CS 232E
Final Examination Solutions
May 11, 2001

1. To eliminate variable x_2 from the last three equations

$$\begin{bmatrix} 4 & -4 & 2 & -1 & 6 \\ 0 & 2 & 7 & 1 & 1 \\ 0 & 0 & 5 & -2 & 4 \\ 0 & 3 & 6 & 0 & -6 \\ 0 & -1 & 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ 1 \\ -12 \\ 5 \end{bmatrix}$$

using the Gaussian elimination algorithm with partial pivoting, we first interchange rows two and four to bring the largest potential pivot element onto the diagonal:

$$\begin{bmatrix} 4 & -4 & 2 & -1 & 6 \\ 0 & 3 & 6 & 0 & -6 \\ 0 & 0 & 5 & -2 & 4 \\ 0 & 2 & 7 & 1 & 1 \\ 0 & -1 & 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -12 \\ 1 \\ -8 \\ 5 \end{bmatrix}$$

Then we use this row to eliminate x_2, obtaining:

$$\begin{bmatrix} 4 & -4 & 2 & -1 & 6 \\ 0 & 3 & 6 & 0 & -6 \\ 0 & 0 & 5 & -2 & 4 \\ 0 & 0 & 3 & 1 & 5 \\ 0 & 0 & 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -12 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

2. With **no** more evaluations of f we have $|\hat{x} - x^*| \le 1.5$ by choosing $\hat{x} = .5$. With k more evaluations of f we have $|\hat{x} - x^*| \le 1.5 \cdot 2^{-k}$. Thus to guarantee that $|\hat{x} - x^*| \le 10^{-5}$, we want $1.5 \cdot 2^{-k} \le 10^{-5}$. This is equivalent to:

$$-k \log(2) = \log(2^{-k}) \le \log(\frac{1}{1.5} 10^{-5})$$

or

$$k \ge -\log(\frac{1}{1.5} 10^{-5}) / \log(2) = 17.1946 .$$

The smallest integer value of k satisfying this is $k = 18$. For this we would have
$|\hat{x} - x^*| \le 1.5 \cdot 2^{-k} = 5.722 \cdot 10^{-6}$

3. With

```
function y = midpoint (f, a, b, n)
h = (b-a)/n;
x = linspace (a+h/2, b-h/2, n);
ff = feval (f, x);
y = h*sum(ff);
```

the invocation

```
midpoint ('exp', 0, 3, 16)
```

results in

```
ans =

   19.05760823435082
```

which is comparable to the exact value of the integral = 19.0855.

4. The invocation of

```
function problem4
y1 = fzero('humps(x)-16', .1)
y2 = fzero('humps(x)-16', .5)
y3 = fzero('humps(x)-16', .7)
y4 = fzero('humps(x)-16', 1)
```

results in

```
y1 =

   0.10318309547341

y2 =

   0.52531420241030

y3 =

   0.77150270211629

y4 =

     1
```

Thus the four solutions are `0.10318309547341, 0.52531420241030, 0.77150270211629,` and `1.`

5.

a. For $i = 1, \ldots, n$:

$$p(x_i) = \sum_{j=1}^{2n} a_j x_i^{j-1} = y_i$$

b. For $i = 1, \ldots, n$:

$$p'(x_i) = \sum_{j=1}^{2n} a_j (j-1) x_i^{j-2} = \sum_{j=2}^{2n} a_j (j-1) x_i^{j-2} = y_i'$$

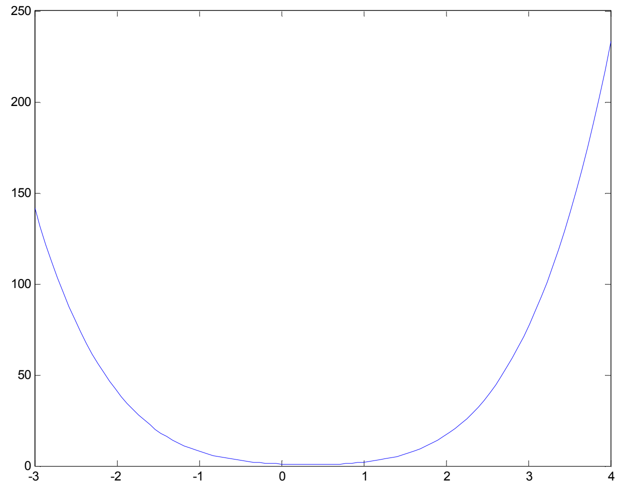
c.

```
function a = osculatory (x, y, yprime)
n = length(x);
A = zeros (2*n);
b = zeros(2*n, 1);
b(1:n) = y;
b(n+1:2*n) = yprime;
for i = 1:n
    A(i, :) = x(i).^(0:2*n-1);
    A(n+i, 2:2*n) = (1:2*n-1).*(x(i).^(0:2*n-2));
end
a = A\b;
```

