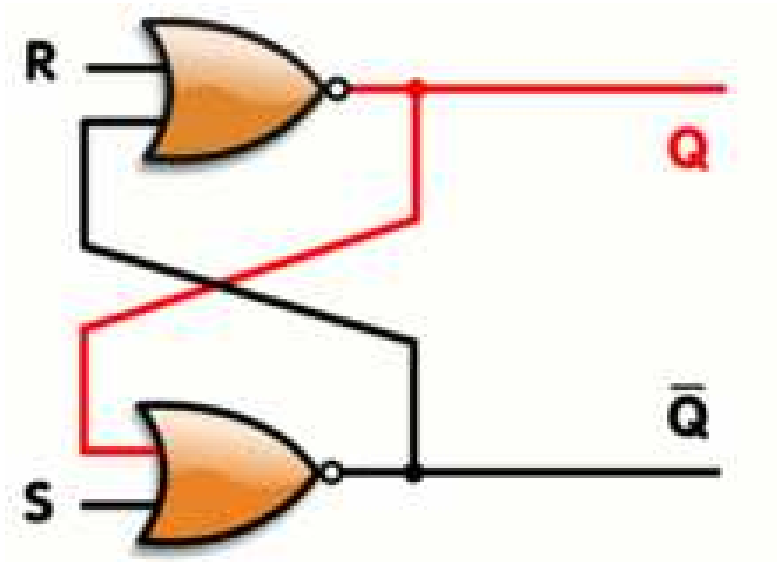
To store a new value:

1. Line `Enable` should be low (0).
2. Place the bit to store on line `Data`.
3. Raise `Enable` to high (1).
4. The value on line `Data` is stored in the device.
5. Lower `Enable` to low (0).
6. Reading `Q` returns the stored bit until next store.

Such "state-holding" devices are called *sequential logic* as opposed to *combinational logic*.

SR Flip Flop: Storing a Bit

An SR flip flop is a step in the direction of a latch.



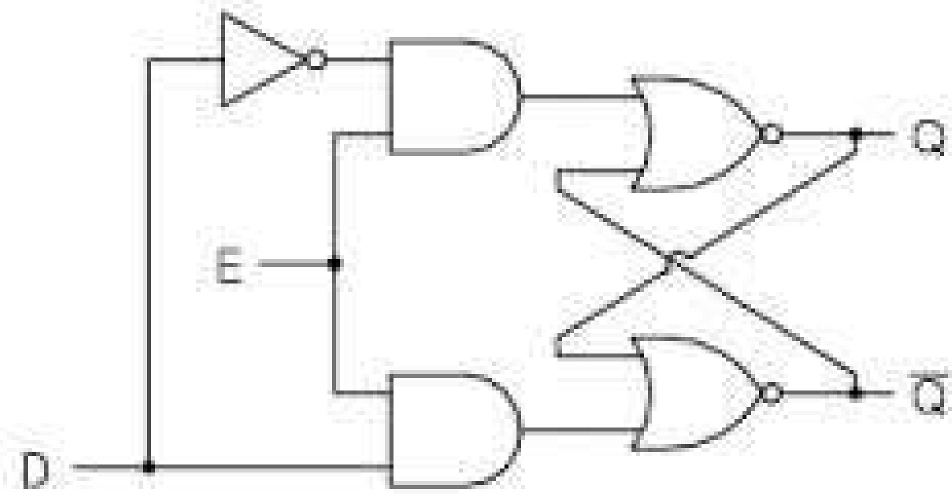
Pulse (temporarily raise) the R (reset) input to record a 0.

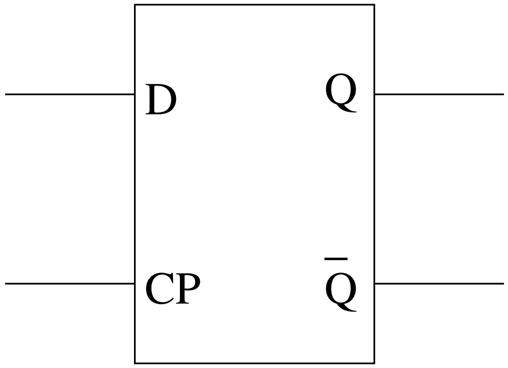
Pulse the S (set) input to record a 1.

