Terminal Velocities of Smooth Spheres

A falling object is obviously being accelerated from rest by gravity. However, in the presence of fluid (e.g., air) resistance there is a counteracting force called drag. If the drag force increases with velocity, then eventually there may be a situation where the two forces are in equilibrium and thus is no net force on the ball so its velocity is thereafter constant. This is called the terminal velocity of the object and the value depends upon the size, roughness, and mass of the object as well as the density of fluid.

The drag force in newtons (= kg-m/sec^2) is

$$F_d = \frac{1}{2} c_d \rho v^2 A,$$

where c_d is the dimensionless coefficient of drag,
ρ is the density of the fluid (in kg/m^2),
v is the velocity of the object (in m/sec), and
A is the cross-sectional area (in m^2).

The coefficient of drag is expressed as a function of the Reynolds number $R_e = \dfrac{\rho v d}{\mu}$,where μ is the dynamic viscosity (in kg /m-sec) and d is diameter (in m) as

$$c_d = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + .4$$

for $0 \le R_e \le 2 \cdot 10^5$, according to F. M. White (*Viscous Fluid Flow*, 2nd ed., 1991, McGraw-Hill, p 182). This is a totally empirical fit to real data for smooth spheres.

Finally, the gravitational force on an object is

$$F_g = mg$$

where m is the mass of the object (in kg), and
g is the acceleration of gravitational on earth (in m/sec^2).

For air $\rho = 1.23$ kg/m^2 and $\mu = 1.78 \cdot 10^{-5}$ kg /m-sec. The cross sectional area of a sphere is $A = \pi d^2 / 4$ and the gravitational constant is $g = 9.8$ m/sec^2 .

1. Construct a MATLAB function `dragerror` that has inputs $v, m, d, \rho,$ and μ and returns the difference $F_d - F_g$.

2. Use `fzero` to compute a zero of `dragerror` to get terminal velocities for each of the following cases:
   - **a**. $m = 2$ gr and $d = 2$ cm.
   - **b**. $m = 2$ kg and $d = 15$ cm.
   - **c**. $m = 200$ kg and $d = 1$ m.

This question is taken from problem 37 on pages 290 and291 of Recktenwald.

(You might be interested in determining the terminal velocity for a ball the size and weight of a regulation baseball $5\frac{1}{8} \pm \frac{1}{8}$ oz. and $9\frac{1}{8} \pm \frac{1}{8}$ in. in circumference. It is claimed by Robert Adair that the terminal velocity is about 95 mph. A baseball is not smooth, however.)