d. Invocation of

```
function problem5
x = [-2; -1; 0; 1; 3];
y = [41; 8; 1; 2; 76];
yprime = [-58; -15; -2; 5; 97];
a = osculatory (x, y, yprime);
z = linspace (-3, 4, 101);
pz = zeros (101, 1);
for i = 1:101
    pz(i) = sum(a'.*(z(i).^(0:9)));
end
plot (z, pz);
```

results in



6. We are given that $\| \bar{b} \| = 600, \| b - \bar{b} \| = .02,$ and $\| \bar{x} \| = .1$. Since

$$\frac{\| x - \bar{x} \|}{\| \bar{x} \|} \leq \kappa(A) \frac{\| b - \bar{b} \|}{\| \bar{b} \|}$$

To obtain $\| x - \bar{x} \| \leq 10^{-4}$, we must have

$$\kappa(A) \leq \frac{\| x - \bar{x} \|}{\| \bar{x} \|} / \frac{\| b - \bar{b} \|}{\| \bar{b} \|} = \frac{10^{-4}}{.1} / \frac{.02}{600} = 30$$

7. Invocation of

```
function problem7
disp(sprintf('        x            cos(x)                    P(x)
                                   error                  relative error'));
x = linspace (-2, 2, 20);
cosx = cos(x);
Px = (12-5*x.^2)./(12+x.^2);
err = Px-cosx;
relerr = err./cosx;
for i = 1:20
    disp(sprintf('%10.5f  %23.15e  %23.15e  %23.15e  %23.15e', x(i),
            cosx(i), Px(i), err(i), relerr(i)));
end
```

results in

x	cos(x)	P(x)	error	relative error
-2.00000	-4.161468365471424e-001	-5.000000000000000e-001	-8.385316345285759e-002	2.014989808611905e-001
-1.78947	-2.169386690491335e-001	-2.638483965014576e-001	-4.690972745232414e-002	2.162349739580073e-001
-1.57895	-8.150951367961911e-003	-3.211009174311932e-002	-2.395914037515741e-002	2.939428699002069e+000
-1.36842	2.009966949880382e-001	1.900958466453674e-001	-1.090084834267077e-002	-5.423396809245799e-002
-1.15789	4.012687548258428e-001	3.970099667774086e-001	-4.258788048434214e-003	-1.061330591334628e-002
-0.94737	5.838216399741467e-001	5.824742268041236e-001	-1.347413170023071e-003	-2.307919196148224e-003
-0.73684	7.405942033261460e-001	7.402826855123673e-001	-3.115178137786945e-004	-4.206322603925472e-004
-0.52632	8.646637019492132e-001	8.646209386281588e-001	-4.276332105435099e-005	-4.945659330668044e-005
-0.31579	9.505514906876966e-001	9.505494505494506e-001	-2.040138246006684e-006	-2.146267999149318e-006
-0.10526	9.944649474795756e-001	9.944649446494464e-001	-2.830129242070711e-009	-2.845881344780970e-009
0.10526	9.944649474795756e-001	9.944649446494464e-001	-2.830129242070711e-009	-2.845881344780970e-009
0.31579	9.505514906876966e-001	9.505494505494506e-001	-2.040138246006684e-006	-2.146267999149318e-006
0.52632	8.646637019492133e-001	8.646209386281588e-001	-4.276332105446201e-005	-4.945659330680884e-005
0.73684	7.405942033261460e-001	7.402826855123673e-001	-3.115178137786945e-004	-4.206322603925472e-004
0.94737	5.838216399741468e-001	5.824742268041239e-001	-1.347413170022960e-003	-2.307919196148034e-003
1.15789	4.012687548258428e-001	3.970099667774086e-001	-4.258788048434214e-003	-1.061330591334628e-002
1.36842	2.009966949880384e-001	1.900958466453677e-001	-1.090084834267072e-002	-5.423396809245765e-002
1.57895	-8.150951367961911e-003	-3.211009174311932e-002	-2.395914037515741e-002	2.939428699002069e+000
1.78947	-2.169386690491333e-001	-2.638483965014575e-001	-4.690972745232419e-002	2.162349739580077e-001
2.00000	-4.161468365471424e-001	-5.000000000000000e-001	-8.385316345285759e-002	2.014989808611905e-001

8. Invocation of

```
function problem8
tmax = fminbnd ('-((cos(x)-2)^2+(.9*sin(x)-3)^2)', 0, 2*pi);
p = [cos(tmax), .9*sin(tmax)]
```

results in

```
p =

  -0.61705911350901  -0.70822512017912
```