Characteristic table

S	R	Qnext	Action
0	0	Q	hold state
0	1	0	reset
1	0	1	set
1	1	X	not allowed

This is not very convenient because it requires pulsing either S or R to record a bit.





Higher level representation
D Latch Truth table

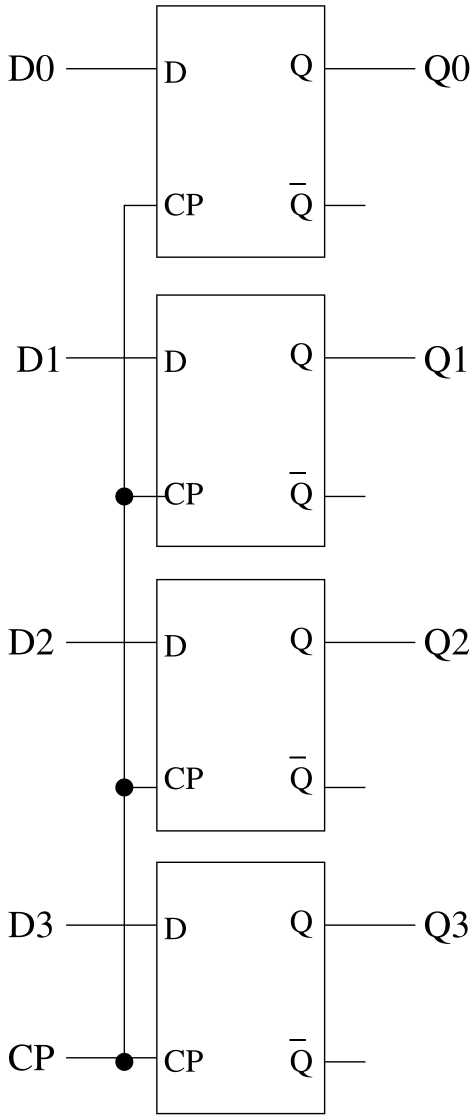
E/CP	D	Q	$\overline{Q}$	Comment
0	X	Q	$\overline{Q}$	No change
1	0	0	1	Reset
1	1	1	0	Set

E (enable) and CP (clock pulse) are just two names for the same input.

A 4-bit Register

4 D latches:

- All share the E/CP (aka WE or Write Enable) input
- D0–D3 are the data input
- Q0–Q3 are the output

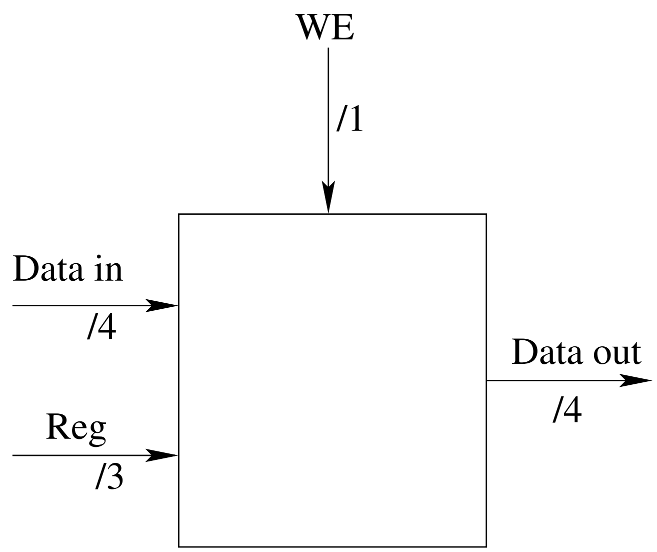


Register File Abstraction

Register file provides the CPU with temporary, fast storage.

- N registers.
- Each of K bits.
- L output ports.

