Terminal Velocities of Smooth Spheres
Solution

1. The drag force in newtons (= kg-m/sec^2) is

$$F_d = \frac{1}{2} c_d \rho v^2 A,$$

where c_d is the dimensionless coefficient of drag,
ρ is the density of the fluid (in kg/m^2),
v is the velocity of the object (in m/sec), and
A is the cross-sectional area (in m^2).

The coefficient of drag is expressed as a function of the Reynolds number $R_e = \frac{\rho v d}{\mu}$,where μ is the dynamic viscosity (in kg /m-sec) and d is diameter (in m) as

$$c_d = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + .4$$

for $0 \le R_e \le 2 \cdot 10^5$. The gravitational force on an object is

$$F_g = mg$$

where m is the mass of the object (in kg), and
g is the acceleration of gravitational on earth (in m/sec^2).

For air $\rho = 1.23$ kg/m^2 and $\mu = 1.78 \cdot 10^{-5}$ kg /m-sec. The cross sectional area of a sphere is $A = \pi d^2 / 4$ and the gravitational constant is $g = 9.8$ m/sec^2 .

1. Construct a MATLAB function dragerror that has inputs $v, m, d, \rho,$ and μ and returns the difference $F_d - F_g$.

```
function y = dragerror(v, m, d, rho, mu)
%
Re = rho*v*d/mu;                    % Reynolds number
cd = 24/Re+6/(1+sqrt(Re))+.4;    % coefficient of drag
Fd = cd*rho*v*v*pi*d*d/8;         % force of drag
y = Fd-9.8*m;
```

2. Use fzero to compute a zero of dragerror to get terminal velocities for each of the following cases:

a. $m = 2$ gr and $d = 2$ cm.

```
va = fzero ('dragerror', 100, optimset, .002, .02, 1.23, 1.78e-5)
```

returns

```
va =  15.14396116880725
```

b. $m = 2$ kg and $d = 15$ cm.

```
vb = fzero ('dragerror', 100, optimset, 2, .15, 1.23, 1.78e-5)
```

returns

```
vb =  66.54623135515701
```

c. $m = 200$ kg and $d = 1$ m.

```
vc = fzero ('dragerror', 100, optimset, 200, 1, 1.23, 1.78e-5)
```

returns

```
vc =  1.004341106536716e+002
```