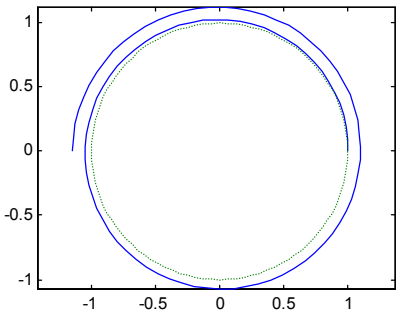
Finding the Dimensions of a Coil

Solutions



Imagine a metal wire wrapped around and around itself forming a spiral. How thick is the coil for a wire of a given length? How long would a wire have to be if the thickness is some given value?

Begin by assuming we have a wire of thickness .1 cm that is wrapped around a circle of radius 1 cm. The spiral (i.e., the center of the wire) then can be represented parametrically as

$$x(t) = (1.05 + \frac{.1}{2\pi} t)\cos(t)$$

$$y(t) = (1.05 + \frac{.1}{2\pi} t)\sin(t)$$

The arclength of the spiral corresponding to the interval [0, T] is $\int_0^T \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ cm. and the thickness in centimeters is $1.1 + \frac{.1}{2\pi} T$.

1.Write a Matlab function `coil` to evaluate the integrand. Make sure it returns a column vector as output if given a column vector as input.

```
function z = coil (t)
fac = .1/(2*pi);
xp = fac*cos(t)-(1.05+fac*t).*sin(t);
yp = fac*sin(t)+(1.05+fac*t).*cos(t);
z = sqrt(xp.*xp+yp.*yp);
```

2. Using quadl with a relative error tolerance of 1.e-8, find the arclength of the spiral corresponding to [0, 600]. You should get about 3,500 cm. .

The statement

```
quadl('coil', 0, 600, 1.e-8)
```

returns

```
ans =

   3.494807373719192e+003
```

3. Using quadl with a relative error tolerance of 1.e-8, find the arclength of the spiral corresponding to [0, 700]. You should get about 4,600 cm. .

The statement

```
quadl('coil', 0, 700, 1.e-8)
```

returns

```
ans =

   4.634315617086571e+003
```

4. Using fzero, find the thickness (to within .01 cm) of the coil of arclength 4,000 cm.

With the function

```
function t = coil_length(T)
t = quadl('coil', 0, T, 1.e-8)-4000;
```

the statement

```
fzero ('coil_length', [600 700], OPTIMSET('TolX',01))
```

returns

```
ans =

   6.443342669279781e+002
```

which corresponds to a thickness in centimeters of

```
1.1+.1*ans/(2*pi)

ans =

  11.38251296231021
```

5. Using fzero, find the arclength (to within .01 cm) of the coil of thickness 100 cm.

Corresponding to the thickness 100, we would have a parameter T of,

```
2*pi*(100-1.1)/.1

ans =

   6.214070268800611e+003
```

The statement

```
quadl('coil', 0, ans, 1.e-8)
```

produces an arclength in centimeters of

```
ans =

   3.138105848287427e+005
```