

## The Effects of Inaccurate Data

- As a simple situation, consider that a belt is to be made about the equator of the earth. You may take the earth's diameter as 7,926 miles. The belt is one foot too large for the equator. In order to make the belt fit, you will put some sand underneath it at an equal height all around the equator. How deep in inches should the sand be? (You made an error of one foot in measuring the circumference. Your problem is to see how that affects the radius of the actual sphere that has this larger value as its circumference.)

The length of the belt is  $7926 \cdot 5280\pi + 1$  feet, thus the diameter of the circle that will make the belt tight is  $(7926 \cdot 5280\pi + 1)/\pi$  feet. The difference in diameters in feet is

$$(7926 \cdot 5280\pi + 1)/\pi - 7926 \cdot 5280 = 1/\pi$$

Thus the difference in radii is half of this (i.e.,  $1/2\pi$  feet  $\approx 1.9098593171$  inches) and this is the depth of the sand wall that must be built all around the equator. Notice that the answer is independent of the diameter of the sphere – the same solution would result if the earth were replaced by the sun or were replaced by a pea.

- This one is trickier. Imagine that we are installing rails for a high speed railroad. One of the rails needs to be replaced. The gap that remains after the old rail is removed is 100 meters. (These rails are quite long to accommodate the smoothness required by the high speed train.) The company that makes the rails is able to guarantee the length to a 1 cm. tolerance. On the low side, if the rail is 99.99 meters then there will obviously be a 1 cm gap between the rails (In fact, they would center the rail so that there is a .5 cm. gaps at each end.) This is tolerable. On the high side however, the rail is 100.01 meters long and it will bow up into a circular arc above the 100 meter gap. Your problem is to determine the maximum height of the resulting bow. (This problem is not easy. Take it as far as you can. It would be nice if you could determine the maximum height as asked but, if you can only reduce it to some equations, do that much. For this problem I will allow co-working.)

We have the situation illustrated at the right. We seek the value of  $x$ . We have from the right triangle

$$r - x = r \cos(\theta),$$

thus

$$x = (1 - \cos(\theta))r.$$

But also from the right triangle

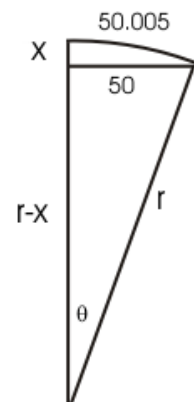
$$\sin(\theta) = \frac{50}{r}$$

and from the length of the arc

$$r\theta = 50.005,$$

so by removing  $r$  we obtain

$$\sin(\theta) = \frac{50}{50.005} \theta.$$



Solving this numerically for  $\theta$  we obtain  $\theta \approx .0244940401572$  and then we get  $x = .6123816211743$  meters. Thus having the rail just 1 centimeter too long results in it bowing up more than 61 centimeters.

[Note: This problem may be solved in many ways – including setting up a differential equation. One easy approximation is to replace the sine function in the nonlinear equation  $\sin(\theta) = \frac{50}{50.005}\theta$  by the first two terms of its Taylor series (i.e.,  $\theta - \theta^3/6$ ). The nonlinear equation

$$\theta - \theta^3/6 = \frac{50}{50.005}\theta$$

has the obvious solution

$$\theta = \sqrt{6\left(1 - \frac{50}{50.005}\right)} \approx .0244936727748$$

This approximate value of  $\theta$  results in an  $x$  of  $.6123724370734985$  (which is only .0015% too low).]