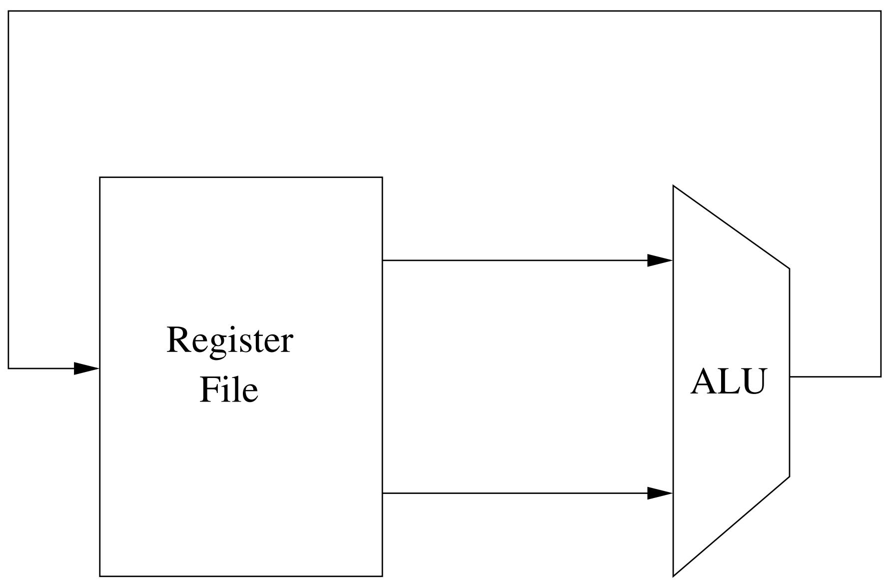
Suppose we want eight 4-bit registers and one output port.



Race-through Condition with D Latches

Write Enable (WE) must be held at "1" long enough to allow:

- Data to be read;
- Operation (e.g., addition) to be performed;
- Result to be stored in target register.



Edge Triggered Flip Flops

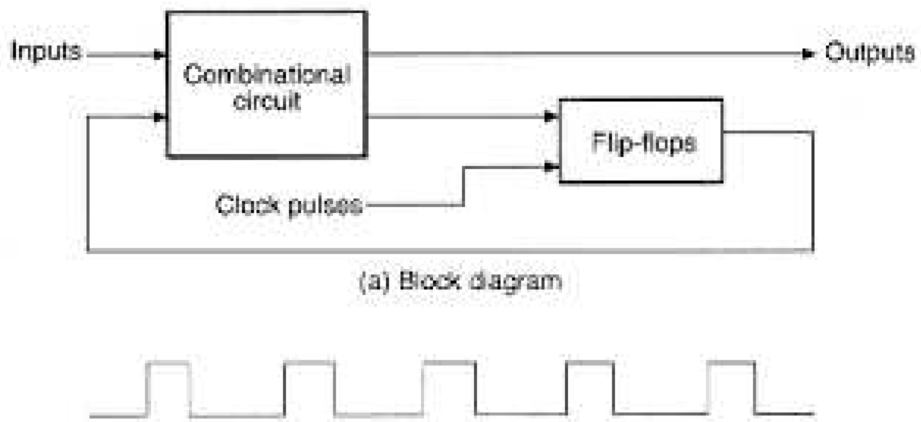
An edge-triggered flip-flop changes states either at the positive edge (rising edge) or at the negative edge (falling edge) of the clock pulse on the control input.

- A register is made up of several flip flops, each providing storage and access for an individual bit.
- A register file is made up of several registers and control logic

Clocking

The clock acts to enforce timing control on the chip.

- An integral part of every synchronous system.
- Can be global



(a) Block diagram

(b) Timing diagram of clock pulses

Clock Frequency = 1 / clock period

- Measured in cycles per second (Hertz)
- 1 KHz = 1000 cycles / second
- 1ns (10^{-9} seconds) = 1GHz (10^{9}) clock frequency
- Higher frequency means faster machine speed.

Random Access Memory (RAM)

Stores many words

- Conceptually, a large array where each row is uniquely addressable.
- In reality, much more complex to increase throughput.
- Multiple chips and banks, interleaved, with multi-word operations.

Many implementations

- Dynamic (DRAM) is large, inexpensive, but relatively slow.
  - 1 transistor and 1 capacitor per bit.
  - Reads are destructive.
  - Requires periodic refresh.
  - Access time takes hundreds of CPU cycles.
- Static (SRAM) is fast but expensive.
  - 6 transistors per bit.
  - Streaming orientation.

Summary

Computation

- Performed by combinational logic.
- Implements boolean functions.
- Continuously reacts to inputs.

Storage

- Registers: part of the CPU.
  - Each holds a single word.
  - Used for temporary results of computation.
  - Loaded on rising clock.
- Memory is much larger.
- Variety of implementation techniques.