Refraction as Minimization Solution

Consider the situation in which a layer with transmission speed c_1 is beneath a layer with transmission speed c_2. The distance from $(0,0)$ to $(x,1)$ is $\sqrt{1+x^2}$ thus the time for the beam of light to travel the distance is $\sqrt{1+x^2}\,/\,c_1$. The distance from $(1,x)$ to $(1,1)$ is $\sqrt{1+(1-x)^2}$ thus the time for the beam of light to travel the distance is $\sqrt{1+(1-x)^2}\,/\,c_2$ and the total transit time is $\sqrt{1+x^2}\,/\,c_1+\sqrt{1+(1-x)^2}\,/\,c_2=(\sqrt{1+x^2}{}_1+r\sqrt{1+(1-x)^2})/c_1$, where r is just the ratio c_1/c_2. In units of inverse c_1 the transit time is $\sqrt{1+x^2}{}_1+r\sqrt{1+(1-x)^2}$, and this will suffice for our purposes here since the units are relevant.

1. Construct a MATLAB function transtime that has inputs x and r and returns the time the ray requires to transit from (0,0) to (1,2).

```
function y = transtime (x, r)
y = sqrt(1+x^2)+r*sqrt(1+(1-x)^2);
```

2. Let r be an array of 101 linearly spaced points from 1 to 10. For each element of the array, use fminbnd to compute a minimum of transtime to get the optimal positions x. This results in an array x as a function of the refraction index r. Plot x versus r (i.e., the dependent variable is x).

```
r = linspace (1, 10, 101);
for k = 1:101
        x(k) = fminbnd ('transtime', 0, 1, optimset, r(k));
end
plot (r, x)
```

3. Consider the angle of incidence θ_1 and angle of refraction θ_2. Recognize that $\sin\theta_1=\dfrac{x}{\sqrt{1+x^2}}$ and $\sin\theta_2=\dfrac{1-x}{\sqrt{1+(1-x)^2}}$. Compute arrays sintheta1 and sintheta2 from x. Create a plot of sintheta1./sintheta2 versus r. You should be able to conclude Snell's Law from this.

```
sintheta1 = x./sqrt(1+x.^2);
sintheta2 = (1-x)./sqrt(1+(1-x).^2);
plot (r, sintheta1./sintheta2)
```