Three Interpolation Problems
Due Tuesday Nov. 12 by 9:30 AM

1. Use MATLAB to determine a function of the form:

$$f(x) = a_1 e^{x} + a_2 e^{2x} + a_3 \sin(4x) + a_4 \cos(4x) + a_5 x^{\pi}$$

that satisfies these conditions:

$$f(0) = f(3)$$

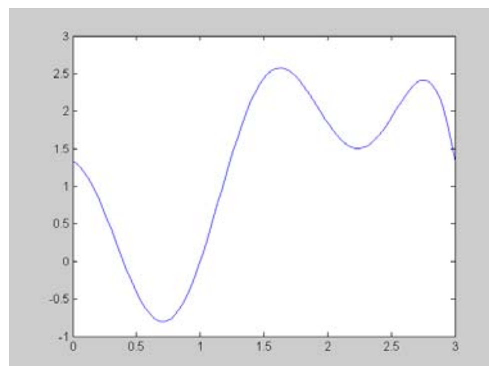
$$\int_0^1 f(x)\,dx = 0$$

$$f'(1) + 2f'(2) = 0$$

$$f''(2) = 5$$

$$f(1) = 0$$

```
A = [exp(0)-exp(3)   exp(2*0)-exp(2*3)             sin(4*0)-sin(4*3)             cos(4*0)-cos(4*3)             0^pi-3^pi;...
      exp(1)-1        1/2*exp(2)-1/2                -1/4*cos(4)+1/4               1/4*sin(4)                    1/(pi+1); ...
     exp(1)+2*exp(2) 2*(exp(2*1)+2*exp(2*2))       4*(cos(4*1)+2*cos(4*2))       -4*(sin(4*1)+2*sin(4*2))      pi*(1^(pi-1)+2*2^(pi-1));
     exp(2)           4*exp(2*2)                    -16*sin(4*2)                  -16*cos(4*2)                  pi*(pi-1)*2^(pi-2); ...
     exp(1)           exp(2*1)                      sin(4*1)                      cos(4*1)                      1^pi];
b = [0; 0; 0; 5; 0];
a = [A\b]
```



results in

```
a =

   0.19957523533614
  -0.03119132080256
  -0.22066848011201
   1.15994852350169
   0.27916318011764
```

Since cond(A) = 2.958240525108441e+003, we can trust all but the last three or four digits of the coefficients.

2. Use fzero to determine a function of the form:

$$f(x) = a_1 e^{x} + a_2 e^{x/2} + a_3 \sin(a_4 x)$$

that satisfies these conditions:

$$f(0) = 2$$

$$f(1) = 1$$

$$f(3) = 0$$

$$f(2) = 3$$

With

```
function y = prob2(a4)
A = [exp(0) exp(.5*0) sin(a4*0); exp(1) exp(.5*1) sin(a4*1); exp(3) exp(.5*3) sin(a4*3)];
b = [2; 1; 0];
a = A\b;
y = a(1)*exp(2)+a(2)*exp(.5*2)+a(3)*sin(a4*2)-3;
```

the call

```
a4 = fzero('prob2', 1)
```

results in

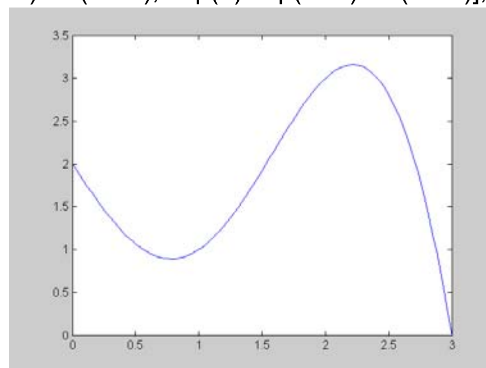
```
a4 =

  1.77473950772067
```

which is a(4). Then by invoking

```
A = [exp(0) exp(.5*0) sin(a4*0); exp(1) exp(.5*1) sin(a4*1); exp(3) exp(.5*3) sin(a4*3)];
b = [2; 1; 0];
a = A\b
```

we get the other three coefficients:

```
a =
 -0.65971127089006
  2.65971127089006
 -1.62552955781056
```



3. Use MATLAB functions to determine a function of the form:

$$f(x) = a_1 + a_2 x^2 + a_3 \sin(x)$$

that satisfies these conditions:

$$f(0) = 2$$

$$f(1) = 1$$

$$\max_{0 \le x \le .5} |f(x)| = 2.5$$

(Hint: You might want to use fzero here and in such a way that fzero invokes fminbnd .)

The command

```
a3 = fzero('prob3f', 1)
```

returns

```
a3 =   2.58363016233572
```

when using

```
function y = prob3f (a3)
A = [1 0; 1 1];
b = [2; 1-a3*sin(1)];
a = A\b;
a(3) = a3;
[x,y] = fminbnd ('prob3', 0, 1/2, optimset, a);
y = -y-2.5;
```

which invokes

```
function y = prob3 (x, a)
y = -abs(a(1)+a(2)*x^2+a(3)*sin(x));
```

Then

```
A = [1 0; 1 1];
b = [2; 1-a3*sin(1)];
a = A\b;
a(3) = a3;
```

yields the solution

```
a =
  2.00000000000000
 -3.17404981708003
  2.58363016233572
```