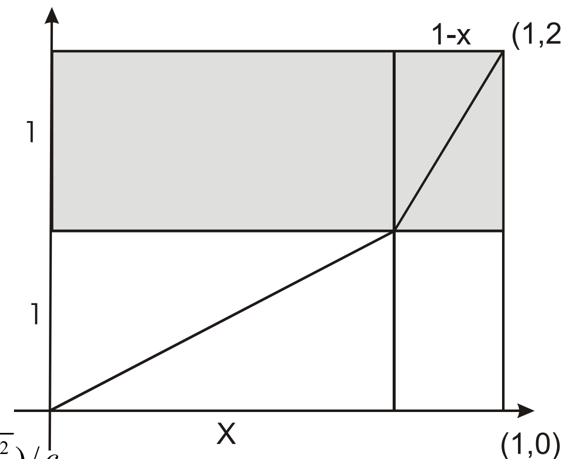
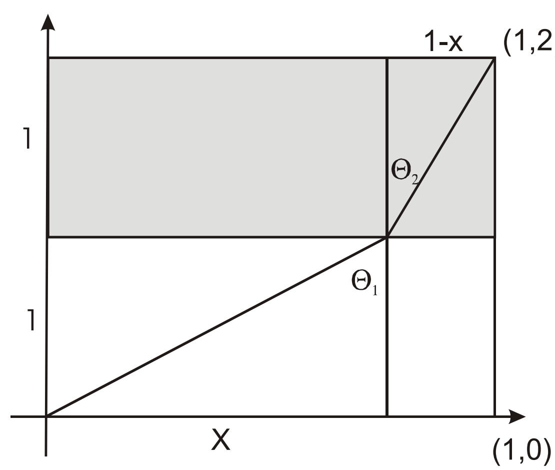
Refraction as Minimization

This problem deals with the direction of a light ray (or a ray of any other wave phenomenon) as it passes through a boundary from a material with one speed of transmission into a second material with a different speed of transmission. The principle that governs the direction of the ray's change of direction as it crosses the boundary is Fermat's principle. The original statement of Fermat's principle was, "The actual path between two points taken by a beam of light is the one which is traversed in the least time."



Consider the situation to the right in which a layer with transmission speed c_1 is beneath a layer with transmission speed c_2. The distance from $(0,0)$ to $(x,1)$ is $\sqrt{1+x^2}$ thus the time for the beam of light to travel the distance is $\sqrt{1+x^2}/c_1$. The distance from $(1,x)$ to $(1,1)$ is $\sqrt{1+(1-x)^2}$ thus the time for the beam of light to travel the distance is $\sqrt{1+(1-x)^2}/c_2$ and the total transit time is $\sqrt{1+x^2}/c_1+\sqrt{1+(1-x)^2}/c_2 = (\sqrt{1+x^2}{}_1 + r\sqrt{1+(1-x)^2})/c_1$, where r is just the ratio c_1/c_2. In units of inverse c_1 the transit time is $\sqrt{1+x^2}{}_1 + r\sqrt{1+(1-x)^2}$, and this will suffice for our purposes here since the units are relevant.

1. Construct a MATLAB function **transtime** that has inputs **x** and **r** and returns the time the ray requires to transit from (0,0) to (1,2).



2. Let **r** be an array of 101 linearly spaced points from 1 to 10. For each element of the array, use **fminbnd** to compute a minimum of **transtime** to get the optimal positions **x**. This results in an array **x** as a function of the refraction index **r**. Plot **x** versus **r** (i.e., the dependent variable is **x**).

3. Consider the angle of incidence θ_1 and angle of refraction θ_2. Recognize that $\sin\theta_1 = \dfrac{x}{\sqrt{1+x^2}}$ and $\sin\theta_2 = \dfrac{1-x}{\sqrt{1+(1-x)^2}}$. Compute arrays **sintheta1** and **sintheta2** from **x**. Create a plot of **sintheta1./sintheta2** versus **r**. You should be able to conclude Snell's Law from this.